

Ripening, TISTOC

Shows that $\frac{1+g}{1+i}$ is interchangeable with $\frac{1}{1+i-g}$, with a small adjustment.

$$\text{I. } S = (1+i-g)^{-1} + (1)^{-2} + \dots$$

$$= \frac{1}{1-r} (1) = (1) \frac{1}{1-1-i+g} = \frac{1}{1+i-g-1} = \frac{1}{i-g}$$

$$\text{II. } S = (1+i)^{-1} + \frac{1+g}{(1+i)^2} + \dots$$

$$= (1+i)^{-1} \frac{1}{1 - \frac{1+g}{1+i}} = \frac{1}{1+i-1-g} = \frac{1}{i-g}$$

Remarkable! The small excess of $\frac{1+g}{1+i} > \frac{1}{1+i-g}$ is ~~not~~ just offset ~~by~~ in (I) by beginning g in $n=1$.

This means I may use whichever form I please!

Just adjust the 1st term properly. I may

interpret $S = \frac{a}{i-g}$ either way.

Try doing same with $(1+i+t) \approx (1+i)(1+t)$
or
$$\frac{1}{1+i+t} \approx \frac{1}{(1+i)(1+t)}$$

Moral:

$\frac{1+g}{1+i}$ cannot be shortcutted in a general case.

But for a usually OK shortcut,

$$\frac{1+g}{1+i} \approx \frac{1}{1+i-g}$$

$$\frac{(1+g)^{n-1}}{(1+i)^n} \quad \text{vs} \quad \frac{1}{(1-g)^{n-1} (1+i)^n} \quad \text{vs} \quad \frac{1}{[1+i-g]^n}$$

