Effects of Δi and τ on bldg. (II)

64. Simpler rule.

$$a = S \cdot i$$

$$\frac{da}{di} = S + i\frac{dS}{di}$$

$$da = S\,di + i\,dS$$

$da < 0$ when $S \cdot di + i\,dS < 0$

$$S\,di < -i\,dS$$

$$\frac{di}{i} < -\frac{dS}{S}$$

But $-\frac{dS}{S}$ has leverage, and $\frac{di}{i}$ does not, so $-\frac{dS}{S} > \frac{di}{i}$,

almost surely.

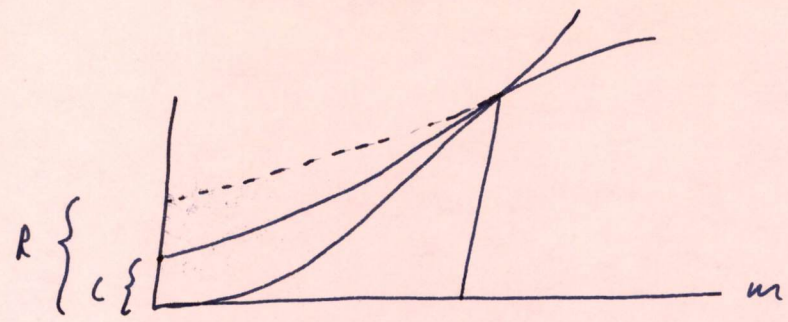
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65. High C/R means intensity, at given m.

But at short m, C/R is high, yet there is little capl. applied, and a not sensitive to t or i. It is like putting lot. Hold it. High C/R then means high labor-intensity. It is still intense, just not capl. intense.

If $m = 1$, $c \to R$, but there is still a big rent share? No, then wage share $\cong \frac{C}{R}$.

~~[See TTTI, (& TT & life paper)]~~



$\frac{C}{R}$ falls ~~with~~ as m rises. It is not that R isn't discounted: it is.

But there is ~~more~~ PV of rent in there: a $\frac{1-e^{-im}}{i}$

$$S = \frac{R-C}{1-e^{-im}}$$

$$S = R - C + Se^{-im}$$

$$R = S(1-e^{-im}) + C = C + a\frac{1-e^{-im}}{i} = C + (PV \text{ of } \Sigma a)$$

$$\frac{C}{R} = \frac{C}{S(1-e^{-im}) + C}$$

So $\frac{C}{R}$ is not a ~~good~~ full index to intensity. $\frac{C}{R} = \frac{Ce^{im}}{FVR}$, and leaves out interest on rent, treats it as a fixed input, understates intensity.

$\frac{C}{S}$ is better? No. It is ~~more of an index to labor intensity~~. It is pretty similar to $\frac{C}{R}$, just larger, ~~and more leveraged~~. But as you move up curve of $\left[Ce^{im} + a\frac{e^{im}-1}{i}\right]$, R rises and S remains same. But so what? No special usefulness.

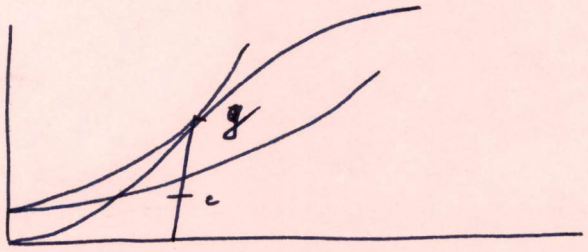
$$\frac{C}{S} = \frac{C}{\frac{R-C}{1-e^{-im}}} = (1-e^{-im})\frac{C}{R-C}$$

Yet ~~for long cycles~~, $\frac{C}{S}$ will be a decr. fn. of ~~smaller due to~~ length, (m).

E.g. for $m=1$, $\frac{C}{S} = 20$ is possible.

For $m=80$ $\frac{C}{S} = \frac{1}{50}$ is possible, yet capl. accumulates so intensity is not that low.

66. Factor proportions – how define intensity?



Intensity. $\frac{Q}{g} > \frac{ce^{im}}{g} = \frac{ce^{im}}{ce^{im} + s(e^{im}-1)} = \frac{1}{1-\frac{s}{c}(1-e^{-im})}$ (1)

$\frac{ce^{im}}{g}$ also $= \frac{c}{ge^{-im}} = \frac{c}{R}$ (2)

This concept is faulty for failing to treat interest on deferred rent as a non-land input.

$Q\frac{}{g} = \frac{ce^{im} + s(e^{im}-1) - sim}{g_m} = \frac{c + s(1-e^{-im}) - ame^{-im}}{PVR}$ (3)

This concept it wd. be nice to ~~simplify by~~ make more <u>self contained</u> by defining s in terms of g & c: $s = \frac{g - ce^{im}}{e^{im}-1}$ (4)

<u>(But note this determines a residually.)</u> (Resulting in understated Q for undermined land)

$Q\frac{}{g}g = ce^{im} + g - ce^{im} - im\frac{g-ce^{im}}{e^{im}-1} = g - (g - ce^{+im})\frac{im}{e^{im}-1}$

$Q\frac{}{g} = 1 - \frac{g-ce^{im}}{g}\frac{im}{e^{im}-1}$. Let $R = ge^{-im}$, and $\frac{Q}{g} = 1 - \frac{R-c}{R}\frac{im}{e^{im}-1}$ (5)

It wd. be even nicer to modify concept of $\frac{Q}{Z}$ for use when revs. come at all times.

$$Q \frac{}{} = \frac{\Sigma\left[C_n e^{i(m-n)}\right] + S(e^{im}-1) - S\,im}{\Sigma\left(R_n e^{i(m-n)}\right) \equiv ZVR} \tag{6}$$

$$S = \frac{ZVR - ZVC}{e^{im}-1} \tag{7}$$

$$Q\, ZVR = ZVC + ZVR - ZVC - \frac{\left(ZVR - {}^{ZV}_{A}C\right) im}{e^{im}-1}$$

$$Q = 1 - \frac{ZVR - ZVC}{ZVR}\,\frac{im}{e^{im}-1} = 1 - \frac{PVR - {}^{PV}_{A}C}{PVR}\,\frac{im}{e^{im}-1} \tag{8}$$

(8) same as (5). Naturally -- it is derived in same way.

This is the defn. of intensity used in Davis paper. App. I, only there it was just given numerically by adding %s that were labor share and capl. share.

p. 78, check it out, see if get same results. I do.

$$Q = 1 - \frac{R-C}{R}\,\frac{im}{e^{im}-1} = 1 - \frac{R-C}{e^{im}(1-e^{-im})}\,\frac{im}{R} = 1 - Se^{-im}\,\frac{im}{R}$$

$$= 1 - \frac{S\,im}{R\,e^{im}}$$

$$= 1 - \frac{S\,im}{ZVR} = 1 - \frac{\sum_{i}^{m} a}{ZVR}$$

67. When revs. not deferred, is it still sensible to treat rent same as with trees? (as in 66.) ? I.e., treat rent as tho not realized, but embodied in capl.

Yes, bec. you're compounding [revs.$_n$] forward to m, thus treating them as though they were deferred to then (and appropriately magnified) and as though no capital had been recovered in the revs., (just like trees. But this makes it hard to apply to tax policy, where you want model tax base as is. (see 71, p. 77)

68. How does my defn. of intensity now cf. with C/R?

If $\frac{im}{e^{im}-1} = 1$, then $Q = 1 - \frac{R-C}{R} = \frac{C}{R}$

But $\frac{im}{e^{im}-1} < 1$, thus reducing 2d term and raising Q.

$\rightarrow Q > \frac{C}{R}$

$\frac{im}{e^{im}-1}$ is decreasing fn. of m, so the higher m, the more it raises Q above $\frac{C}{R}$. Thus C/R alone

✓ understates intensity of longer cycles relative to short, and overstates intensity of short cycles relative to long.

So, from (65), p. 70, when $m=1$, $R_1 = Ce^i + a$

and $PVR_0 = C + ae^{-i}$

so $\frac{C}{R} = \frac{C}{C + ae^{-i}}$ This → ? Not if a is high ÷ C, as in Champaign City, Ill.

$\frac{im}{e^{im}-1}$ doesn't change it much, when $m=1$.

If $m=1$

$$\frac{im}{e^{im}-1} = \frac{i}{e^{i}-1} \cong \frac{i}{i} = 1$$

But when $m = 100$ (no, not nearly at all)

$\frac{c}{R}$ very small, but $\frac{mi}{e^{im}-1} = (\text{say}) \frac{100 \cdot .05}{e^{5}-1} = \frac{5}{147}$

is also small.

$$Q = 1 - \frac{R-C}{R} \times (.03)$$

Now since $\frac{c}{R}$ is small, $\frac{R-C}{R} = 1 - \frac{c}{R} \to 1$. This alone wd. make $Q \to 0$. But the .03 coefficient nearly wipes this out, leaving a high intensity.

It is still true that long cycles are less intensive. This merely avoids overstating the point.

Long cycles are especially less labor intensive. The present adjustment refers only to capital. (land-derived capital). But note how intensity rises, even if $C = 0$! If you let capl. pile up on land you get intensity. Also, rent falls.

69. There is something direct & self-evident about defining

$$y = \frac{c}{C + a\frac{1-e^{-im}}{i}}$$

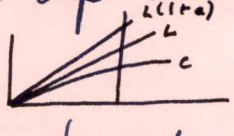
But it's wrong.

Because $R = C + S(1-e^{-im}) = C + a\frac{1-e^{-im}}{i}$

(This is consistent with all the above)? Or am I back to $\frac{c}{R}$?

Yes.

70. Doubts about use of $\underline{a}$ in def. are resolved by device of normalizing, as in Bewis app. $\bar{I}$, and III. (III refers to depreg capl.)

71. Good intuitive explan needed for why Q rises with $\underline{m}$ when capl. is depreg. {  In this model, $Q = \frac{L}{LC(1+a)} = \frac{1}{1+a}$ ... If longer $\underline{m}$ makes a fall, it raises Q. }

Unimportant

You could say the early land inputs were dual purpose: to yield early revs., and also to carry the carcass or shell of bldg. into future to yield future revs. Thus part of the rent is plowed back, is an invt. for the future, just as with trees. But this is expressed in depreciation & lacks of same.

Probably this is even a normal condition: early years, the revs. do not cover int. on S+C; but the invt. is justified over whole life. ~~(I've noticed this in R+M's)~~ If so, the bldg. wd. appreciate. See 84, p. 88

If C is very low, $\frac{R-C}{R} \to 1$, and value of Q could have an arbitrary element since there is no unambiguous value for $\underline{m}$. This is solved by specifying a time period one wishes to evaluate --- the cycle one feels responsible for, or the period one feels is dominated by C_0. [Or, the period might be forever, but my formula is premised on one finite cycle]

71.

~~70~~. Intensity $\equiv Q = 1 - \frac{R-C}{R} \frac{im}{e^{im}-1}$

$i = .05$

Let $C = 200$

$S = 800$

$$PVR = \frac{(C+S)e^{im} - S}{e^{im}} = C + S - Se^{-im} = C + S(1 - e^{-im}) = C + a\frac{1-e^{-im}}{i}$$

m	R		$\frac{R-C}{R}$	im	$e^{im}-1$	$\frac{im}{e^{im}-1}$	$\frac{R-C}{R}\frac{im}{e^{im}-1}$
7	$\frac{7.81}{1.41} = 431.9$		.536	.35	.41	.853	.457
14	$\frac{14.84}{1.97} = 596.9$		.665	.70	.97	.72	.479
56	$\frac{56.261}{15.4} = 949.1$		.789	2.80	14.4	.194	.15

This checks with Davis paper, App. 1.

Q = 1 - [last column above], and this = i.q. in Davis paper, p. 4.

72. Now I have a new defn. of intensity, to replace $\frac{c}{R}$.

Intensity, quotient,

$$Q = 1 - \frac{R-c}{R}\,\frac{im}{e^{im}-1}$$

Better: $Q = \left(\frac{c}{R}-1\right)\frac{mi}{e^{mi}-1} + 1$

$$\frac{im}{e^{im}-1} \equiv \psi$$

✓ It is of course simple interest ÷ compound int.

✓ It is also the % of the surplus that is rent. ("Surplus" $= (g - Ce^{im})$)

✓ Note that this % falls from 100% when $m=1$, down very low where $m=60$.

73. When $m=1$ $\quad \frac{im}{(1+i)^m - 1} = \frac{i}{i} = 1$

But $\quad \frac{im}{e^{im}-1} = \frac{i}{e^{i}-1} < 1$

Because $e^i > 1+i$

or $\quad i > \ln(1+i)$

This is because $(1+i)^1$ represents simple interest for one year, due to annual compounding.

$e^i > 1+i$ because e^i represents daily or continuous compounding.

74. $e = 2.718\,281\,828$

$$\left(1 - \frac{1}{10000}\right)^{-10000} = \frac{1}{.9999}^{10000}$$

$$= \left(\frac{1}{.9994}\right)^{10000}$$

Values of $\frac{mi}{e^{mi}-1}$ at 5%

EXPERIMENT DATA SHEET

DEPARTMENT _____

SUBJECT _____

EXPERIMENT NO. _____ **ASSIGNMENT NO.** _____ **DATE** _____

m	.05m	$e^{.05m}$	$(e^{.05m}-1)$	$\frac{.05m}{e^{.05m}-1}$							
1	.05	1.05127 ~~1.0513~~	.0512	~~.9986~~ .98 ~~.9752~~	.9752 ~~.9986~~						
2	.10	1.1052	.105	.952							
3	.15										
4	.20	1.2214	.221	.905							
5	.25	1.284	.284	.880							
8	.40										
10	.50	1.6489	.648	.772							
15	.75	2.117	1.117	.671							
16	.80										
20	1.00	2.718	1.718	.582							
25	1.25	3.490	2.490	.502							
30	1.50	4.481	3.481	.431							
35	1.65	5.753	4.753	.3487							
40	2.0	7.388	6.388	.313							
50	2.5	12.18	11.18	.224							
60	3.0	20.08	19.08	.157							
70	3.5	33.11	32.11	.109							
80	4.0	54.58	53.58	.075							
90	4.5	89.48	88.48	.051							
100	5.0	148.35	147.35	.034							

SIGNED _____

75. Simplify

$$Q = 1 - \frac{R-C}{R}\,\frac{mi}{e^{mi}-1} \equiv 1 - \frac{R-C}{R}\,\psi$$

$$\boxed{Q = 1 + \left(\frac{C}{R} - 1\right)\psi} \leftarrow \text{This is simpler, because } \psi \text{ appears only once.}$$

or

$$Q = \frac{C}{R}\psi + (1-\psi)$$

~~76~~

Note that this is based on internal valuation of $\underline{S}$ & $\underline{a}$. That is why $Q \to 1$ when $m \to \infty$ and $\psi \to 0$. What's happening is that $a \to 0$, or at least $\underline{a}$ is declining.

