Managerial Economics Supplemental Notes
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Managerial Economics, Supplemental Notes, 23 Apr 96

Topics: 1) Faustmann as a Performance Standard
2) Faustmann as a springboard to all the math of finance

1. Short derivation of Faustmann

Divide Present Value (P) in 2 parts, S and K. K is embodied in the product, F. S is like land or a catalyst, needed to make processes work, but not embodied in the products.

$$(K+S)\cdot I^n = F+S$$

Solving for S:

$$S(I^n-1) = F-KI^n$$

$$S = \frac{-KI^n + F}{I^n - 1} \qquad (1)$$

2. All the math of finance follows from (1). Examples follow - and more come during the rest of the course.

3. Let n=1. This means F comes annually. Following our conventions, that makes it a flow, instead of a point (or series of lumpy points). Our convention is to denote that flow as a (for annual).

$$S = \frac{-K(1+i) + a}{i} \qquad (2)$$

Note how closely this resembles one of our 3 basic building blocks of finance, namely: S=a/i. In (2) the numerator is still basically cash flow, which we have called a. Here, however, we have treated some of the expenses explicitly as capital, K. This capital is exhausted within a year, so we call it "working capital." The formula deducts not just the outlay on capital, but interest on it, too.[1]

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[1]Since the capital outlay here is exhausted within the year, it is an annual flow, and we should change K to k. However, since we are doing nothing further with it here, we omit this

Where deducting of K(1+i) is not done explicitly, as in (2), you should consider it done silently behind the scenes in calculating a. This deduction is, as the usage goes, "subsumed" in a. "Subsumed" means hidden, but silently understood. Such hiding is done not to conceal, but to simplify the numerator while we manipulate and analyze the rest of Equation (1). However, be warned, it may also lead to forgetting what is hidden, hence to error. Therefore, **REMEMBER!!!**: a, by itself, is net of expenses, and expenses include interest on working capital.

That is why we can, and routinely do, simplify (2) into $S = a/i$.

Before we do, however, let's see why we must never forget this concealed cost of interest on K. §s 4-9 deal with it. Whenever n>1, this factor begins chipping away at a with strength that grows with n and i. With n>1, also, we stop calling it working capital; it becomes hardware capital (and has other descriptors like heavy capital, fixed capital, hard capital, capital goods, clay capital, buildings, machinery, equipment, etc.)

4. When n=2, we observe two changes in (2):

$$S = \frac{-KI^2 + F}{2i + i^2} \qquad (3)$$

The major change is in the base, which has doubled (i^2 is negligible at this point - it represents the effect of compounding, which later looms much larger). However, compound interest on K has also begun to erode the numerator.

To visualize what happens as n grows, draw a figure with n as the abscissa, with two curves: a logistic growth curve of F, and a curve of exponential growth, beginning from K on the ordinate. There should be a lenticular area between the two curves, beginning where they cross, and F becomes higher than KI^n.

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step.

Figure 1: Elements Numerator of Equation (1), visualized

As n rises, two things change: so its average value per year falls.

1) KI^n rises. This part is obvious.
2) F ~~and K~~ occur; less often, ~~Visualize the vertical line between them moving to the right. This vertical is the numerator of (1)~~. Algebraically, the effect of this on S is shown in the growth of the base of (1).

5. Note the dual sensitivity of S to i. i appears twice in (1), and both times with leverage. This means S is highly sensitive to $\pm\Delta i$. This is why land prices, historically, have been so unstable. For example, in Japan it is why they were able to rise fantastically, 1979-89, and then fall by 80%, 1989-96.

To offset this risk, plan land uses with lower n values. You have some control over n; but very little control over i, which is ruled by world markets.