$$(1-g)(1+i) = \cancel{1-gi-g-gi-1-2gi-g}$$

$$= 1+i-g-gi < 1+i-g$$

$$\frac{1+g}{1+i} \stackrel{?}{=} \frac{1}{(1+i)(1-g)}$$

$$1+g \stackrel{?}{=} \frac{1}{1-g}$$

$$(1+g)(1-g) \stackrel{?}{=} 1$$

$$1-g+g-g^2 < 1$$

$$1+g < \frac{1}{1-g}$$

$$1 + g < \frac{1}{1-g}$$

$$(1-g)(1+i) < 1+i-g$$

$$\frac{1}{(1-g)(1+i)} > \frac{1}{1+i-g}$$

$$\frac{1+g}{1+i} \stackrel{?}{=} \frac{1}{1+i-g}$$

$$1+i-g+g+gi-g^2 \stackrel{?}{=} 1+i$$

$$gi-g^2 \stackrel{?}{=} 0$$

$$i \stackrel{?}{=} g$$

$$i > g$$

∴

$$\frac{1+g}{1+i} > \frac{1}{1+i-g}$$

$$g = .05$$

$$i = .10$$

$$\frac{1+g}{1+i} = \frac{1.05}{1.10} = .954545$$

$$\frac{1}{1+i-g} = \frac{1}{1.05} = .95238$$

$$g = 0$$

$$\frac{1+g}{1+i} = \frac{1}{1+i}$$

$$\frac{1}{1+i-g} = \frac{1}{1+i}$$

$$g = i$$

$$\frac{1+i}{1+i} = 1$$

$$\frac{1}{1+i-i} = 1$$

$$g = 1.00$$

$$i = 1.05$$

$$\frac{1+g}{1+i} = \frac{2}{2.05}$$

$$\frac{1}{1+i-g} = \frac{1}{1.05}$$

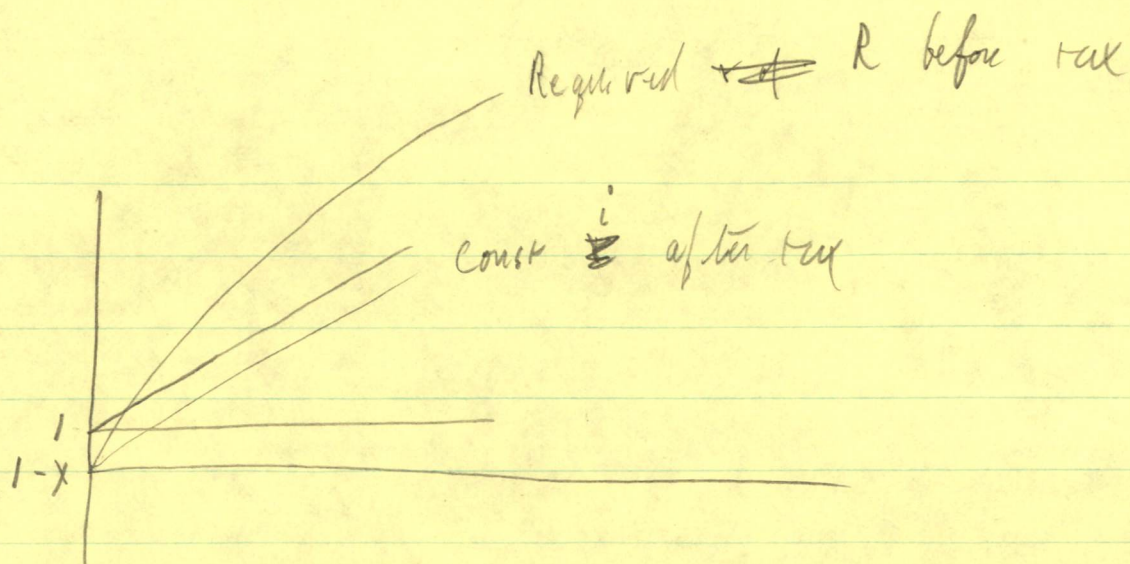
Tax on gross selling price of land raises only the gain, but S_0 drops down to make it so. But then the gain on ~~short~~ early sales must be extreme.

How about sequences of sales? S_0 can drop as low as necessary!

But suppose we assume there is a functional reason for holding land vacant, and the alt. is an ^{immediate} ~~immediate~~ lower use.

a. The σ for a short wait is depressed more than a long? ..

✓ b. a useful model will now show that the holding option PV is reduced less than the ordy. inc. option



Results of tax on gain on land sales

1. S_0 drops to $1 - x$
2. Fast prospective sales, or nearly ripe land must fall more than very green land.
So, S_0 will fall by different %s, : more for ripe, less for less ripe.

$$\sigma = \frac{(R_n - \sigma)(1 - \epsilon)}{(1+i)^n}$$

$$\sigma = \frac{R_n - \epsilon(R_n - \sigma)}{(1+i)^n} = \frac{R_n(1 - \epsilon) + \epsilon\sigma}{(1+i)^n}$$

$$\sigma \left[1 - \frac{\epsilon}{(1+i)^n} \right] = \frac{R_n(1 - \epsilon)}{(1+i)^n}$$

$$\sigma = \frac{R_n(1 - \epsilon)}{(1+i)^n - \epsilon}$$

$$S = \frac{R_n}{(1+i)^n}$$

$$\frac{\sigma}{S} = \frac{(1-t)}{1 - \frac{t}{(1+i)^n}}$$

a decr'g fu. of n .

?

1. Assume r , (after tax return) remains const.
2. a. " i , (hfr " ") " "
- b. " R_n " "

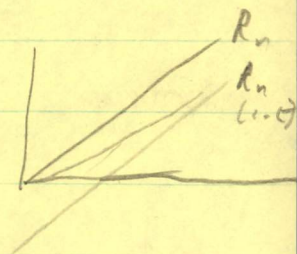
3. Tax is absorbed in lower σ

$$4. R_n = (1+i)^n$$

$$5. S = R_n (1+i)^{-n} = 1$$

$$6. \frac{\sigma}{S} = \sigma$$

$$7. \sigma = \frac{R_n (1-t) + t\sigma}{(1+i)^n}$$



$$\sigma[(1+i)^n - t] = R_n(1-t)$$

$$\sigma = \frac{(1+i)^n (1-t)}{(1+i)^n - t} = \frac{1-t}{1 - \frac{t}{(1+i)^n}}$$