So if you let a cycle drift on past economical cut-off point, you are diluting $\underline{a}$, hence for this reason $Q \to 1$.

Solution: assume cut-off comes when $R_m \leq a$.

75a. ~~Read~~ If the cycle in question is the wrong one, take a higher $\underline{a}$ from another cycle and apply alternative def. where

$$Q = 1 - \frac{\sum_1^m a}{TVR} \qquad (p.\ 74)$$

~~71~~

This gives a higher $\underline{a}$ and lower Q; but Q still rises with $\underline{m}$, and it shdn't, it shd. fall as you add more land and get no return for it. Hold it!

→ If $\underline{a}$ is fixed, then i has to fall, so TVR doesn't rise.

16. But can my Q be arbitrary sometimes?

E.g. parking lot.

$$Q = 1 + \left(\frac{c}{R} - 1\right)\frac{mi}{e^{mi} - 1}$$

If C small, m is short, so $Q \to 0$, on parking lot.

~~It's similarly~~ w. trees, ~~that~~ ψ makes big difference. $\frac{c}{R}$ is small, which wd. make Q ~~small~~, but low value of ψ prevents it, raises Q.

But w. bldgs. say $\frac{c}{R} = .75$, a long life can raise ~~that up~~ Q by lowering ψ. This suggests rents are plowed back w. bldgs. too.

17. And does it matter if I deduct c_n from R or add to C?

Yes. Add to C, for full picture.

18. $TVR = Ce^{im} + S(e^{im}-1)$

Labor input = C

Land input = Sim

Cap. input = $C(e^{im}-1) + S(e^{im}-1) - Sim$

$= TVR - C - Sim$

Labor intensity = $\frac{C}{TVR}$

Land intensity = $\frac{Sim}{TVR}$

Capl. intensity = $\frac{Ce^{im} + S(e^{im}-1) - C - Sim}{Ce^{im} + S(e^{im}-1)}$

$= 1 - \frac{C + Sim}{TVR}$

Intensity, $Q = \frac{Ce^{im} + S(e^{im}-1) - Sim}{Ce^{im} + S(e^{im}-1)} = 1 - \frac{Sim}{Ce^{im} + S(e^{im}-1)}$

$= 1 - \frac{\frac{R-C}{1-e^{-im}}\, im}{Re^{im}} = 1 - \frac{R-C}{R}\,\frac{im}{e^{im}-1}$

Again, intensity, Q, ~~falls with m~~ rises with m because longer m means lower a and S. But if a is externally fixed, as on o.c., then $Q = 1 - \frac{\Sigma a}{TVR}$ and ... then i is residual and falls if m drags on too long.

Capt. intensity

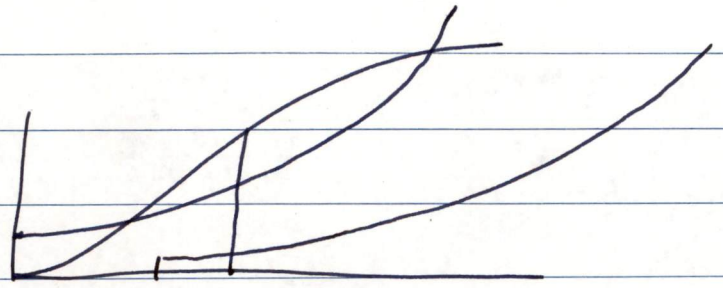
$$1 - \frac{C + Sim}{TVR} = 1 - \frac{C + \frac{R-C}{1-e^{-im}} im}{Re^{im}}$$

$$= 1 - \frac{c - Ce^{-im} + Rim - Cim}{R(e^{im} - 1)}$$

$$= 1 - \frac{C(1 - e^{-im}) + (R-C)im}{Re^{im}(1 - e^{-im})}$$

$$= 1 - \frac{C}{Re^{im}} + \frac{R-C}{Re^{im}} \frac{im}{1-e^{-im}} = 1 - \frac{C}{Re^{im}} + \frac{R-C}{R} \frac{im}{e^{im}-1}$$

79.



$Q \equiv \frac{\text{Labor \& capl. input}}{\text{Output}}$

Output = Σ all inputs, if inputs are used well. So on this assumption, $Q = \frac{\text{Labor \& Capl.}}{\text{Labor \& Capl. \& Land}}$

✓ If land badly used, then Output < Σ all inputs,

and using Output in denom. overstates Q.

To avoid use Σ inputs in denom.

But this changes numerator, too.

Ce^{in} not affected. ? (Yes it is, if i is residual, & is reduced)

But $\left[a\frac{e^{in}-1}{i} - ma\right]$ ∂ a. But ∂ i, and i falls)

If you Δ denom. by using o.c. a, must also increase the capl. input in num., by using o.c. a. (But reduce it by using lower i)

Stinging out m increases Q when a is internally determined.
But when a is exogenous, o.c., externally imposed, then ---
then i must be internal, lowering capl. input, lowering intensity.

Thus, excessive life:
1. Raises intensity if a is residual
2. Lowers " " i " "

80. Likewise taxes
If a residual, taxes lower a, thus increase intensity, by ~~it~~ ratio measure. If a becomes zero, intensity becomes 1, bcs. then is no land input.
to watch that. It is still ok to compare intensities, and how taxes change relative bidding powers.
Taxes t, and i, are alike in reducing intensity, but making it looks like higher intensity by reducing S & a. & see 86, p. 89.

81. In defining intensity, watch the netting out.
Take $Q = \dfrac{(C+S)e^{im} - S(1+im)}{(C+S)e^{im} - S}$ L 1
Denom. is stumpage. But harvest labor, C_m, is netted out. This lowers ratio. If added a like amount to top & bottom, wd. raise Q.

82. Tax bias.

Simple first pass was to say $\underline{t}$ and $\underline{i}$ are biased vs. high $\frac{C}{R}$, (or high $\frac{C}{S}$).

That is true at any given $\underline{m}$.

But at short $\underline{m}$, high $\frac{C}{R}$ means high labor-intensity, not capital, i & t have little effect, ... but that little is against higher $\frac{C}{R}$.

(and so for fast deprecn. regardless of $\underline{m}$.)

At long $\underline{m}$, and low early revs., high $\frac{C}{R}$ means high capital-intensity --- but ~~is not an adequate measure of~~ understates intensity. $\underline{i}$ and $\underline{t}$ are biased against these uses ... long life, capl.-intensive.

Short cycles do not nec'ly have high $\frac{C}{R}$. On good corn land, $m=1$, $C = \frac{1}{2}R$.

On parking lot, m is irrelevant, C & R are const. flows, $C = \frac{1}{10}R$.

82A. ✓ But a bldg. that employs or uses much labor over its life represents a labor-using land use, when all inputs & outputs are counted. This is where netting out is very misleading. Capl. even of slow deprecn. is complementary to labor, usually.

83. Since $\underline{t}$, like $\underline{i}$, hits capl. - int-emive mixes, is it good?

Not unqualified good, no. Bcs. it reduces opt. int-emity, and that can hurt labor too.

That is, it doesn't just cause labor to subst. for capl, it also causes land to sub. for capl., and the labor that complements capl., like building trades, and workers inside bldgs.

(And of course funnels capl. into p.w; & export.)(& distorting)

84. a cause of investing capl. for remote future returns.

When the ~~inst~~ early revs. do not cover int. on (C+S); and are justified by appreen.

Appreen. of capl. like trees, is ok.

Appreen. of l.v. caused by early devmt. is a social waste.

1. If due to attracting p.w. leverage of pvt. over pub. inv., waste is obvious.

2. Even if no public money involved, benefits dubious. Say it's to attract buyers to a retail site. This is zero-sum. They come from others. (It may be permanent, by getting them to build near you.)

3. When l.v. are rising anyway, chances are good that

some will think they have to cause the rise, and invest to do so ... even though it wd. occur anyway.

But the larger fact is: when demand rises, opps. to have demand & p.w. your way rise, sparking "lightning-rod" invts all over.

4. There are many ways to invest yr. money to make yr. own l.v. rise.

This is exaggerated by:

Large holdings

Leverage over p.w.

Tax advantages to Δ l.v.

It justifies longest-lived invts, bcs. product is of perpetual life, and rising value.

85. Part of fn. of land input is to keep bldg. from depreg.

Physically

Environmentally or locationally

86. Higher i & t speed up turnover some. but by reducing rent, not near as much as otherwise.

The speedup is greatest on urgl. land, but it goes out of use.

On rent-yielding land, I once showed in green book that there was still a speedup. But check it. Leverage might even reverse it.

Contents, highlights, p. 1

Page	No.		
11	15	$d\ln(Si) = \frac{dS}{S} + \frac{di}{i}$	And see p. 54, bottom, p. 55 p. 26, # 25. p. 39, # 36 p. 57 # 50 p. 61. # 55
12	16	$\frac{d\ln(1-e^{-im})}{di} = \frac{m}{e^{im}-1}$	
15	17	When $S = a/i$, is not bas. $c = 0$, but $m = 1$.	
30	30	Need for keying PVR to depress. pattern, in showing effects of t on $\underline{a}$ a θ.	
31, 32 53	32 48	Derive $a = \frac{i+t}{1-e^{-(i+t)m}}\left[\Sigma\left(R_n e^{-(i+t)n}\right) - C\right]$	And see p. 98 #100 p. 56 p. 94 #96 p. 100 p. 93 #93
59 88	51 84	Summary of biases where t is more biased than i. And see p. 67; p. 49 #44; p. 102 #104; p. 125, #121.	
72 79 83	66 72 78	Define intensity, $Q = 1 - \frac{R-C}{R}\frac{mi}{e^{mi}-1} = 1 + \left(\frac{C}{R} - 1\right)\frac{mi}{e^{mi}-1}$ and p. 81 #75; and p. 82 #76.	
75	68	$Q > \frac{C}{R}$	
79	73	$e^i > 1+i$	
80		Values of $\frac{mi}{e^{mi}-1}$ at 5%	
81	75a	Q when bldg. is valued as residual	
83	78, last line.		
85	79		
86	80		
86	81		
90	87	(Rising $\underline{a}$)	

Contents, highlights, p. 2

Page	No.	
87 92 102	82 89 102	t & i are biased against capl. intensity & long life, not against high C/R alone. (But where capl. complements labor, bias is against labor.) p. 92 #89; p. 102, #102; 105 #105;
87	82A	Intensity must include all inputs & outputs over life.
88	83	Netting out, or "clearing", can understate labor-intensity, (e.g. by using cash flow to rep. gross; or using stumpage to rep. gross.)
89 112	85 111	Is rent stored up in bldgs., libra trees?
89 114	86 113	How much does decline of rent from i & t offset speedup of turnover due to i & t?
90	87	Modify Q for rising $\underline{a}$.
94 109	96 109B	Try to find a_t by deducting PVT from S, and reconcile with other method.
~~92~~ 91	98	Prop. tax not biased ~~vs.~~ forestry in theory; biased for it in practice.
100	100	Def. of $\underline{a}$ after tax derived in PIPO case, intuitively clear.
101	101	Levying t forces us to have 2 concepts of intensity. Lowers S_t below social value. Thus understates land input, overstates capl. input, overstates true intensity.
104 106	107 108	Tax on appreig capl. erodes some of own base, (like land tax.)
105	107	$\frac{s-\theta}{s}$ by new approach

Contents, Highlights, p. 3

Page	No.	
106 112	108 111	Is tax on appreig capl. different? p. 107 concludes "No".
109 94	109B 96	Quest for Θ by method of $S - \Sigma PVT$
110	110A	$T_n = PV_n g_m - PV_n \sum_n^m a$ and variations
111	110B	$B_n = PV_n \sum_n^m R - PV \sum_n^m a$
112 89 124	111 85 120	Bldgs. can store up unripe rents, like trees, and interest, too. This is just what cash flow analysis does, p. 120.
112	110B	Rising rents can cause overbldg.
114 89	113 86	owing to leverage, might i or t reduce a by more than increase of $i \cdot B_n$ and thus wholly or more than wholely offset effect on life, and lengthen life?
122 115 ~~116~~	118 114 ~~115~~	Paradox: $\Theta\left(e^{n(i+t)}-1\right) \stackrel{?}{=} \frac{a}{i+t}\left(e^{n(i+t)}-1\right)$ p. 119 may help.
116 ~~117~~	114A ~~112~~	
120	116	Cash flow analysis
121	117	$\ln(1+r) = r\left(1-\frac{r}{2}\right)$
123	119	Agenda: Normalize.

Why st on bldg. is harder on bldgs. than si. Little of value pp. 1 - 7 (1

Summary of findings, including tools developed along the way.

~~See p. 4 for clean statement re the problem that si doesn't lower rent when $i \leq 0$.~~

ok 1. If $S = \frac{a}{i}$ and a not $f(i)$

$$\frac{dS}{di} = -\frac{a}{i^2} = -\frac{S}{i}$$

$$dS = -S\frac{di}{i}$$

$$\frac{dS}{S} = -\frac{di}{i}$$

$$S + dS = S\left(1 - \frac{di}{i}\right)$$

ok 2. $$\frac{d(Si)}{di} = S + i\frac{dS}{di}$$

If $\frac{dS}{di} = -\frac{S}{i}$, as in (1), then

$$\frac{d(Si)}{di} = S - i\frac{S}{i} = 0$$

ok 3. $d(Si) = S\,di + i\,dS$

$= 0$ when $\frac{di}{i} = -\frac{dS}{S}$

4. The object of this exercise is to show that a bldg. tax is more biased vs. bldgs. than is an ~~compar~~ equal rise of i, or otherwise comparable rise of i.

"Otherwise comparable" could mean ~~"bdgs~~ producing same decline in S. But the Δi required to produce the same decline in S declines ~~with C~~ as $\frac{C}{R}$ rises, and that may be one way of getting at this.

When $\frac{C}{R} = 0$, t produces no decline of S, so the comparable $\Delta i = 0$.

When $\frac{C}{R} = 1$, t & Δi produce (about) same decline of S, bcs. they have same base, so comparable $\Delta i = t$.