6. (1) is an excellent Performance Standard. It is much more comprehensive than most. It lets you juggle many balls in the air at once.

a. F is gross revenue less current running costs. These costs are traditonally called "expenses," as opposed to hardware purchases, called "capital outlays." The deduction of expenses is "subsumed" in F.

b. K is hardware cost. We treat it separately from working capital, following universal tradition and custom. Thus, both K (capitalized outlays) and e (expenses) are deducted, although in different ways and places.

c. Interest on capital is deducted: interest on K, explicitly; interest on working capital, silently or implicitly.

d. The value of time appears twice; both times negatively. It is subtracted from the numerator; and added to the base.

e. Competing S-values derived from competing uses of capital and land are made **commensurable**, regardless of different

n-values, because they are all assumed to repeat forever. We will learn the importance of this point via examples, problems, and illustrations, as we proceed. See §11.

7. The above sets up a good quiz question or problem. Given the following facts, which is a better use of land, A or B?

	F	n	i	K
A	100	5	.06	50
B	200	7	.08	115

Comparing F and K, choice B looks much better. However, choice A "lays its egg" in 5 years instead of 7. Use B is also handicapped by the premise of a higher discount rate.[2]

Plugging values into (1), we find use A yields a higher S-value.

$$\text{A.}\quad S = \frac{-50\cdot 1.06^5 + 100}{1.06^5 - 1} = \frac{-67 + 100}{.338} = \frac{33}{.338} = 98$$

$$\text{B.}\quad S = \frac{-115\cdot 1.08^7 + 200}{1.08^7 - 1} = \frac{-197 + 200}{.714} = \frac{3}{.714} = 4.2$$

Note how (1) steers you away from heavy use of hardware, and deferred yields, and high i.r.s.

Notice how (1) guides you where to focus your attention, to maximize your wealth. E.g., at 8%, it tells you to raise F by spending more on o&m[3] in later years, rather than K up front.

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[2]The premise fits the facts of many cases, where the lender has a steep "yield curve preference," meaning she charges a higher rate on longer-term loans.

[3]o&m = operation and maintenance. It is traditionally given in upper case, but I use lower case to stick with my convention of using lower case letters to denote flow values. o&m costs are part of expenses, not capitalized.

Notice how (1) screams at you to lower n, if you can. Cf. §5, and 6,d.

8. The problem given in §7 stacked the cards against B by giving it a longer period, n, and a higher i.r., .08. Let's do it again using the same values for both A and B: n=5, and i=.06.

A. S = 98 (same as before)

$$\text{B. } S = \frac{-115\cdot 1.338 + 200}{.338} = \frac{-154 + 200}{.338} = \frac{46}{.338} = 136$$

Now, B is the better use (136>98).

9. Considering the great benefit of lowering n, one might invest K specifically to speed the arrival of F, rather than to raise its value. An example is investing in marketing; another is planting nursery stock, instead of seeds, to get a new forest off to a fast start.

Let's invest another 15, to lower n from 5 to 4, and evaluate B now.

$$\text{B. } S = \frac{-130\cdot 1.262 + 200}{.262} = \frac{36}{.262} = 137$$

137>136, so in this case, the gain is only trivial. You can play "what if," and identify other cases where the gain would be great. What if the quicker payback let you finance at 5% instead of 6%? Over to you.

For a stronger example, let's invest an added $50 of K to lower n from 10 (choice C) to 8 (choice D).

	F	n	i	K	I^n	$F-KI^n$	I^n-1	S
C	800	10	.06	400	1.791	80	.791	101
D	800	8	.06	450	1.594	83	.594	139

A good way to summarize these results is in terms of fractional changes. We raised K by 12.5%. This lowered n by

20%. S rose by 38%. Almost all the rise of S is due to the fall of the base (I^n-1).

If we could lower n from 10 to 8 without any rise of K, e.g. simply by changing some part of our philosophy of management, we would raise S all the way up to $273.

The moral is: to maximize your wealth, **SPEED!**

PART II: Faustmann as a springboard to all math of finance

10. Now, let's conflate[4] the numerator of (1) into one figure or symbol, F. That is, let it be silently understood here that F is net of KI^n. We can do this because F and KI^n are **commensurable.** They are point values at the same point in time, viz. n. To be sure, K occurs at time 0, but we converted it to the equivalent value at time n by compounding it.