1. What if $S = \frac{R_n}{(1+i)^4 - 1}$?

2. What if $\sigma = S$, so R_n had to adjust?



$$(1+r)^n = (1+j)^n (1-t) + t$$

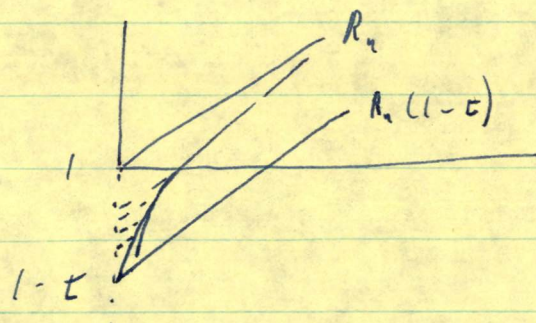
$$(1+j)^n (1-t) = (1+r)^n - t$$

$$(1+j)^n = \frac{(1+r)^n - t}{1-t}$$

1. If r is fixed, and C_0 is fixed at one, then j varies, a decr'g fu. of n . This is a bit like fwd. shifting.

Discounting FV at r , the sooner, earlier ones have lower PV.

2. Now let r still be fixed, but R_n are also fixed. PVs fall.



This curve will do it, t is probly right.

- a. For $n=1$, to make r after a gross tax (to make it simpler), the same as it was before

$$S = R_1 (1+i)^{-1} = 1$$

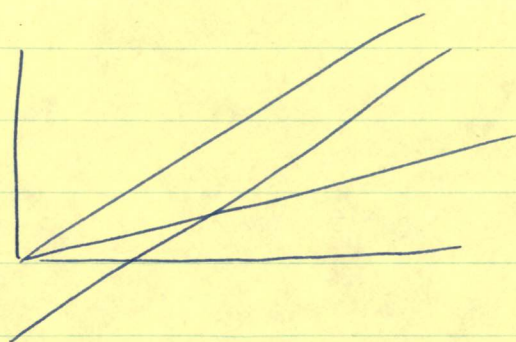
$$r = R_1 (1-t) (1+i)^{-1} = 1-t \quad \text{for all } n.$$

2

Using this method, the gross tax looks neutral!
Yet it has taxed mostly principal $\$$, for $u=1$, and
mostly income, later years. Must be something
wrong w. the method.

j, the before-tax r of r , is now 100% or more
when $u=1$. This is very hard to achieve.
Rather than take this cut in value, owners will
shift to selling in other yrs?

Buy vacant land at $t=0$; sell at n , for $R(1-\epsilon)$



1. If i remains same, $S = R(1+i)^{-n}$

$$\sigma = \frac{R(1-\epsilon) + \epsilon R}{(1+i)^n} = \frac{R(1-\epsilon)}{(1+i)^n}$$

$$\sigma = \frac{R - \epsilon(R - \sigma)}{(1+i)^n} = \frac{R(1-\epsilon) + \epsilon\sigma}{(1+i)^n}$$

$$\sigma(1 - \epsilon(1+i)^n) = \frac{R(1-\epsilon)}{(1+i)^n}$$

$$\sigma = \frac{R(1-\epsilon)}{(1+i)^n [1 - \epsilon(1+i)^n]} = \frac{R(1-\epsilon)}{(1+i)^n - \epsilon}$$

$$\frac{\sigma}{S} = \frac{1-\epsilon}{1 - \epsilon(1+i)^{-n}} \quad \text{a decr. fn. of } n.$$

But this does not acc. for lower later resale values.

$$S = R(1+i)^{-n}$$

$$? \quad \sigma(1+i(1+t))^n = R$$

A Häjg-Simon tax on Δ wd. require a pre-tax Δ to be higher, to cover both i and Tax.

$$\hat{i} = i + t\hat{i}$$

$$\hat{i}(1-t) = i$$

$$\hat{i} = \frac{i}{1-t}$$

$$\sigma \times \left(1 + \frac{i}{1-t}\right)^n = R$$

$$\sigma = \frac{R}{\left(1 + \frac{i}{1-t}\right)^n}$$

$$\frac{\sigma}{S} =$$

Ec. 132, June 8, 1981

(20 copies, pls)

I am guilty of underdramatizing the benefit of deferring tax payments. Recall our basic model for capital gains:

$(1+i)^n =$ ~~the~~ growth of land value before tax.