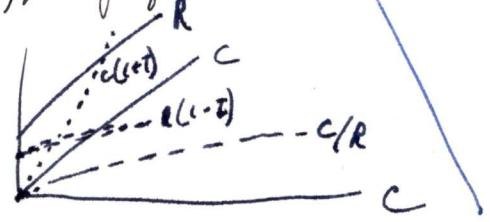
In between, the comparable Δi increases with $\frac{C}{R}$.

$$\frac{\text{Comp. } \Delta i}{t} = \frac{C}{R}$$

So if $\Delta i = t$, Δi is higher than the comparable t

Comparable $\Delta i = t\,\frac{C}{R}$

Thus the efficacy of t (in lowering S) relative to Δi increases with $\frac{C}{R}$



see p. 43

5. If Δi lowers S to 0, as when $\frac{c}{R} \to 1$, then the Δi will not raise income any.

And whenever S is made small, the Δi has little effect.

~~Eg.~~ E.g. Δi, Δi has greater effect on lower $\frac{c}{R}$. The compensatory effect of Δi does not compensate for that, because it is on the base of $(S - \Delta S_i)$

Ok, but so what?

6. PVR comes from 2 parts

$$PVR = \underset{S}{\frac{a}{i-g}} + \underset{c}{b\,\frac{1-e^{-(i+d)n}}{i+d}}$$

Interest on PVR is

$$i\,PVR = a\,\frac{i}{i-g} + b\left(1-e^{-(i+d)n}\right)\frac{i}{i+d}$$

If $g=0$ & $d=0$ that devolves to

$$i\,PVR = a + b - b\,e^{-in} \quad \text{or}$$

Rent + Cap. service less depreciation, or rent & interest

When Δi, a & b are const. & ~~b~~ depreciation falls, so $(i \cdot PVR)$ rises.

not really const. There is a drop, a cap. loss because $b\,\frac{1-e^{-in}}{i}$ drops.

Review

~~How~~ The heavier ~~the weight~~ of b vs. a, the more the rise. If $b=0$, ~~there is~~ no rise at all. This says that when Δi, the extra inc. to capl. comes out of lower depreciation, so rent remains same. Thus Δi doesn't reduce rent. That's hard to believe! Review.

Now add $d>0$ and $g>0$.

d reduces effect of Δi; g adds to it.

So the more capl., & the faster it decays, the less sensitive is PVR to Δi. But the more capl. does not reduce the effect of tax on PVR, but Δs it.

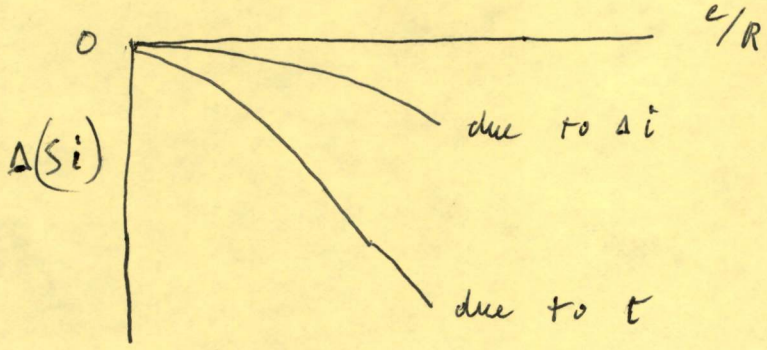
1. $\frac{d(Si)}{di}$ and $\frac{d(Si)}{dt}$ both go to 0 when $C = 0$.

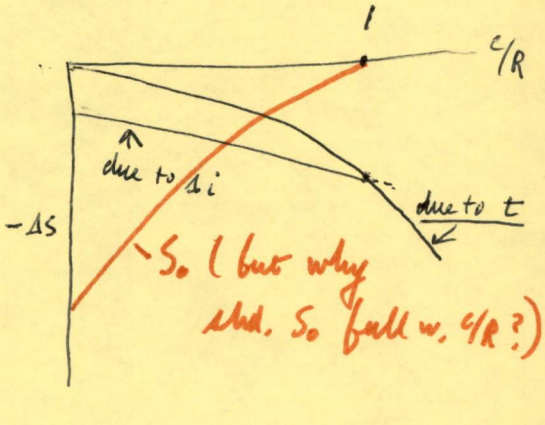
But when $C > 0$, decline of land rent = Si due to Δi > decline of rent due to t. Why? I doubt this.

When $C \to R$, $-\Delta S_t \to -\Delta S_i$ (except for influence of denom., but it has nothing to influence when S is near zero.)

→ { so if rent = Si, Δi is relatively much more favorable than t to high $\frac{C}{R}$, because they lower S the same (caution: for given S, R − C remains const., and they do not lower S the same. See p. 43-4) but Δi raises the mult. So

$-\Delta(Si)_i \ll -\Delta(Si)_t$



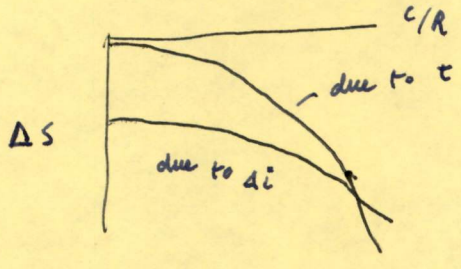


Algebra of ratio of $\frac{-\Delta(Si)_t}{-\Delta(Si)_i}$ needed.

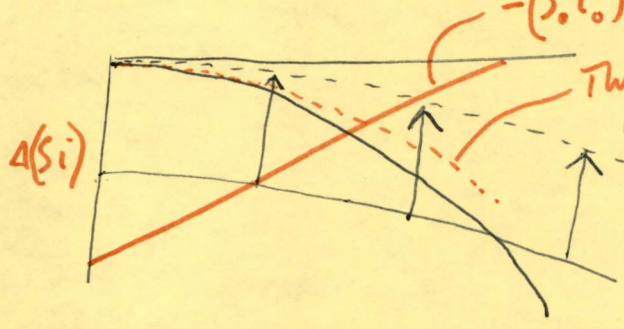
See p. 44

= 39, for clearer model — casting doubt on this one.

Stage I: Effect of $-\Delta S$ alone.



Stage II: Effect of Δi on income, shown by ↑



8. Having the basic statement of (7), there's lots of frosting to put on the cake. E.g. that $-\frac{dR}{di}$ is a decrg. $f(\frac{c}{R})$

9. To Prove: $\frac{-d(\$i)}{di} \Big/ \frac{-d(Si)}{dt}$ is a decreasing $f(c)$

$-\Delta S_i > -\Delta S_t$

$\frac{-\Delta S_i}{-\Delta S_t} = \Theta \frac{R}{c}$

~~$\frac{d\$}{di}$~~

Too vague. To make precise, must specify all exactly, and key T to Θ

$\frac{-\Delta(Si)_i}{-\Delta(Si)_t} < \Theta \frac{R}{c}$

~~$-\Delta(Si)_i < -(Si)_t$~~ Not proven.

$\frac{-\Delta(Si)_i}{-\Delta(Si)_t} = \frac{-(\Delta i\, S_i + i\,\Delta S_i)}{-\Delta S_t \cdot i} = \Theta\frac{R}{c} + \underbrace{\frac{\Delta i}{i}\frac{S_i}{\Delta S_t}}_{\text{neg, bcs. } \Delta S_t < 0}$

$= \frac{\Theta R}{c} + \frac{\Delta i}{i}\frac{S_i}{-TC} = \frac{+TCi\Theta R - \cancel{c}\,\Delta i\, S_i}{+Tc^2 i} = \frac{Ti\Theta R - \Delta i S_i}{Tci}$

~~$+\Theta\frac{R}{c} - \frac{\Delta i}{i}\frac{S_i}{TC}$~~ $= \Theta\frac{R}{c} - \frac{\Delta i}{i}\frac{S_i}{TC} = \frac{1}{c}\left[\Theta R - \frac{\Delta i}{iT} S_i\right]$

This ratio, like $\frac{R}{c}$, is a decreasing $f(\frac{c}{R})$, [redacted]

but only if $Ti\Theta R > \Delta i\, S_i$

or $\frac{T\Theta(c+s)}{S_i} > \frac{\Delta i}{i}$

$T \cong \frac{t}{i}$ if m is very long.

If $t = \Delta i$

$T \cong \frac{\Delta i}{i}$

So $\Phi(C+S) > S_i$

$\Phi \cong 1$

∴ Inequality is almost certainly true. Φ may even > 1

∴ $\frac{-d(S_i)}{di} \Big/ \frac{-d(S_i)}{dt}$ is a decreasing $f\left(\frac{C}{R}\right)$. QED? Too Crude.

10. $\frac{-dS}{dt} = \Phi \frac{1}{i} C = T C$

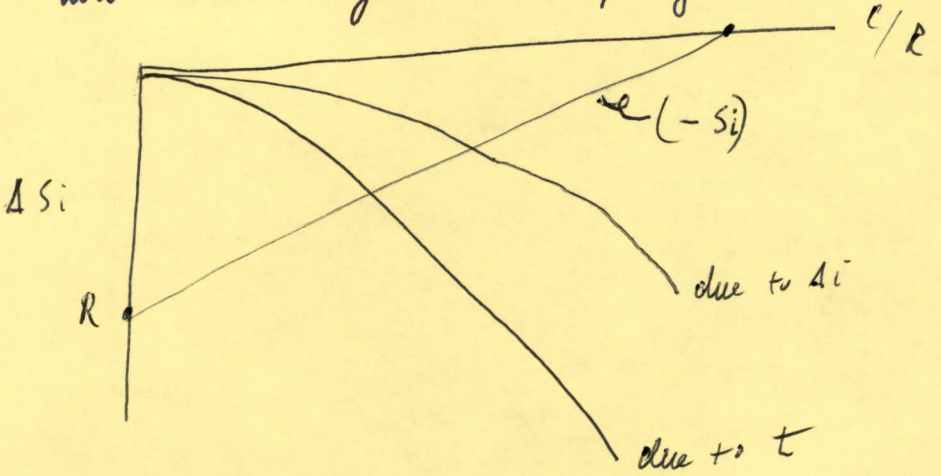
where $T \equiv \frac{\Phi}{i}$ and $\Phi = f(t)$

$\frac{-dS}{} = \Phi \frac{dt}{i} C = dt\, T\, C$

$\frac{-dS}{di} = \Psi \frac{1}{i} R$

$\frac{\frac{-dS}{dt}}{\frac{-dS}{di}} = \frac{\frac{\Phi}{i} C}{\frac{\Psi}{i} R} = \frac{\Phi C}{\Psi R}$

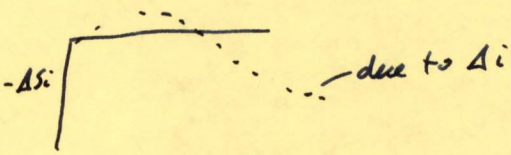
11. Adding to complexity, as C/R rises, S falls — but this is non-essential. Says who? Seems pretty essential.



12. Much trouble, no clear result.

Not only is $-\Delta(Si)_i < -\Delta(Si)_t$, it is ~~even possible~~ barely conceivable

for $-\Delta(Si)_i < 0$



i.e. $\Delta(Si) > 0$

That is when R_n come early, so PVR falls only a little, despite leverage.

Two ways of perceiving this.

A. $Si = a = \frac{R\,i}{1-e^{-im}} - \frac{C\,i}{1-e^{-im}}$

If $R = \frac{1-e^{-im}}{i}$ then first term = 1, and

$a = 1 - C\frac{i}{1-e^{-im}}$ a decrg. $f(i)$

or

$$\frac{d\ln(R-C)}{di} > -\left[\frac{1}{i} - \frac{m}{e^{im}-1}\right]$$

$$\frac{dR}{di} = \frac{1}{i+d}\left(m e^{-(i+d)m} - R\right)$$

$$\frac{\frac{dR}{di}}{R-C} = \frac{dR/di}{\frac{1-e^{-(i+d)m} - C(i+d)}{i+d}} = \frac{im}{e^{(i+d)m} - 1 - C(i+d)e^{(i+d)m}} - \frac{R}{(R-C)(i+d)}$$

So

$$\frac{m}{e^{(i+d)m}(1-C(i+d)) - 1} - \frac{R}{(R-C)(i+d)} > -\left[\frac{1}{i} - \frac{m}{e^{im}-1}\right]$$

Maybe someday I can simplify, if needed.

B. A simpler derivation is

$$a = (R-C) \times cf$$

$$\frac{da}{di} > 0 \text{ if } -\frac{d\ln(R-C)}{di} \lessgtr + \frac{d\ln cf}{di} = \frac{1}{i} - \frac{m}{e^{im}-1}$$

or $$-\frac{dR}{di} < (R-C)\left[\frac{1}{i} - \frac{m}{e^{im}-1}\right]$$

~~Not~~

13. $\frac{1}{i} - \frac{m}{e^{im}-1} > 0$ for all $m > 0$, and $= 0$ for $m = 0$

It is Δg $f(m)$, and $\to \frac{1}{i}$ when $m \to \infty$

$$\frac{1}{i} - \frac{m}{e^{im}-1} = \frac{e^{im} - (1 + mi)}{i(e^{im}-1)} = \frac{\text{Growth at comp. int.} - \text{growth at simple int.}}{i(e^{im}-1)}$$

→ 14. Higher $\frac{C}{R}$ mean R comes more from earlier years, and is less

~~Rent~~ sensitive to Δi. $\frac{-dR}{di}$ declines.

~~So $S \cdot i$ is more likely to rise when Δi~~

But $R - C$ also declines, rep's increased leverage.

It is inherently implausible that higher i should make rent rise. Probably the most this factor can do is to temper its fall.

~~Rent~~ suppose a parking lot, with decaying income. $S = \frac{a_0}{i+d}$

If rent $= a = S \cdot i = a_0 \frac{i}{i+d}$, then rent is an Δg $f(i)$

→ But when land appres. or depres., Si is ~~rent~~ the annual income, while a is not.

$Si = a_0 - Sd$

$a_0 = S(i+d)$ The cash flow covers int. plus depreen.

Also ~~it~~ when $C > 0$, decaying R_n means higher depreen. Check that.

$$S = \frac{R-C}{1-e^{-im}}$$

$$S_i = (R-C)\frac{i}{1-e^{-im}}$$

Now suppose all R comes at $\underline{0}$, same as C_0, so $-\frac{dR}{di} = 0$

Then $\frac{d(S_i)}{di} > 0$.

But it is impossible for all R to come at 0, or anything near it.

Bes. then:

a. C_0 is fully depred. & renewal is due

b. Land is also fully decayed. That lowers S_i

and lowers $\frac{d(S_i)}{di}$

Probably ? cd. prove, w. an effort, that $\frac{d(S_i)}{di}$ can ever > 0.

But it is not necessary, so drop it.