We are doing this to cut down the clutter, while we manipulate the rest of the equation. However, it has another, greater value. Note that when there are intermediate costs and revenues, at times other than n or 0, we can also compound them forward to time n, and conflate them with F. By this technique, we make (1) much more general and adaptable in application.

(1) now becomes:

$$S = F/(I^n-1) \qquad (4)$$

$$S I^n - S = F$$

$$S = \frac{F+S}{I^n}$$

11. That pesky little "-1" in the base of (4) makes all S-values **commensurable**, because they all have the same time frame: infinity. That little "-1" distinguishes (4) from simple discounting of F. When n is quite long, and/or if i is high, the "-1" is swamped by the high value of I^n, and hardly affects S. However, when n and i are low, the "-1" exerts great leverage, and raises S a lot. Here is a numerical comparison of 2 rival uses, E and F, for a parcel of land.

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[4]"Conflate" means to treat a collection of things as a unit. Sometimes it means to commingle thngs irreversibly - to fuse, meld, or scramble. My intent here is to hold the parts separate for future analysis. The melding is only temporary.

Let $P = F/I^n$.

	F	i	n	I^n	I^n-1	P	S	S/P
E	600	.11	10	2.84	1.84	211	326	1.54
F	300	.11	5	1.68	.68	179	441	2.46

Thus, $300 coming every 5 years (that's F) is worth more than $600 coming every 10 years (that's E). 441>326.

More is involved here than simple discounting. Using simple discounting, E is worth a lot more than F (211>179). The other, greater factor is making the two choices **commensurable**, by using the same time-span for each.

Like many important truths, once you get it, it seems so obvious you wonder if you have learned anything difficult. If you find it obvious, congratulations! The reason it is worth stressing, though, is that many writers on management, unthinking, give you P as a performance standard. They would make E the better choice. Now you know better than to fall for that.

12. Now, let's do some tricks to spring from (4) to all the mathematics of finance. We already know that a=Si. This is one of the 3 basic building-blocks of finance. It follows that S=a/i. Substitute this in (4), and solve for a:

$$a/i = F/(I^n-1)$$

$$a = F \frac{i}{I^n - 1} \qquad (5)$$

(5) converts F into its level annual equivalent (a). This is called **annualizing.** It gives us the rate of exchange between point values (like F) and flow values (like a).

a is also a comprehensive Performance Standard, just like S. S has disappeared. We are now dealing with a finite flow, n years long. However, all a values are commensurable, whatever the length of the n from which they are derived. They could be repeated to from here to eternity.

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$$F = a\,\frac{I^n - 1}{i} \qquad (6)^5$$

Now substitute $P=FI^{-n}$:

$$P = FI^{-n} = a\,\frac{1 - I^{-n}}{i} \qquad (7)$$

(7) gives the Discounted Cash Flow (DCF) of a for n years. The ratio on the right side, which is P/a, is the Discounted Cash Flow Factor (DCFF). It gives the Present Value of a finite flow of cash of $1 per year.

In Reading #21, at the end of Volume I of Readings, this is called the DCF of a Finite Flow, and is derived in another way.[6] The point here is that Faustmann's Formula is as good an entrée as any into the harmonious, mutually consistent world of finance, all of which fits together with all parts supporting and confirming all other parts. With Reading #32 and ff., we will see how all of finance is contained in (7).

To close, here are some lines from UCR's Poet Laureate, The Great Highlander, Bobbie Burns.

To catch Dame Fortune's golden smile
  Assiduous wait upon her;
And gather gear by every wile
  That's justified by honour:
Not for to hide it in a hedge
  Nor for a train-attendant
But for the glorious privilege
  Of being independent.

In ploughman phrase, "God send you speed,"
  Still daily to grow wiser;
And may ye better reck the rede,[7]
  Than ever did th'adviser!

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[5]For future reference, this is the Compounded Cash Flow Factor, or CCFF.

[6]If a principle is true, you can derive it in many ways, because the laws of the universe are harmonious. If you can't derive it more ways than one, you probably made an error the first time.

[7]"Reck the rede" means heed the advice.