$(1+i(1-t))^n =$ ~~the~~ growth of taxpayer's after-tax wealth, after a neutral (or "Hug-Simons") tax.

$(1+i)^n(1-t) + t =$ ~~the~~ growth of taxpayer's after-tax wealth, after ~~our present kind of~~ a capital gains tax deferred until "realization" of gains by sale, as per Eisner v. Macomber (1920).

r = after-tax rate of return to taxpayer.

I underdramatized the contrast by focusing on r, where the differences look slight. This time, let's calculate Future Values (FV), a slight difference in r corresponds to a huge difference in FV.

Let: $i = 20\%$
 $t = 50\%$
 $n = 50 \text{ years}$

Then:

$$(1+i)^n = 1.20^{50} = \$9,100.44 \quad (1)$$

(That's right: we start with \$1 and at 20% compounded we end with nearly \$10,000 in 50 years. Would you rather pay interest or receive?)
Now extract a 50% tax annually, a Haig-Simons tax:

$$(1+i(1-t))^{50} = 1.10^{50} = \$117.39 \quad (2)$$

(Cutting the annual growth rate in half cuts the FV to 117/9100 or 1.3% of what it is at 20%. That is the power of exponentiation.)

Next, apply a tax upon realization, as per Macomber:

$$(1+i)^{50} (1-t) + t = \$9100 \times \frac{1}{2} + \frac{1}{2} = \$4551 \quad (3)$$

Applying the Macomber rule, our taxpayer accumulates $4551/117.39$ or 39 times as much as under Haig-Simons, in 50 years.

In addition, of course, his gain is ^{at least} 60% exempt.
(or 100% if he plays his cards right). At 60% he does better, anyway, as follows.
Adjusting for that:

$$(1+i)^n(1-.45) + .45 = \$9100 \times (1-.20) + .20 = \$7280.20$$

(4)

(Not bad: he gets $7280/117.39$ or 62 times more than if he put his money in the bank at 20%, subject to tax as "ordinary" income at the full rate, payable yearly.)

As to little old "r", the after-tax return is up to 19.47%. You might forget a small thing like going from ~~10%~~ .10 up to .1947, but remember that 62-fold advantage in accumulating wealth! Now, if you can only live 50 years longer, --- Well, you may never live to spend it, but your heirs can step up the basis on your death, reducing the tax rate to zero. If you have a family dynasty, this helps. You might even borrow cash from your heirs and live it up. "With a little bit of luck, you can 'ave it all and not get hooked."

But is this good public policy?

Perp. Apprec. Model

Should σ be taxed?

Basic problem:

1. If $\sigma = \frac{a(1-\tau)}{i-g}$

Then $\frac{\sigma}{S} = 1 - \tau$

2. If $S = \frac{FV}{(1+i)^n}$

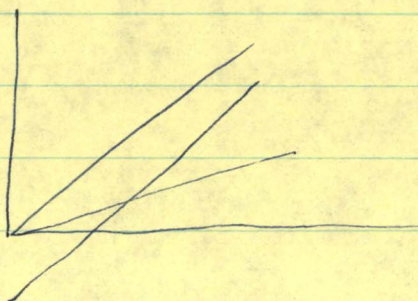
and $FV = \frac{a}{i}$

$$S = \frac{a}{i} (1+i)^{-n}$$

$$\sigma = \frac{a(1-\tau)}{i} (1+i)^{-n}$$

$$\frac{\sigma}{S} = 1 - \tau$$

So, of this w. TISTOC



In TISTOC, you keep same beginning, + change τ .
In NPV, you reduce PV of S_0 to σ .