15. $\frac{d\,S \cdot i}{di} = S + i\frac{dS}{di}$

$d(S \cdot i) = S\,di + i\,dS$

$\ln(S \cdot i) = \ln S + \ln i$

$\frac{d \ln(Si)}{di} = \frac{\frac{dS}{di}}{S} + \frac{1}{i}$

$d \ln(Si) = \frac{dS}{S} + \frac{di}{i}$

In my case, all the (dS)s are neg. and $di > 0$.

Adding 2d term reduces the % drop, but does not change the rankings, because it is the same for all.

p. 70

$\frac{d(S \cdot i)}{di}$ is neg. so long as

$S\,di + i\,dS < 0$

$S\,di < -i\,dS$

$\frac{di}{i} < -\frac{dS}{S}$

That is so long as S drops by a greater % than i rises.

Which it does bec. of leverage.

16 - I have already shown where? that land income, $S \cdot i$, is more reduced by a tax than a Δi; and how the excess reduction increases with intensity, c/R.

$$S = \frac{R - c}{1 - e^{-im}}$$

$$-\frac{dS}{dt} = T(c)$$

$$-\frac{dS}{di} = T(R)$$ but also increases value of denom., which also reduces S.

The denom. reps. future cycles. Thus, $-\frac{dS}{di} > -\frac{dS}{dt}$

16 A. Now the ~~reactive~~ sensitivity of denom. to Δi ~~increases~~ decreases with m. ***

Digress to show ~~that~~ above sentence.

$$\frac{d}{di}\left(1 - e^{-im}\right) = -(-me^{-im}) = me^{-im}$$ An increasing and then decreasing $T(m)$. ~~A decreasing $T(m)$?~~

$$\frac{d \ln(1 - e^{-im})}{di} = \frac{me^{-im}}{1 - e^{-im}} = \frac{m}{e^{im} - 1}$$ A decreasing $T(m)$. (proved C-13)

~~I~~ (So the smaller m is the greater the % change in denom. $(1 - e^{-im})$ with i.

$$\frac{d}{dm}\left(me^{-im}\right) = m(-ie^{-im}) + e^{-im} = \frac{1 - mi}{e^{im}}$$

This is > 0 for $1 > mi$ or $m < \frac{1}{i}$

and < 0 for $1 < mi$ or $m > \frac{1}{i}$

So for small m, $(1 - e^{-im})$ increases with i.

And for large m, " " decreases " "

~~$\frac{d}{dm}\left(\frac{m}{e^{im}-1}\right)$~~

$$\frac{d}{dm}\left(\frac{m}{e^{im}-1}\right) = \frac{(e^{im}-1) - mie^{im}}{(e^{im}-1)^2}$$

~~$\frac{d}{dm}$~~ > 0 if

$$1 - e^{-im} > mi$$

$$\frac{1-e^{-im}}{i} > m$$

But $m > \frac{1-e^{-im}}{i}$

So deriv. is negative.

So the % change in $(1-e^{-im})$ with Δi gets smaller as m gets larger.

~~End digression to power sets~~

~~the Numerator~~

$$S = \frac{R-C}{1-e^{-im}}$$

Denom. rises with m and with i.

Its % change w. Δi decreases as m increases.

(Its $\frac{\%}{R}$ sensitivity to Δm also decreases as i increases)

When m is small, R gets less sensitive to Δi; but denom. gets more sensitive.

So if the shorter values of m do not nearly make S less sensitive to i. They make S_2 loom larger. Actually, we know they do make S less sensitive to Δi; this is just a compensatory factor, and fairly weak.

16, resumed.

As $\frac{C}{R}$ rises, S_i does not decline as much ~~as~~ from s_i as from Δt.

That is ~~the basic.~~ not proven, and probably not so. ~~Also~~ The fracting, may be the only case we have. and there is no fracting here.

N.G. a. Higher $\frac{C}{R}$ means R is less sensitive to s_i, because revs. come earlier. So $S_i = (R-C)\frac{i}{1-e^{-in}}$ is less reduced by s_i.

The factor of $C\frac{i}{1-e^{-in}}$ is unaffected by when revs. come.

But $R\frac{i}{1-e^{-in}}$ is affected.

If $R = \frac{1-e^{-in}}{i}$, then $R\frac{i}{1-e^{-in}} = 1$, unaffected by s_i

But that is a nondecaying cash flow. If C is high, c.f. decays,

so $R\frac{i}{1-e^{-in}}$ rises with i.

(Of course, a parallel effect is at work with tax impact. ~~So this doesn't much distinguish it from s_i~~)

~~But that factor is independent of $\frac{C}{R}$, pretty much.~~

N.I. b. Higher $\frac{C}{R}$ means decaying c.f., but also, probly a <u>longer n</u>.

<u>Longer n</u> makes denom. less sensitive to s_i, ~~so~~ wise

So as $\frac{C}{R}$ rises, and n lengthens, S is reduced less by s_i, and S_i is reduced less.

Looking at that in terms of crf, when m is long, crf is all interest, and Δ of num. w. Δi is little reduced by small Δ of denom. When m is short, denom. → i, crf → 1, and less increased by Δi. When m is short, rent is little reduced by Δi. But this is equally true of t.

Looking at it in terms of salvage of future site, when m is long, that is a small compared to other factors and has little effect on present value. Even tho e^{-im} is more sensitive to Δi when m is long, $1 - e^{-im}$ is less sensitive.

---

17. When $S = \frac{a}{i}$, that is not becs. $C = 0$, but becs. $m = 1$

Thus a land use using much C_0, but recovering it quickly, is akin to the parking lot. PVR is little reduced by Δi; and S is little reduced by a tax on C. But Δi does reduce S a lot, bec. $(1 - e^{-im}) = \frac{i}{1+i} \doteq i$

And Si is only slightly reduced, if at all, by Δi. Likewise, it is little reduced by Δt.

However, the ratio of the effects, small as they are, still depends on $\frac{C}{R}$. ? ~~And should be easy to describe~~. See 41, p. 45.

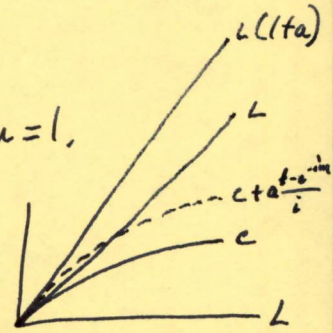
It seems not: both are based on C.

If we look just at $S = \frac{R-C}{1-e^{-in}}$, we'd say short life makes ~~it~~ num. less sensitive to int. or taxes — & that's where leverage is.

and makes denom. more sensitive to ~~taxes~~ int., where no leverage is, i.e. indep. of $\frac{C}{R}$.

Thus at short m si retains its power to lower S, but ~~t~~ t loses its power. Just like parking lot. ~~So si is more neutral between long m & short, t is harder on long m.~~

~~See p. 41, ## 42, confirming.~~



18. Counter-dominants

Hi $\frac{C}{R}$ means long life, why? C can = R when m = 1. means slow depreciation.

In graph, $\frac{c}{i}$ ~~falls with life~~ $\frac{c}{i} = \frac{C}{R}$ is const. with m. See 84, p. 90

But Hi C/R means faster decay of R_n, because R_n includes investment.

Which prevails? See p. 20.

But so what? Even if C = R, can have m = 1. I.e. if R_n decays fast, that brings future gen. closer.

17. (resumed).

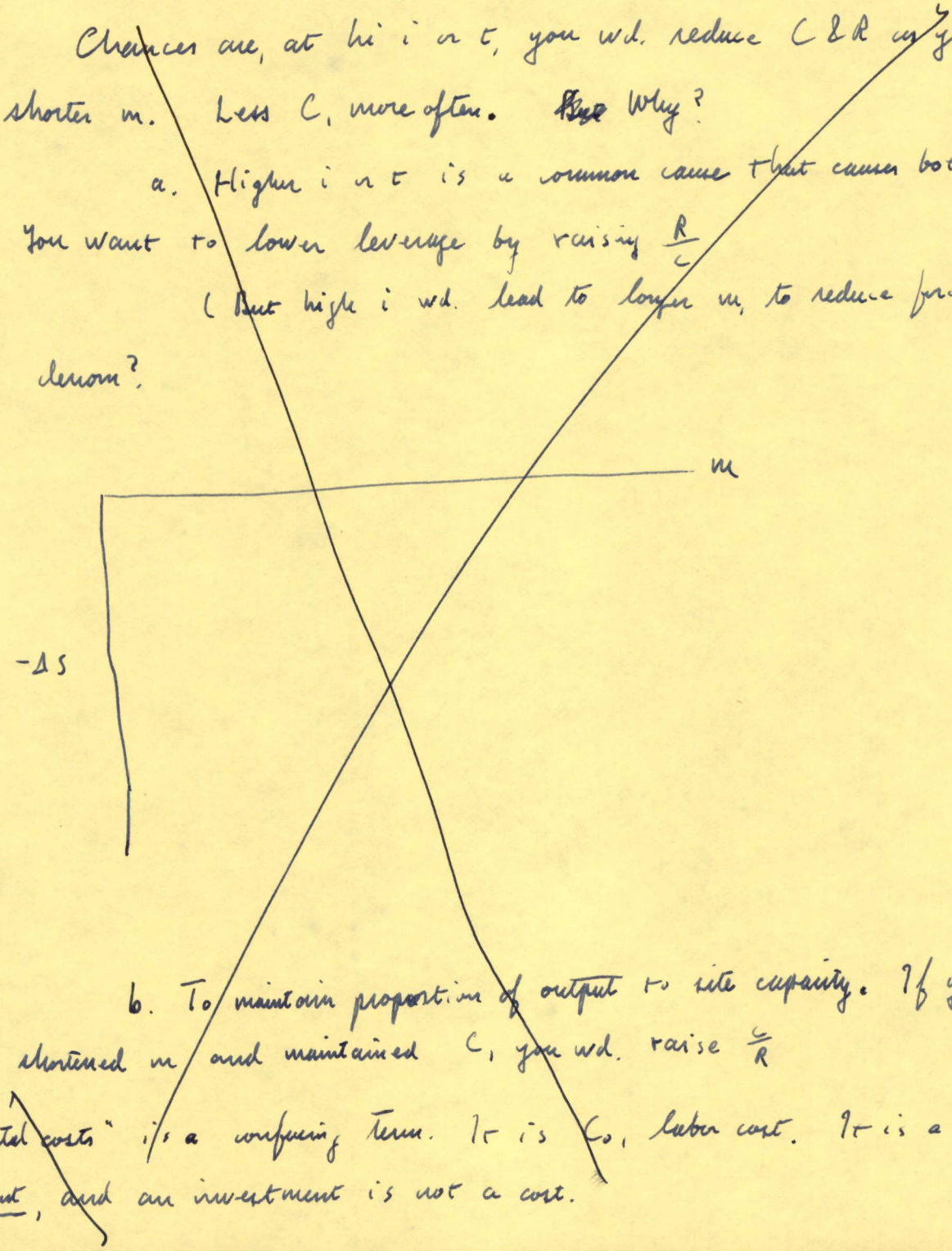
At low m, capital is not a large input compared to labor. So the net cost of capital — interest — loses force. The leverage of R-C is still there, but ~~capital~~ costs of capital‡ — interest & propty. tax — don't dig into the R-C gap so deep.

Chances are, at hi i or t, you wd. reduce C & R as you go to shorter m. Less C, more often. ~~But~~ Why?

a. Higher i or t is a common cause that causes both

You want to lower leverage by raising $\frac{R}{C}$

(But high i wd. lead to longer m, to reduce force of i in denom?



b. To maintain proportion of output to site capacity. If you shortened m and maintained C, you wd. raise $\frac{C}{R}$

‡ "capital costs" is a confusing term. It is C_0, labor cost. It is a capl. investment, and an investment is not a cost.

$$S = \frac{R-C}{1-e^{-im}}$$

The denom. is a minor compensating factor, for m over say 20.

? But as m gets short, it brings to an end the ~~gains possibility~~ benefit of responding to higher i by lowering m. After that, you must concentrate on lowering C/R.

With Δt, more of response can come from shortening m.

$$\frac{d}{di}\frac{(R-C)}{1-e^{-im}} = \frac{(1-e^{-im})\frac{dR}{di} - (R-C)(--me^{-im})}{(1-e^{-im})^2}$$

$$= \frac{\frac{dR}{di} - (R-C)\frac{m}{e^{im}-1}}{1-e^{-im}}$$

$$\frac{d}{di}\ln\frac{R-C}{1-e^{-im}} = \frac{\frac{dR}{di}}{R-C} - \frac{m}{e^{im}-1}$$

The 2d term is large, even up to $m = 60$, $\frac{60}{63} = .95$ or so.

But ~~Watch~~ di ~~Part~~ cuts it way down.

$$d\ln\frac{R-C}{1-e^{-im}} = \frac{dR}{R-C} - di\,\frac{m}{e^{im}-1}$$

For Δt, $-\frac{\Delta S}{S}$ is just $\frac{dR}{R-C}$, the first term, as denom. remains same.

~~Puts it all together.~~

20. The general idea is that higher C/R removes the difference between effects of τ & Δi, by:

(a) Making them same value

(b) Making their time-derivatives more alike --- C gets longer & R gets shorter. (?)

(c) And now a 3rd factor. $1 - e^{-i\omega}$ becomes less sensitive to Δi as ω gets longer. This denom. is another factor making S more sensitive to Δi than to τ. Δi reduces R & also raises denom. But with longer ω, the difference becomes smaller.

21. $(1+i)(1+d) = 1+i+d+id > 1+i+d$

But

$e^i e^d = e^{i+d}$ The small stuff drops out.

$(1-d)(1+d) = 1-d^2 < 1$

$e^{-d} e^d = 1$

Again, small stuff drops out.

So a decaying series, $\frac{1}{1+i} + \frac{1-d}{(1+i)^2} + \cdots$ can be

summed up as

$$\Sigma = \frac{1 - e^{-(i+d)m}}{i}$$

Because $e^{-d} = \frac{1}{e^d}$

and $\frac{1}{e^d e^i} = \frac{1}{e^{d+i}}$

22. Let R come from a decaying fn., and see if $\frac{dR}{di}$ can be 0 or +.

$$R = \frac{1 - e^{-(i+d)m}}{i+d}$$

$$-\frac{dR}{di} = \frac{R}{i+d} - \frac{m e^{-(i+d)m}}{i+d}$$

$$\frac{-\frac{dR}{di}}{R} = \frac{1}{i+d} - \frac{m}{e^{(i+d)m} - 1}$$

This kind of depreciation is exactly like raising $\underline{i}$. ~~12~~

Or levying $\underline{I}$.

$$\frac{da}{di} > 0 \quad \text{if}$$

$$\frac{1}{i+d} - \frac{m}{e^{(i+d)m}-1} < \frac{R-C}{R}\left[\frac{1}{i} - \frac{m}{e^{im}-1}\right]$$

or

$$\frac{\frac{1}{i+d} - \frac{m}{e^{(i+d)m}-1}}{\frac{1}{i} - \frac{m}{e^{im}-1}} < \frac{R-C}{R} = 1 - \frac{C}{R}$$

If $m = \infty$

$$\frac{i}{i+d} < \frac{R-C}{R}$$ ~~but $d > 0$~~

Now $m = \infty$ could mean $d = 0$, in which case $\frac{i}{i+d} = 1$

But you could have long life despite depreciation (e.g. something that decays into a usable heap of stable material, a land fill say.)