$$\sigma [1 + i(1-t)]^n = R$$

$$\sigma [1 + \hat{i}]^n = R$$

$\hat{i}$ is new pre-tax return, hi enough so i remains i

$$i(1-t)$$

$$\hat{i}(1-t) = i$$

$$\hat{i} = i(1-t)^{-1}$$

$$\sigma \left[1 + \frac{i}{1-t} \right]^n = R$$

$$S = R(1+i)^{-n}$$

$$\frac{\sigma}{S} = \frac{(1+i)^n}{\left[1 + \frac{i}{1-t} \right]^n} \quad \text{a decr'g fn. of } \underline{n} \text{ ! ?}$$

So, a Haig-Simons tax is biased vs. Mowen sales; it seems? But then, is a prop'ty. tax?

Prop. Tax {

$$S = \frac{a}{i}$$

$$\theta = \frac{a}{i+t}$$

$$\frac{\theta}{S} = \frac{i}{i+t}$$

$$\text{Next try } S = \frac{(1+i)^n}{(1+i)^n}$$

Hair-s Tax

$$S = 1 = \frac{(1+i)^n}{(1+i)^n}$$



$$\sigma \left(1 + \frac{i}{1-t}\right)^n = R_n = (1+i)^n$$

$$\frac{\sigma}{S} = \frac{(1+i)^n}{\left(1 + \frac{i}{1-t}\right)^n} \Bigg/ 1 = \sigma$$

Prop. Tax

$$S = \frac{(1+i)^n}{(1+i)^n}$$

$$\theta = -t \frac{(1+i)}{1+i} - t \frac{(1+i)^2}{(1+i)^2}$$

$$\theta = -t \theta \frac{(1+i)}{(1+i)} - t \theta \frac{(1+i)^2}{(1+i)^2} - \dots - t \theta \frac{(1+i)^n}{(1+i)^n} + \frac{(1+i)^n}{(1+i)^n}$$

$$= -n t \theta + 1$$

$$\theta(1 + n t) = 1$$

$$\theta = \frac{1}{1 + n t}$$

Is prop. tax then better than H-s?

$$S_i = a + S_g = a + \Delta S$$

$$r_i = a(1-t) + \overset{\sigma_f}{\Delta S}$$

✓ Key qn. here is the Ben Smith qn: is the tax on income before-tax or after tax?

The ΔS is after tax, in same way that property tax hits income after tax.

ΔS is the Δ of l.v. before effect of tax. Now, fisc has $t\%$ of it; landholder keeps $(1-t)\%$, or σ_f . But this is after-tax income.

Treat prop. tax like z

$$S = \frac{a}{i+t}$$

$$S_i = a - tS$$

1. I have alleged, that annual ΔS in P.A. model should be taxed, in T15704,

Rationale: $S_i = a + S_g$

$$S_i > a.$$

$$S_i - a = S_g$$

S_g necessary to cover yearly c.c.

S_g being treated as current income.

2. In Ag. Ec. class notes, I further allege that S includes capd. annual Δ .

$$S = \frac{a + S_g}{i}$$

The mystery here is, S_g is for one year only; all later years are higher. So, is this a valid perception? Does it lead to errors?

And what happens to tax capd. when

$$S = \frac{a + S_g - S_k}{i} \quad ?$$

Taxes on unrealized gains, P.A. model,

~~$S = \frac{a}{i-i}$~~

The immunity of unrealized gains from income taxes, and the immunity of land from phantom profits taxation, ~~may be shown~~ ^{show} strikingly in the following model.

Let:

S = Site Value

i = nominal interest rate

a = annual ordinary income of land

g = annual % growth of a , in current \$
(this can represent ^{general} inflation, or a ~~real~~ rise of real land income)

τ = income tax rate.