✓ In general seems unlikely. But main point is, PV $R \cdot i$ can rise some when $\frac{C}{R}$ high & C decays.

Try $m = 10$

$d = .93$

$i = .07$

$R = .99$

$C = .50$

Δi drives you toward faster recovery of site; how? ~~it~~ doesn't.

For given C, faster recovery of capt. means higher R, lower C/R. t favors that. Δi differs by Δg the denom., and making it less sensitive to m. So you can't benefit as much by Δm. Checks that

23. $S \cdot i$ is const. if $\underline{a}$ is not a $f(i)$

a is not $f(i)$ if $\frac{-dS}{S} = \frac{di}{i}$ or $\frac{+\frac{dS}{di}}{S} = -\frac{1}{i}$

$$S = \frac{R-C}{1-e^{-im}}$$

$$\ln S = \ln(R-C) - \ln(1-e^{-im})$$

$$\frac{d \ln S}{di} = \frac{\frac{dR}{di}}{R-C} - \frac{--me^{-im}}{1-e^{-im}}$$

Si is const. if above $= \frac{-1}{i}$

or $\frac{dR}{di} = (R-C)\left[\frac{me^{-im}}{1-e^{-im}} - \frac{1}{i}\right]$ We've been here before.

24. $\Delta(Si) = 0$ when

$Si = (S+\Delta S)(i+\Delta i)$

or

$\frac{S}{S+\Delta S} = \frac{i+\Delta i}{i}$

25. t adds to costs.

Δi " " ", but also discounts future rents. Since it discounts all the same, it won't change ranking [— unless maybe the future rents have different time patterns. See p. 49, #44, this comes to nought.]

26. If a is const., then $\frac{a}{i}$ falls with Δi just enough so $\frac{a}{i} \cdot i$ remains const.

So if a declines, $\frac{a}{i} \cdot i$ will decline. Hardly more than a truism.

27. $S \stackrel{?}{=} \frac{Rm}{e^{im}-1} - \frac{Ce^{im}}{e^{(i+t)m}-1} = \frac{Rm}{e^{im}-1} - \frac{C}{e^{tm}-e^{-im}}$ No

$S \stackrel{?}{=} \frac{Rm - Ce^{(i+t)m}}{e^{im}-1} = \frac{Rm}{e^{im}-1} - \frac{C}{e^{-tm}-e^{-(i+t)m}}$ ~~maybe~~ No

$S \stackrel{?}{=} \frac{Rm}{e^{im}-1} - \frac{Ce^{(i+t)m}}{e^{(i+t)m}-1} = \frac{Rm}{e^{im}-1} - \frac{C}{1-e^{-(i+t)m}}$ No

But it's ~~probably~~ complexer. Res. tax base is more than $Ce^{(i+t)m}$

28. ~~Return to this one?~~

$$a \overset{?}{=} R \cdot crf - C \frac{i+t}{1-e^{-(i+t)n}} \qquad (28)$$

Probably wrong. Eq. 2d term makes tax burden independent of how fast C depreciates if so,

$$S = \frac{a}{i} = \frac{R}{1-e^{-in}} - C \frac{i+t}{i} \frac{1}{1-e^{-(i+t)n}}$$

What if C appreciates?

This is treating bldg. tax like an ~~added~~ addition to interest cost on capl., but not on land. That is close to being correct.

Then $\dfrac{-da/dt}{-da/di} = \dfrac{\xi}{\varepsilon} \dfrac{C}{R}$

You'd only have to work out 2 details:

a. Effect of depreen. paths on precision of $C \frac{i+t}{1-e^{-(i+t)n}}$ see #30.

b. $\frac{\xi}{\varepsilon}$

29. Again if 28 is ok, adds new wrinkles.

$-\dfrac{dR}{di}$ can be reduced by early payout, (a fast decay path)

? But $\dfrac{d}{di} \dfrac{i+t}{1-e^{-(i+t)n}}$ not reduced by faster decay path.

Thus faster decay is a way to avoid impact of Δi, but not of t.

$$V_0 = y\left[\frac{1}{(1+i)(1+t)} + \frac{1}{(1+i)^2(1+t)^2} + \cdots\right]$$

$$V_0 = y\,\frac{1-e^{-(i+t)n}}{i+t}$$

Here, y is a level instalment. It covers [int + tax + depreciation] on V_0. Int. & tax components decline each year, as base declines. Depreciation rises each yr.

If income decays,

$$V_0 = z\left[\frac{1}{(1+i)(1+t)} + \frac{(1-d)}{(1+i)(1+t)} + \cdots\right]$$

$$V_0 = z\,\frac{1-e^{-(i+t+d)n}}{i+t+d}$$

Here, z is the first term of a series that declines in a known pattern.

$$z_n = z_0(1-d)^n$$

It too covers int., taxes, & depreciation. Depreciation ~~declines~~ rises as much each year, and may decline.

~~Do~~ Get the level equivalent having same PV, and deduct that from gross levelled revs. to get levelled rent.

But PV is merely V_0, or C_0. Yet if C_0 depreciates faster, the int. & tax burden should be less.

As to int. burden, faster recovery of C_0 shows up in earlier R_n

As to tax burden, earlier R_n only reduce it if asset value falls. Assume it does, in general.

? Then earlier R_n gain you i% a year — but not ~~i~~ $(i+t)$%.

The formula

$$a = \frac{R\,i}{1-e^{-in}} - C_0 \frac{i+\tau}{1-e^{-(i+t)n}}$$

✓ overstates tax burden if C depreciates fast, without overstating benefit of early R to compensate.

✓ So this approach is hazardous.

It even makes one wonder about c+f. as statement of recovery + int. burden, bec. here int. is based on undepred. base, depreciated at s.f. rate. But answer is that faster depreciation raises R. And this in turn explains mystery of how R·c+f can increase w. i. It is a surrogate for decline of int. burden. But this can lead to error re Δt. when R is recovered very fast.

✓ I.e., instead of a drop in C·c+f, you get a rise in R·c+f.

31. Effect of a_i is to make you want revs. early, but not care about deprecn. of bldg. value.

Effect of t is nil on when revs come.

32. Annual cost of servicing C_0 is unaffected (~~the~~ in my eqn.) by how fast C_0 depreciates. The benefit shows up in section R_n, hence higher PVR. You discount the R_n at i, not $(i+t)$, because they are not the tax base – they are > tax base.

Taxes must be related to C, and deprecn. allowed.

Ways of doing:

a. a separate entry, based on deprecn. sched. (keyed to R_n path)

b. Discount $(R_n - a)$ at $i+t$. Since $a = f(R, t)$ this gets into a simul. eqn. & possibly a neat solution.

To 48, p. 53 ~~$S = \frac{PVR - C}{1 - e^{-im}} - \left[\sum_1^m R_n e^{-(i+t)n} \; a \frac{1 - e^{-(i+t)m}}{i+t}\right]$~~

$$(1 - e^{-im})S = -C_0 + \sum_1^m (R_n - a_n) e^{-(i+t)n} + * \sum_1^m a_n e^{-in}$$

If a ~~const~~ level

$$= -C + \sum_1^m R_n e^{-(i+t)n} - a\frac{1 - e^{-(i+t)m}}{i+t} + a\frac{1 - e^{-im}}{i}$$

$$0 = -C + \sum R_n e^{-(i+t)n} - a\,\frac{1-e^{-(i+t)m}}{i+t} \qquad +0$$

$$a = \frac{i+t}{1-e^{-(i+t)m}}\left[\sum R_n e^{-(i+t)n} - C\right]$$

$$S = \frac{a}{i}$$

Very neat, all right.

Now I have site value after bldg taxes, with bldg depreciation perfectly keyed to parts of R_n.

In time I will modify it for growth of a.

33. Now, compare 32 with a', rent following $\Delta i = j$

$$S' = \frac{-C_0 + \sum_{1}^{m} R_n e^{-(i+j)n}}{1-e^{-(i+j)m}} \qquad a' = (i+j)\,\frac{\sum R_n e^{-(i+j)n} - C_0}{1-e^{-(i+j)m}}$$

Let $j = t$

$S' < S$

$a' = a$

$$\frac{S'}{S} = \frac{1}{\frac{i+t}{i}} = \frac{i}{i+t}$$

But how can everything drop out?

⇒ We know that $S' < S$ when $C = 0$, and $S' \to S$ when $C = R$, so C/R should appear.

Their ratio changes when i changes? Seems arbitrary, bec. t in Δi can be any small amt.

But their increases are always $= 1$

$$\frac{S'(i+j)}{S \cdot i} = \frac{i}{i+t}\,\frac{i+j}{i} = 1$$

Impossible!

~~Hybrid~~

Define S_i original value

$S_t \equiv S$ after tax, t

$S_i \equiv S$ after $\Delta i = t$

$S_t > S_i$ unless $S = 0$ — then both are negative anyway.

$$\frac{S_i}{S_t} \overset{?}{=} \frac{i}{i+t}$$

But S_i becomes neg. while S_t still positive? No, as PVR → c ~~[illegible]~~ the effects of Δi & t become the same. But those are only ungd. effects.

This seems to say that S_i & S_t will reach zero together.

But I am sure S_i reaches zero first. — see p. 4.

So there is something wrong here. Check it.

(This peters out) (on p. 38)

35. Can Δi wipe out S at a lower rate than Δt?

As PVR → C, signif. effects of Δi and Δt become the same (except for denom., $(1-e^{-in})$

$$(1-e^{-in})S = \Sigma(R_n e^{-in}) - C_0$$

But we're asking about:

a. a big increment. A $t = .04$ compared to $\Delta i = .04$

b. Comparisons of effects too small to wipe out S.

Still, it's not true that effects are the same when $C = R$.

Because Δi lowers R below C, so then further effects of Δi are less than effects of Δt.

More generally, when $R > C$ and remains so, Δi brings R closer to C and reduces difference of further Δi. Δt also brings them closer. Δt also reduces R, only not by so much.

Spell that out.
Under Δt,

$$R_t \text{ or } PVR_t = \sum_1^m (R_n - a_n) e^{-(i+t)n} + \sum_1^m a_n e^{-in}$$

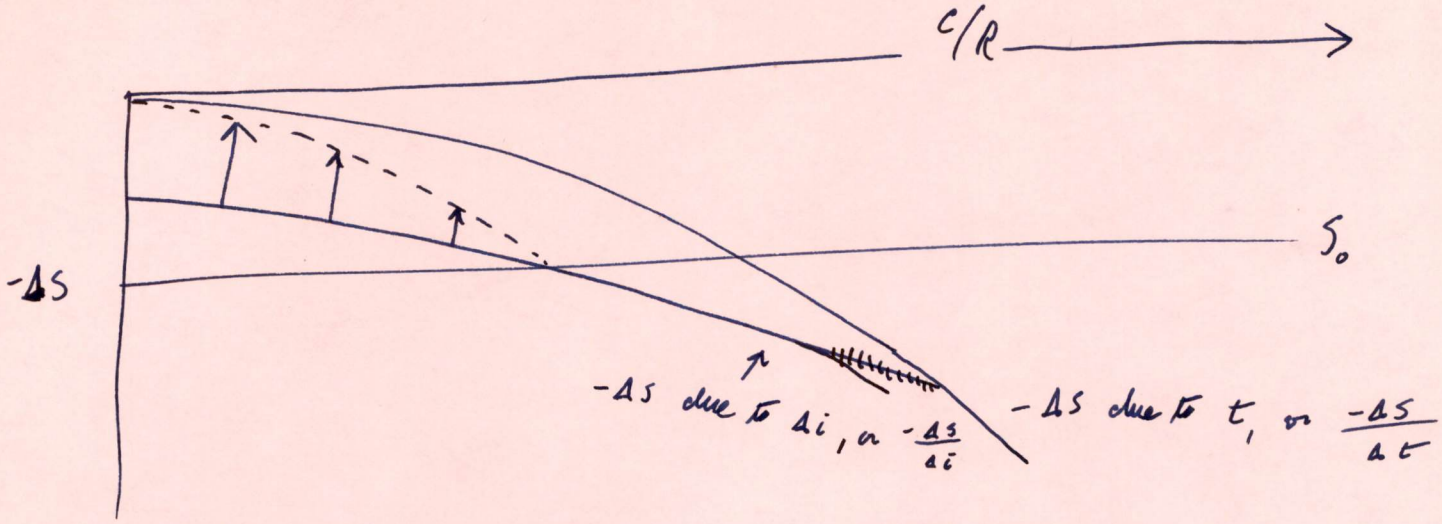
$$= \Sigma(R_n e^{-(i+t)n}) - a\left[\frac{1-e^{-(i+t)n}}{i+t} - \frac{1-e^{-in}}{i}\right]$$

$$= \Sigma(R_n e^{-(i+t)n}) - a\left[\frac{i - ie^{-(i+t)n} - i - t + (i+t)e^{-in}}{i(i+t)}\right]$$

$$PVR_t = \sum\left(R_n e^{-(i+t)n}\right) + \frac{a e^{-(i+t)m}}{i+t} + \frac{a t}{i(i+t)} - \frac{a e^{-im}}{i}$$

Under $i = j = t$

$PVR_i = \sum R_n e^{-(i+j)m} < PVR_t$ if $a > 0$



C/R never reaches 1. Assume a given S_0, and an array of C/R that yield it. (Why do I show curves meeting? $-\Delta S_i > -\Delta S_t$ always?) (No, not when S_i becomes negative. Since Δi & Δt are large, they make S cross the equator.)

↑ show effect of Δi on income. When $S_0 - \Delta S = 0$, Δi has no effect on income.

$\frac{C}{R} = \frac{C}{C+S} = \frac{C}{C+K}$. If $S = K = S_0$, then R rises as C/R rises; and $\frac{-\Delta S}{\Delta i} > -\frac{\Delta S}{\Delta t}$ because $R > C$

If $C = 0$, Δi will never make $S = 0$, no matter how large the Δi. Because base falls, & $S = \frac{a}{i}$

This may be heart of difference of t & Δi .... the capn. effect.

If $C > 0$, Δi & t both lower $\underline{a}$. Δi lowers $\underline{a}$ more? BVR can never be ≤ 0, a kind of capn. effect, so larger discrete jumps in i have progressively less increased effect on S, while base of Δt remains C.

Say there is one cycle, & $S = \frac{1}{i} - C(1+T) = \frac{1}{i} - C\left(1+\frac{t}{i}\right)$

$= \frac{1}{i}(1 - Ct) - C$

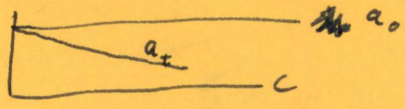
$\text{Rent} = 1 - Ct - Ci = 1 - C(t+i)$

This has t & i reduce rent by same amount in same way.

36. The question is, will s_i and t shift the ranks of bidders differently?

The contradiction is, s_i and t lower land <u>rent</u> the same; but they lower land <u>value</u> differently.

At given S_0, both lower rent more for higher C ($R = C/(C+s)(1-e^{-it})$) (hence simply higher C)



But s_i also lowers land <u>value</u> by an added factor, due to higher cap. rate (or discount of future rents). This added factor is nil if $C = R_1$, or $C(1+T) = R_1$, or $C = R(1-T)$. But if rent remains, s_i lowers its P.V.

However, it lowers it by a constant proportion, so the highest rent is capd. into the highest bid.

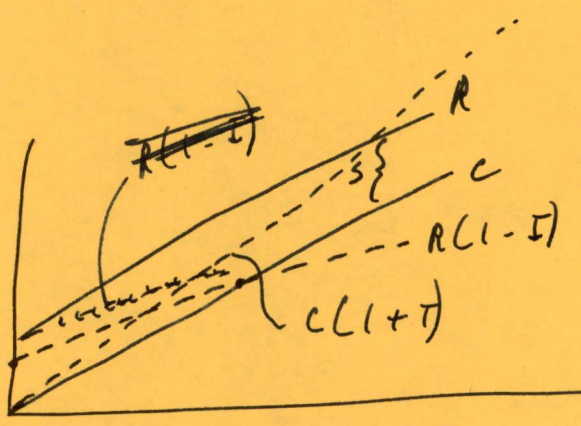
Thus the value of the parking lot declines. But if its rent is higher than another use, it is still higher. The other rent is also capd. at a higher rate.

I see no way out of this.

---

(36) is ok. Not much else in here.

as C/R moves from 1 to 0, the Δi necessary to reduce S by as much as a given t moves from t to 0.



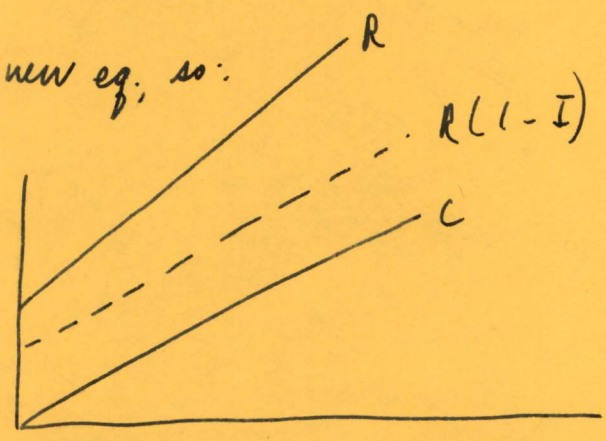
Warning - watch the unit

Before, all have same S_0

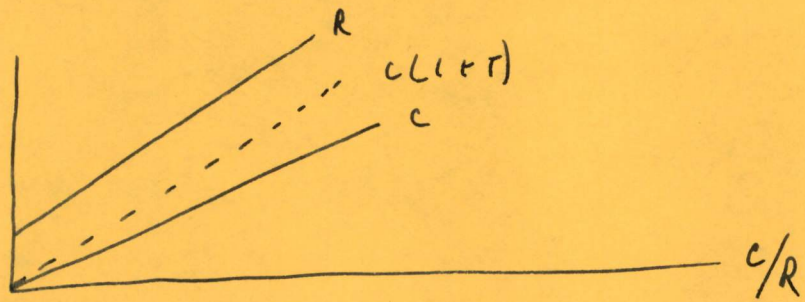
after, all the lower C/R look better.

But that can't mean C=0 takes over every site. Demand for low-C uses is saturated, & for hi-C uses frustrated.

a new eq., so:



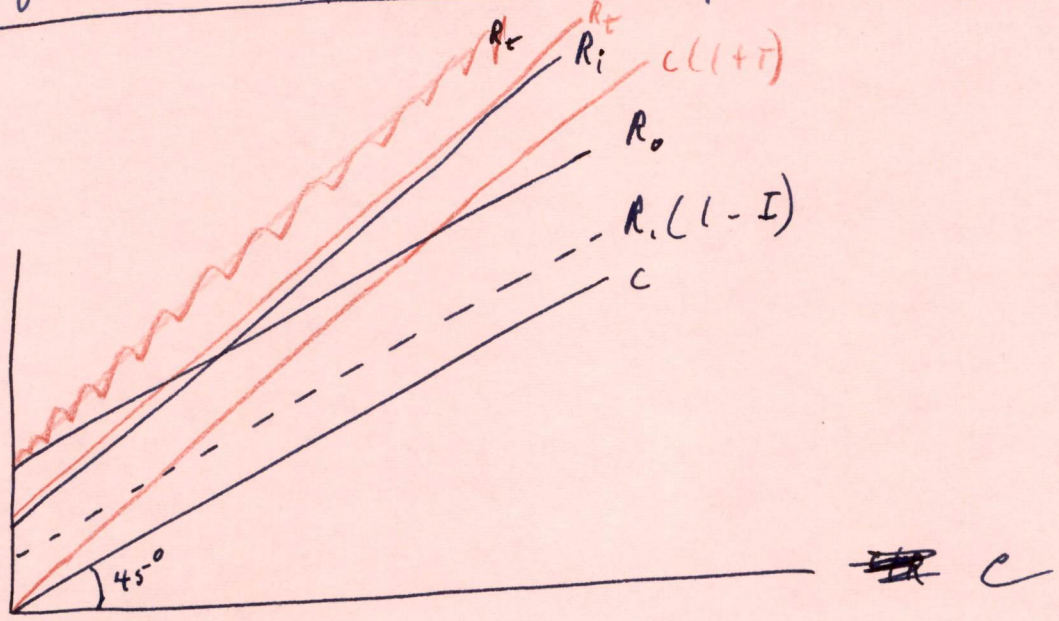
or



If it's $C(1+T)$, the $C=0$ user needs it yield enuy more ue.

to maintain same S_i Others must.

If it's $R(1-I)$, all user must yield more ue.

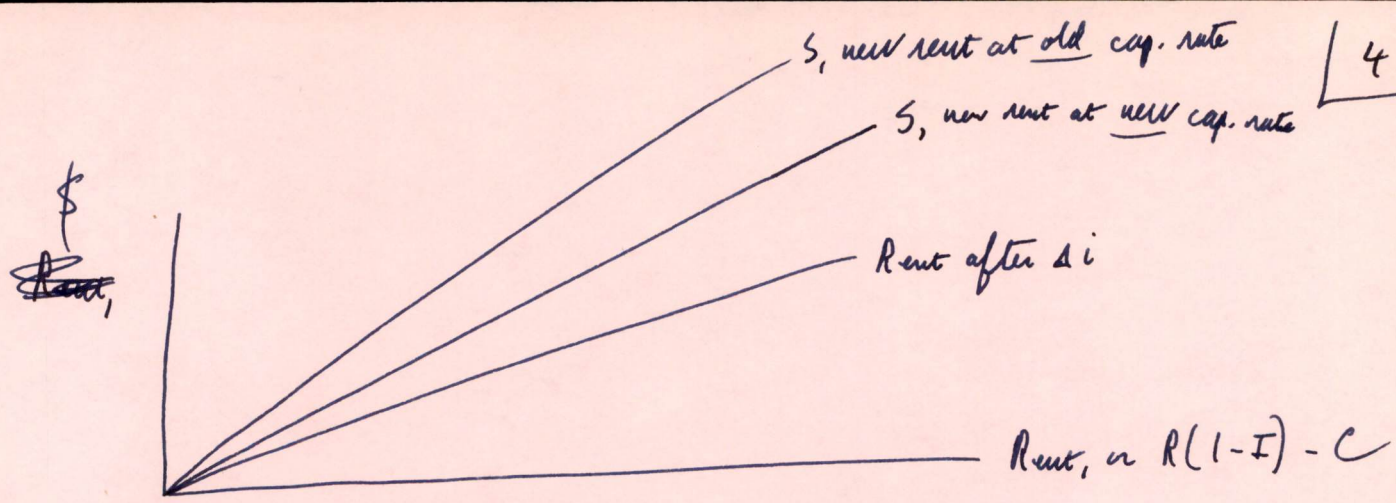


True, but I don't see this beau on difference of Δi & Δt.

~~Low C ones~~

Users of O C have less demand per unit bes. there are more of them.

37.



On above, the new cap. rate fails to change ordering. (But C doesn't appear on this graph)

Agenda

1. Maybe the zero-point is different, for Δi and t.

2. To find rent, do you first find S, then mult. by i?
Or do you deduct yrly. cap. costs from cash flow?
It might matter. Loose end.

---

1. $S = R - C$

$S = 0$ when $R(1-T) = C$

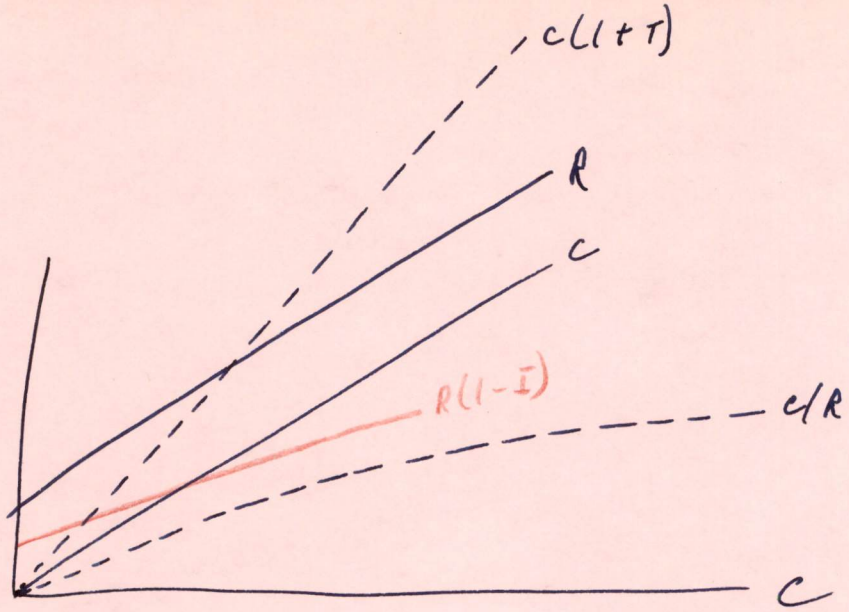
or $R = C(1+T)$

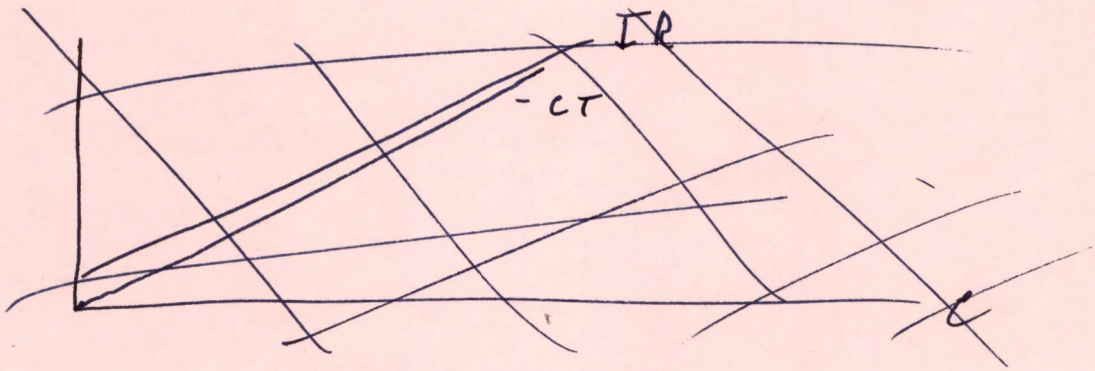
If $I = T$, the 0-point comes at a lower value of Δi than of t, bec. $R > C$.

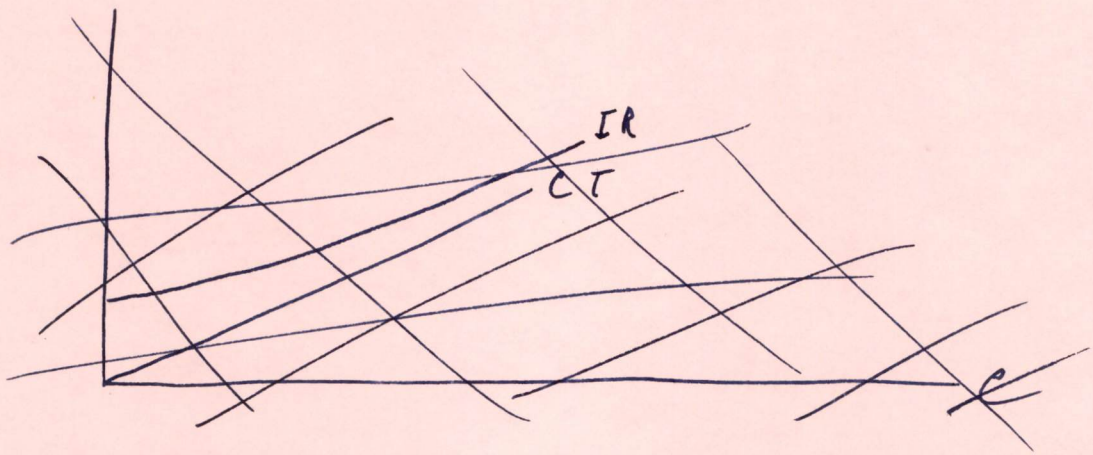
Loose end

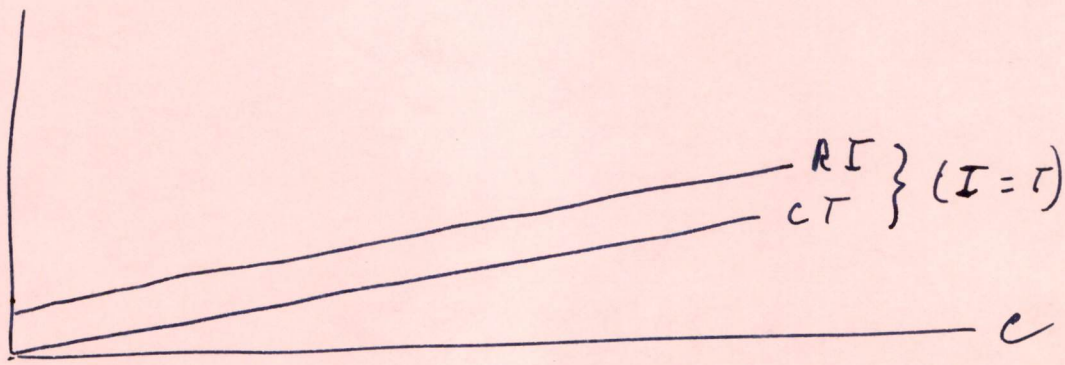
~~A.~~

38.