If $\tau = 0$,

$$S = \frac{a}{i-g} \quad (1)$$

(1) is found as follows.

$$S = a \left[\frac{1}{(1+i)} + \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \dots + \frac{(1+g)^{n-1}}{(1+i)^n} \right] \quad (2)$$

But for land, $n = \infty$, so:

$$S = a(1+i)^{-1} \left[\frac{1}{1 - \frac{1+g}{1+i}} \right] = \frac{a}{1+i-1-g} = \frac{a}{i-g} \quad (3)$$

Rearranging terms, we have:

$$S = \frac{a + Sg}{i} \quad (4)$$

Sg is the ^{current year's} ~~annual~~ unrealized increment of the site value. Note that it is being capitalized, just as though received in cash. ^{*}

Now impose an income tax. Let τ be site value after the tax.

$$\tau = \frac{a(1-\tau) + \tau Sg}{i} \quad (5)$$

a is taxable; τSg is not. Therefore, site value does not decline by the full amt. of the tax rate, unless $g = 0$.

* Don't overgeneralize from this, however. The next years' increments will all be larger, but only the current year's increment is capitalized.

One may wonder if the ~~tax on a~~ does not ~~reduce proportionately~~. formula adequately ~~expresses the tax~~ ~~deducts for~~ shows the deduction of taxes on increments to ordinary income after year one. Let us derive it longhand.

$$T = a \left[\frac{1-t}{1+i} + \frac{(1+g)(1-t)}{(1+i)^2} + \frac{(1+g)^2(1-t)}{(1+i)^3} + \dots \right]$$

$$= a(1-t) \left[(1+i)^{-1} + \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \dots \right]$$

$$= a(1-t) \times \frac{1}{i-g} \quad (6)$$

$$\sigma_i - \sigma_g = a(1-t)$$

$$\sigma = \frac{a(1-t) + \sigma_g}{i} \quad (7)$$

(7) is the same as (5).

$$\frac{\frac{s}{s}}{\frac{\frac{a(1-\epsilon)}{i-g}}{\frac{a}{i-g}}} = 1-\epsilon \quad (8)$$

Whoops! This began as a beautiful demonstration, but then fell into a trap, above. How to recapture original idea?

$$S = a \left[(1+i)^{-1} + \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \dots + \frac{(1+g)^{n-1}}{(1+i)^n} \right]$$

For $n = \infty$,

$$S = a \left[(1+i)^{-1} \frac{1}{1 - \frac{1+g}{1+i}} \right] = a \frac{1}{1+i-1-g} = \frac{a}{i-g}$$

$$Si = \frac{a + Sg}{i}$$

$$S = \frac{a + Sg}{i}$$

But after Tax

$$S = \frac{a(1-t) + Sg}{i}$$

bes. a is Taxable; Sg is not.
Not right? No, right.

$$S = a \left[\frac{(1-t)}{1+i} + \frac{(1+g)(1-t)}{(1+i)^2} + \dots \right]$$

$$= \frac{a(1-t)}{i-g}$$

$$Si - Sg = a(1-t)$$

$$S = \frac{a(1-t) + Sg}{i}$$

$$S = a \frac{1 - (1+g)^{-n}}{i} + \cancel{2.82} S (1+i)^{-n} (1+g)^{n-1}$$

$$S \left[1 - \frac{(1+g)^{n-1}}{(1+i)^n} \right] = a \frac{1 - (1+i)^{-n}}{i}$$

$$S = \frac{a}{i} \frac{1 - (1+i)^{-n}}{1 - \frac{(1+g)^{n-1}}{(1+i)^n}}$$

$$\cancel{[S(1+g)^{n-1} - S](1-.4t)}$$

$$S = a(1-t) \frac{1-(1+i)^{-n}}{i} + S(1-.4t) \frac{(1+g)^{n-1}}{(1+i)^n}$$

$$S = \frac{a}{i} [1-t] \left[\frac{1-(1+i)^{-n}}{1 - (1-.4t) \frac{(1+g)^{n-1}}{(1+i)^n}} \right]$$