$CT < RI$, but $\frac{CT}{RI} = C/R$ and $\to 1$ as $C/R \to 1$. $(R - C = K)$

$\frac{-\Delta S}{\Delta t} = CT$ and $\frac{-\Delta S}{\Delta i} = RI$

Let $C/R = \Theta$

$C = R\Theta$

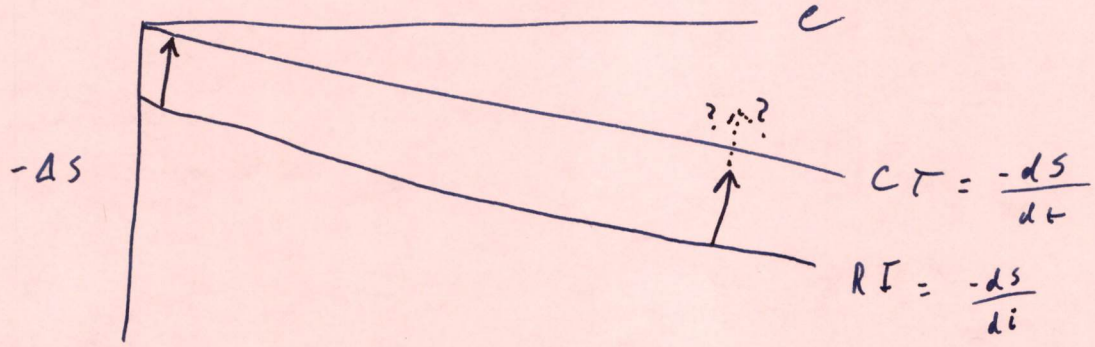
$CT = RI\Theta$

$$\frac{-\Delta S/\Delta t}{S} = \frac{CT}{S} = \frac{RI\Theta}{S} = \Theta\left(\frac{-\Delta S/\Delta i}{S}\right)$$

$$\frac{-\Delta S/\Delta t}{-\Delta S/\Delta i} = \Theta = C/R$$

OK, but repetitive and inconclusive.

39.



40.

If $t > 0$, c+f. is payment yielding recov. int. & propty. taxes.

Chances are, c+f. $= \dfrac{i+t}{1-e^{-(i+t)m}}$

If $PV = \dfrac{1-e^{-(i+t)m}}{i+t}$

Then $a = PV \dfrac{i+t}{1-e^{-(i+t)m}}$ so I guess that is it.

41. If $m = 1$

$a \stackrel{?}{=} R - C(1+i)$

or

$a \stackrel{?}{=} R(1+i)^{-1} - C$

Makes quite a difference, which end of the year you choose!

But only if $S \to 0$? If S is large, $R >> C$, still matters.

So, what is $\underline{a}$?

$$S = \frac{a}{1+i} + \frac{a}{(1+i)^2} + \cdots + \frac{a}{(1+i)^n} = \frac{a}{1+i} \frac{1-(1+i)^{-n}}{1-\frac{1}{1+i}} = a \frac{1-0}{i} = \frac{a}{i}$$

So when $S = \frac{a}{i}$, a is rent received at ~~year~~ end of one year.

Just before payment due, $S = \frac{a}{i} + a$

Valued at end of year: If $a = R - C(1+i)$

Then $S = \frac{a}{i} = \frac{R - C(1+i)}{i}$.

Valued at first of year: If $a = R(1+i)^{-1} - C$

Then $S = \frac{a}{i} + a = \frac{R(1+i)^{-1} - C}{i} + R(1+i)^{-1} - C$

or $S = a\left(\frac{1}{i} + 1\right) = a\left(\frac{1+i}{i}\right) = \frac{a}{i}(1+i) = \frac{R(1+i)^{-1} - C}{i}(1+i)$

$= \frac{R - C(1+i)}{i}$

So these are consistent

~~But if Δi, a ~~as first~~ lowers value of rent~~

Assume rent received at end of yr.; C_0 sunk on first of yr.

Δi lowers value of a at first of yr. by more than t does. (But tenant can pay more if he pays at end of year.)

Suppose cap. turns ~~qtly~~ daily but stays at one level. Then rent is Rev. less ~~loss~~ $C(i+t)$.

Discounting the rent from end of yr. depends on its being deferred to end of yr. But if it comes steadily, no difference of i & t.

See p. 53 for neat solution

42.

If $n = 1$

$$S = \frac{R - C}{1 - e^{-i}} = \frac{R - C}{i}$$

Δi lowers S bec. denom. is sensitive to Δi.
t hardly affects S, even though R & C maybe high.

If $n = \infty$

$S = R - C$

Δi lowers S, based on R
t " " , but not as much, bec. based on C

So effect of t is always less, but increases relative to Δi as n is longer.

Thus t is more biased against long n than Δi is. (as far as effect on S goes. Effect on rent ~~may be~~ is the same)

~~Thus~~

Longer n makes effects of i & t more alike, just as higher C/R does. But unlike high C/R, they never become identical, even in the extreme of $n = \infty$

43.

$\frac{-dS}{di} > \frac{-dS}{dt}$ for 2 reasons.

a. $R > C$

b. R comes from further in future than tax base.

When $m = 1$, (b) is obliterated bec. tax base is constantly renewed.

When $m = \infty$, C doesn't depres., and (b) is obscured.

For $m = 35$ or so, as cap. depres, current cap/land ratio falls, so R declines more from a Δi than $C(1+\tau)$ rises from a rise of τ, even if they are equal.

Then you can escape from τ by fast recovery of cap. coupled w. slow recovery of site. But you can't escape Δi so well that way, w.r.t. effect on S.

But this adds little to what we already know about escaping by lowering C/R.

44. cf. 2 uses.

1. $S_1 = \frac{1}{i - g}$

2. $S_2 = \frac{1.1}{i}$

$S_1 > S_2$ when

$$\frac{1}{i-g} > \frac{1.1}{i}$$

$$i > 1.1i - 1.1g$$

$$1.1g > .1i$$

$$\frac{1.1g}{.1} > i$$

$$11g > i$$

as i rises, S_2 ~~rises faster~~ falls slower than S_1 and takes over.

So here is a case where Δi changes bids in a way that t doesn't. But this is premised on annual rent, $\therefore$ annual flexibility of land allocn.

Not $\therefore$ very relevant? Yes, if modified.

It's relevant ~~when~~ with timber, but then I have not really treated the rent as deferred. A is deferred, but I define rent as what you wd. pay a landowner if the firm paid him annually

There is no deferred rent.

Rent is a flow, and not deferrable by nature. It is the annual value of having the land on disposal. Wages and rent are not deferrable. Consumption, liquidation, realizn. are deferrable. Capl. is stored labor & rent.

45. a. si works on a higher base than t, when it comes to affecting ~~S~~ S = R - C.

(But same base, C, wrt affecting rent.)

When R(1-I) = C, further si have same base as t. But up to that point, si has greater effect, on S.

✓ b. If m = 1, it is same, almost, as if C = 0, w.r.t. Capital intensity.

c. If there is a different effect, of si & t, it may depend on the hardness of C.

This wd. be observed for small si, where it makes no diff. ~~in~~ Using calculus may be a trap.

For large si & t, si erodes its base and t does not. (But this won't change their

effects are sent)

46. $a = PVR \frac{i}{1-e^{-im}} - C \frac{i+t}{1-e^{-(i+t)m}}$

If $m = \infty$

$a = R \cdot i - C(i+t)$

and $R = \frac{1}{i}$, so

$a = b - C(i+t)$ and i & t have same effects.

If $m = m_1$ and b const,

$PVR = b \frac{1-e^{-im}}{i}$

and

$a = b \frac{1-e^{-im}}{i} \frac{i}{1-e^{-im}} - C \frac{i+t}{1-e^{-(i+t)m}}$

But if b decays, then R is less sensitive to Δi than Δt, and 1st term becomes an Δ g $f(i)$

$a = b \frac{1-e^{-(i+d)m}}{i+d} \frac{i}{1-e^{-im}} - C \frac{i+t}{1-e^{-(i+t)m}}$

Now, Δi reduces a by less than Δt. The trouble with

this is that the 2d term is premised on a different depreen. pattern of C_1, a slow sinking fund pattern.

41. But why can you apply crf regardless of actual depreen. to int., ~~& not to tax~~ but say taxes must depend on actual depreen? Is this a differenc?

crf is a level payment covering int. & recov. of C_0. If C_0 is actually recovered faster, that shows up in rev. side as higher R.

But re taxes, if d is very high, & most comes back in year 1, then tax burden is small, while crf stays same. So ? question mixing ~~on~~ is it & like that.

✓

I don't quite understand why not. but I don't think it is correct.

~~I~~ You must take PV of taxes, then level them w. crf, I guess - - -

48. Codify #32

$S_t \equiv$ S after tax on capl.

$$S_t\left(1-e^{-i\infty}\right) = -C_0 + \sum_1^m\left[\left(R_n - a_n\right)e^{-(i+t)n}\right] + \sum_1^m\left[a_n e^{-in}\right] \qquad (1)$$

$\underline{a}$ is level, so last term is $a\,\dfrac{1-e^{-i\infty}}{i} = S_t\left(1-e^{-i\infty}\right)$

cancelling, and leaving:

$$C = \sum_1^m\left(R_n e^{-(i+t)n}\right) - a\,\frac{1-e^{-(i+t)m}}{i+t} \qquad (2)$$

$$a_t = \frac{i+t}{1-e^{-(i+t)m}}\left[\sum_1^m\left(R_n e^{-(i+t)n}\right) - C\right] \qquad (3)$$

See p. 94, #96. This expression keys bldg. depreca. to patch of R_n. (& three approach) (p. 100)

~~But does it apply to trees? Then, $R_n = be^{in} + ae^{in}$ and I wonder about dis. R by t.~~

(It would need to be modified if $\underline{a}$ not level, but save that until sure it's right.) (You wd. sub. $a(1+g)^n$ or ae^{ng} for $\underline{a}$)

Now note that (3) is the same rent as if t were Δi.

That is, base rent is:

$a = c+f\left[PVR - C_0\right]$, and (3) is just that,

replacing i by $(i+t)$, same as you would replace it by $(i+\Delta i)$.

Another derivation of (3).

a bldg. tax is levied.

$$C_o = \sum_1^m \left[(R_n - a_n) e^{-(i+\tau)n} \right] \qquad (4)$$

(4) simply says that the excess of Revs. over rents
a. will be taxed
b. must cover C_o, after taxes.

There is no give in anything but $\underline{a}$.
Solve for $\underline{a}$.

$$C_o = \sum_1^m \left(R_n e^{-(i+\tau)n} \right) - a \frac{1 - e^{-(i+\tau)m}}{i+\tau} \qquad (5)$$

$$a = \frac{i+\tau}{1 - e^{-(i+\tau)m}} \left[\sum_1^m \left(R_n e^{-(i+\tau)n} \right) - C_o \right] \qquad (6)$$

Same as (3)

N.B. To figure effect of Δi on rent, one would proceed just the same way, using Δi for τ. So they work out the same because they start the same.

so they lower rent the same.

Now ~~if~~ Δi lowers S more, because $S = \frac{a}{i}$

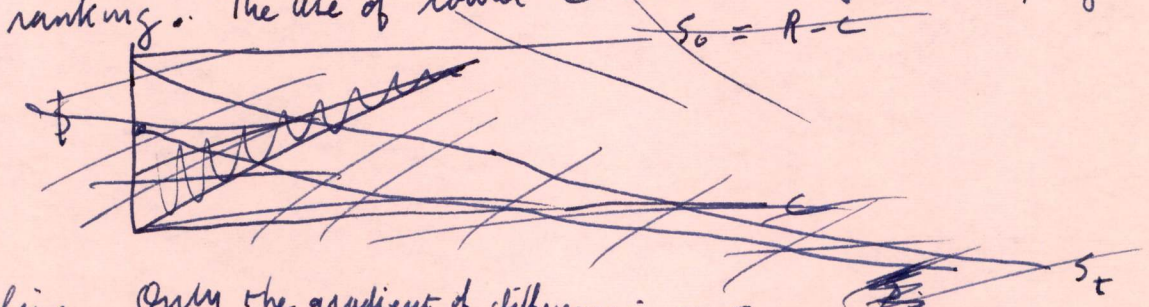
$$\ln S = \ln a - \ln i$$

$$\frac{d \ln S}{di} = \frac{da/di}{a} - \frac{1}{i}$$

$$d \ln S = \frac{da}{a} - \frac{di}{i} \qquad (da < 0)$$

If Δi and t lower rent the same, they will have same effect on shifting to a new high bidder. The one yielding highest rent.

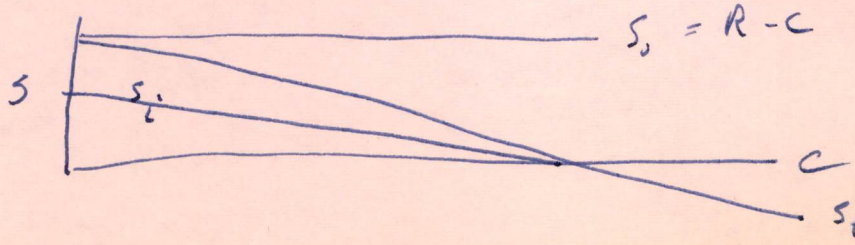
It doesn't matter that the % drop of S ~~becomes~~ for parking lots is 0 for t and material for Δi. It won't change the ranking. The use of lower C becomes a higher bidder, right down the



line. Only the gradient of difference is greater when t than when Δi. (This may make a difference)

~~But~~ S_i & S_t both become 0 when rent becomes 0.