On sale, seller gets:

$$\begin{aligned} S(1+g)^{n-1} - t \times .4 \times \left\{ S[(1+g)^{n-1} - 1] \right\} \\ = S(1+g)^{n-1} (1-.4t) + .4tS \end{aligned}$$

$$S = a(1-t) \frac{1-(1+i)^{-n}}{i} + \frac{S}{(1+i)^n} \left[(1+g)^{n-1} (1-.4t) + .4t \right]$$

$$S = \frac{a(1-t) \frac{1-(1+i)^{-n}}{i}}{1 - \frac{(1+g)^{n-1} (1-.4t) + .4t}{(1+i)^n}}$$

$$= \frac{a(1-t) \left[\frac{(1+i)^n - 1}{i} \right]}{(1+i)^n - [(1+g)^{n-1} (1-.4t) + .4t]}$$

alternative forms for income tax on P.A. retaining neutrality.
~~the inc. taxes, to be neutral~~

$$1. \quad \sigma = \frac{a(1-t) + \sigma g(1-t)}{i}$$

$$2. \quad \sigma = \frac{a(1-t) + \cancel{\sigma g} - g s t}{i}$$

$$\sigma = \frac{a}{i - g + t}$$

$$\sigma = \frac{a + g\sigma - t\sigma}{i}$$

Another approach: a is level for 20 years; then sell for $6 \times S$.

$$S = a \frac{1 - (1+i)^{-20}}{i} + 6S (1+i)^{-20}$$

$$\sigma = a(1-t) \frac{1 - (1+i)^{-20}}{i} + 6\sigma (1+i)^{-20} (1 - \cancel{t}) + t\sigma$$

remove pref. rate, focus on timing

Forgot to add $t\sigma$

$$\sigma [1 - 6 (1+i)^{-20} (1-t)] = a(1-t) \frac{1 - (1+i)^{-20}}{i}$$

$$a = 2$$

$$t = .5$$

$$i = .10$$

$$\sigma = \frac{8.57}{1 - 3 \times .15} = \frac{8.57}{.55} = 15.47$$

~~This wants completing.~~

Rather than sell for $6 \times S$, sell for $(1+g)^n \times S$.

$$S = a \frac{1 - (1+i)^{-n}}{i} + S \frac{(1+g)^n}{(1+i)^n}$$

$$S = \frac{a}{i} \frac{1 - (1+i)^{-n}}{1 - \left(\frac{1+g}{1+i}\right)^n} = \frac{.951}{.}$$

$$n = 20$$

$$i = .10$$

g		Adds to value
.01	1.040	4% 40.90
.02	1.093	9.3%
.05	1.406	40.6%
.09	5.100	410%

cf. P.A., $S = \frac{a}{i} + S \frac{g}{i}$, seems to say that if $g = i$, it adds only 100% to value, but this is absurd. As higher $g \rightarrow \Delta S \rightarrow \Delta S \frac{g}{i} \rightarrow \Delta S$ etc.

~~$S = \frac{a}{i-g}$~~

$$\frac{\frac{a}{i-g} - \frac{a}{i}}{\frac{a}{i}} = \frac{i}{i-g} - 1 = \frac{i - i + g}{i-g} = \frac{g}{i-g}$$

Prop. tax cap. and growth

$$S = \frac{a}{i - g + t}$$

$$Si = a + Sg - St \quad (1)$$

$$\text{If } g = 0, \quad S = \frac{a - St}{i} \quad (2)$$

In (2), taxes are simply capd. in perp. at a const. level. The introduction of g raises g , of why St is still cap. at const. level. S is now higher, so St is now a tax on Sg as well as $\underline{a}$, and that begins to explain it.

Derive the formula, longhand:

$$S = a \left[\frac{1}{1+i} \right] ?$$

Begin w. $\underline{a}$, due at end of one year.

$$PV(1+i) = a$$

pd. at end of year.

if $t=0$. i is int. on PV,

$$PV(1+i+t) = a$$

t is on base of PV,

pd. at end of yr.

Wants completing

Agenda -

See if phantom profit effect actually reduces
effective tax rate. Now, that would be
news!