So they must reach 0 at same C. (Test this on $S = R - C$, it may be a weak point)



Now compare S_t and S_i

$$S_t = \frac{a_t}{i}$$

$$S_i = \frac{a_i}{i + \Delta i}$$

Let $\Delta i = t \equiv j$

assume $a_t = a_i$

$$\frac{S_i}{S_t} = \frac{\frac{a_i}{i+j}}{\frac{a_t}{i}} = \frac{i}{i+j}$$

When $a = 0$, they can be the same.

When $a < 0$, and S_i & S_t are both < 0, S_t is more negative.

✓ The fact that C/R doesn't appear in the ratio means its effects are used up in determining a

Test (3) by letting R_n be level

Then:

$$a_\tau = \frac{i+\tau}{1-e^{-(i+\tau)m}} \left[R_n \frac{1-e^{-(i+\tau)m}}{i+\tau} - C \right]$$

$$a_\tau = R_n - C_0 \frac{i+\tau}{1-e^{-(i+\tau)m}} \qquad (7)$$

Looks ok. C_0 here is treated as though it were depreciation on s.f. basis, in keeping w. assumption of level R.

Test by $m = \infty$

$$a = R_n - C(i+\tau) \qquad \text{ok}$$

Test by $m = 1$

$$a = R_n - C \frac{i+\tau}{\frac{i+\tau}{1+i+\tau}} = R_n - C(1+i+\tau) \qquad \text{ok.}$$

p. 59, Summary of biases, where $\underline{t}$ is worse than $\underline{i}$ | 57 | 58

p. 60 - 64, pretty sterile.

50. With t, S of pkg. lot, C=0, drops by 0%.

So it can now outbid some uses previously higher.

With Δi, S of parking lot drops by $\frac{\Delta i}{i + \Delta i}$ %, say 10%. It might seem that this would lower its relative power. But the higher uses now drop by that extra 10%, too.

Take one that dropped just below the parking lot by dropping 20%, from a t. Now it drops 20% + 10%, or 30%, and pkg. lot drops 10% instead of 0%. The pkg. lot will still be on top. ~~That~~

This is allowing that the last 10% drop for the higher use is from the lower base, so it is less than 30% off the original base.

51. Higher i cuts down all uses where aim is to establish rents in the future. Higher t does not. There is a difference.

Thus, as to $S = \frac{a}{i-g}$. Higher i swamps the g.

$S_i = a + Sg$

$S = \frac{a}{i} + \frac{Sg}{i} = \frac{a + Sg}{i}$ Here, higher i means lower

S_i means lower Sg cfd. to a. Thus higher i reduces value of holding land for future uses — and t does not, but the reverse.

This applies to ripening; and preemption; & leverage over p.w.

Δi works against programs that leave land w/o capl. for a while to get deferred yields. But Δt does not, — unless they are corporeal, like timber, and in practice assns. miss that, too. Δt and Δi are equally hard on high C_o of slow payout.

But Δt doesn't make the land itself pay out faster, while Δi does.

Thus t wd. not penalize:
- ripening minerals; or with holding ripe minerals.
- interim shoddy uses, while waiting for higher ones
- preemptive holding by monopolist, to control markets
- min. time to get volunteer reforestation, subbing land for capl. low & long
- letting time pass to avoid cap. gains tax.
- holding land waiting for p.w.
- holding land to catch spillovers from one's other land

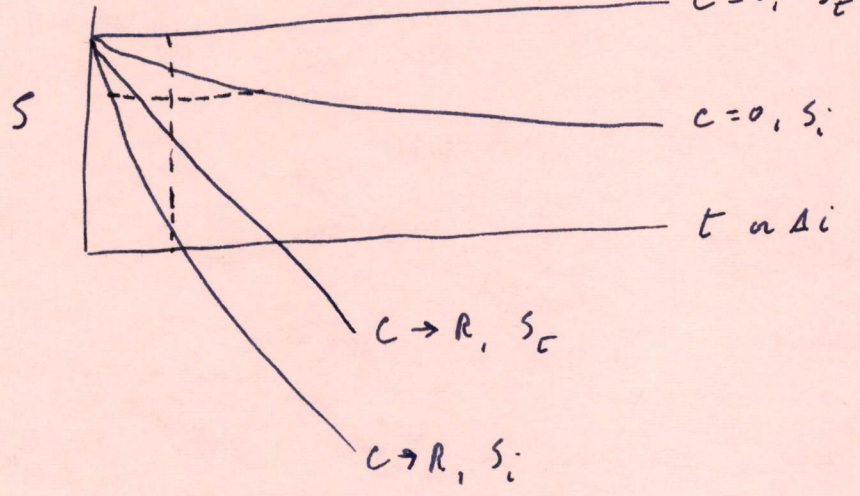
Also, i favors deferred applm. of capl.? Forests as vs. bldgs. Tax base is less than PV of tree, all along. Now there is a bias? — (and the reverse of what foresters complain) See p. 94

? now doubt this.

52. When $C/R \to 1$, τ shd. lower rent by more than Δi, because they lower S almost the same, but Δi compensates for lowering S, since rent = $S \cdot i$.

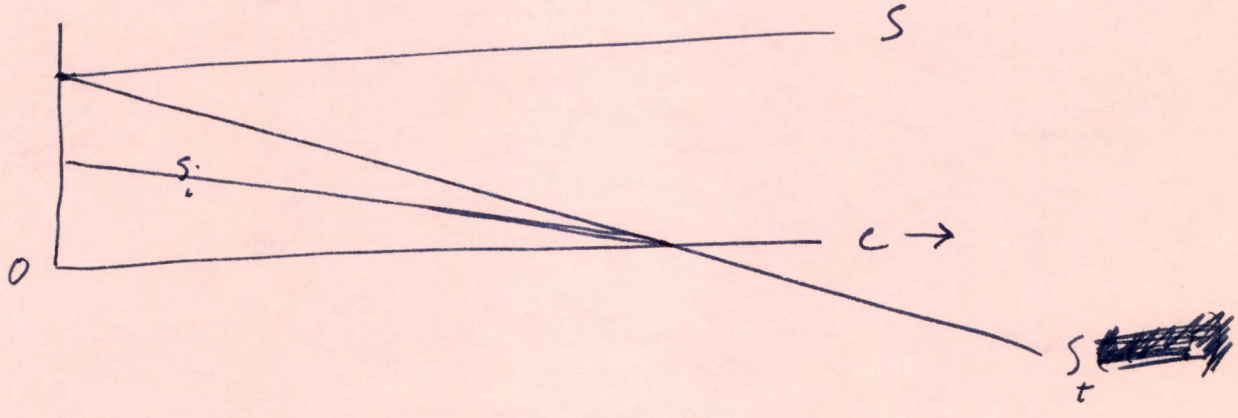
Can I refute this?

53.



So?

54.



55. If i goes up, that lowers $\frac{a}{i}$ (and $\frac{a}{i-g}$) more than uses of faster payout, earlier revs.

Yet if rev. comes very early, most S comes from later generations, and their rent is disc., making S again sensitive to Δi. If $m=1$, $S=\frac{a}{i}$ again.

And no matter where rev. comes, ~~from~~ land goes on in perp., so I end up w. a value for $\underline{a}$, then divide by i, and get same $-\Delta S$ as w. putting lot on top of the $-\Delta a$.

If m is shorter, so PVR goes down less, why then $(1-e^{-im})$ in denom. is more sensitive to Δi, and when $m=1$, denom. $\equiv i$.

55a. If $R = b\,\frac{1-e^{-im}}{i}$ $\quad (d=0)$

$$S = \frac{b}{i} - C/(1-e^{-im}) \text{ , (and } S \text{ is } \Delta g \text{ fn. of } \underline{m})$$

and S is sens. to Δi which lowers b/i (but also lowers 2d term)

55b. If $R = b\,\frac{1-e^{-(i+d)m}}{i+d}$

$$S = \frac{b}{i+d}\,\frac{1-e^{-(i+d)m}}{1-e^{-im}} - \frac{C}{1-e^{-im}}$$

as $d \to 1$, and $m \to 1$, first term $\to b/i$, again.

56. Approach: calc. $\frac{dS/di}{dS/dt}$ show as $f\left(\frac{R}{c}\right)$.

But only do if necy.

$$S = \frac{\Sigma(R_n e^{-in}) - C}{1 - e^{-im}} = \frac{b\frac{1 - e^{-(i+d)m}}{i+d} - C}{1 - e^{-im}}$$

$\frac{dS}{di}$ = ouch!

$$\ln S = \ln N - \ln D$$

$$\frac{d(\ln S)}{di} =$$

Do above later if needed, or to check. ($\frac{dS}{dt}$ wd. require new eqn.)

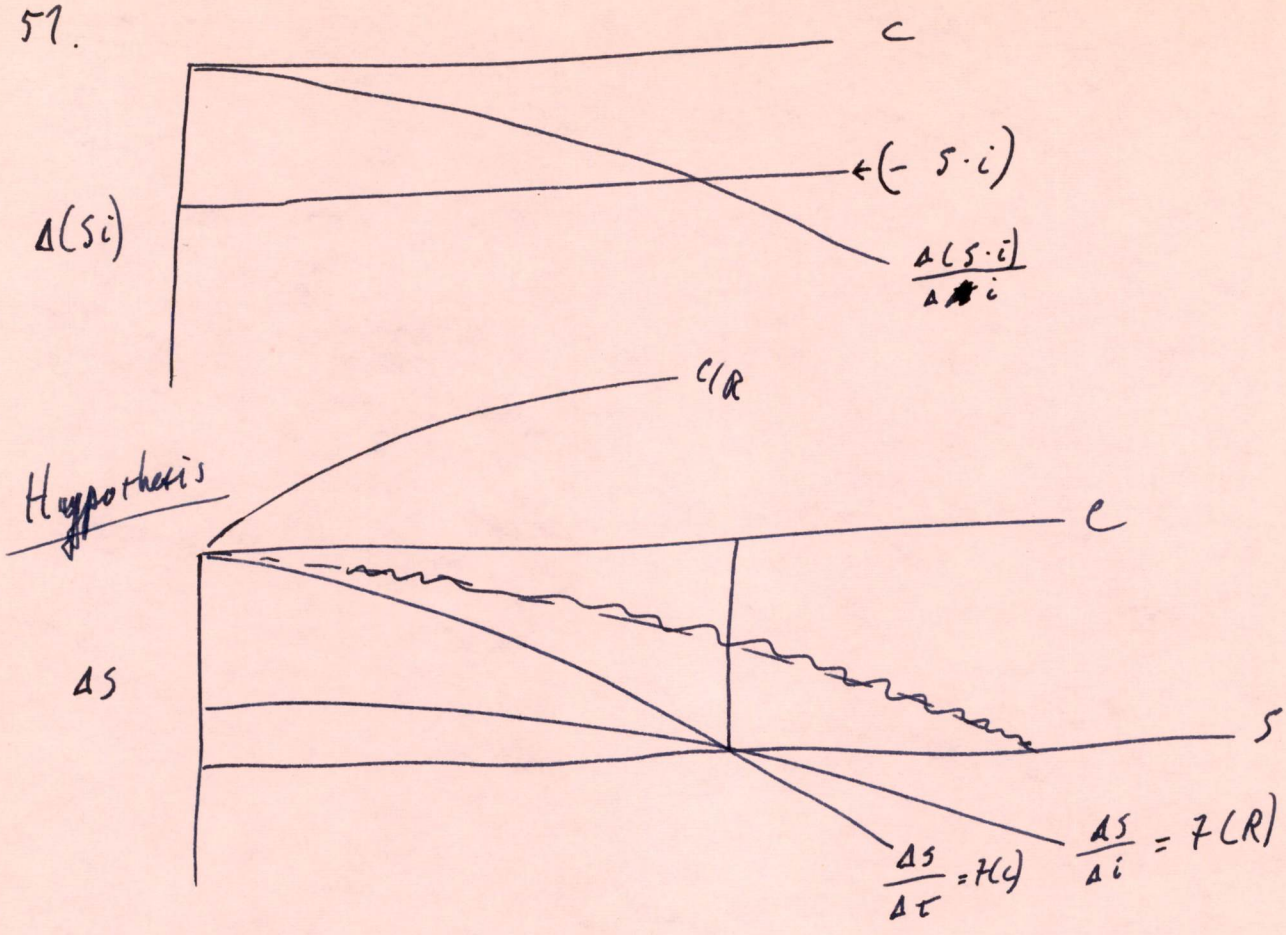
$$\frac{d\left(\frac{1 - e^{-im}}{i}\right)}{di} = \frac{i(-(-me^{-im})) - (1 - e^{-im})}{i^2} = \frac{me^{-im}}{i} - \frac{1 - e^{-im}}{i^2}$$

$$\frac{d\left(\ln\frac{1 - e^{-im}}{i}\right)}{di} = \frac{m}{e^{im} - 1} - \frac{1}{i}$$

$$-d\ln\frac{1 - e^{-im}}{i} = \frac{di}{i} - \frac{di\, m}{e^{im} - 1}$$

Finish only if necy. (See #9, p. 5, for crude approach)

57.



When both t and Δi reduce S to 0, they reduce rent to 0.

When $i = 0$, neither t nor Δi reduce rent at all.

So they probably affect it the same in between, too.

✓ This pretty well kills p. 4

61. Summarize & Codify

Can $da/di > 0$? maybe, under special & unlikely premises.

$$a = (R - C) \cdot cnf \tag{1}$$

$$\frac{da}{di} > 0 \quad \text{if} \quad \frac{d(\ln a)}{di} > 0$$

$$\ln a = \ln(R - C) + \ln i - \ln(1 - e^{-im}) \tag{2}$$

$$\frac{d(\ln a)}{di} = \frac{dR/di}{R - C} + \frac{1}{i} - \frac{m}{e^{im} - 1} \tag{3}$$

$$\frac{d(\ln a)}{di} > 0 \quad \text{if} \quad \frac{dR/di}{R - C} > -\left[\frac{1}{i} - \frac{m}{e^{im} - 1}\right]$$

or

$$\frac{-dR/di}{R - C} < \frac{1}{i} - \frac{m}{e^{im} - 1} \tag{4}$$

$\frac{da}{di} > 0$ if:

$$\frac{-dR/di}{R-C} < \frac{1}{i} - \frac{m}{e^{im}-1} \qquad (4)$$

(4) is most likely if:

a) $C = 0$, because C makes left side larger.

~~b) d is small, because~~

b) d is large, i.e. depreciation is fast, bcs. fast depreciation makes left side smaller (less sensitive to di)

c) m is large, bcs. rt. side increases with m. (even though left side does too; left side increases less with m when d is large)

(a), (b), & (c) are an unlikely combo.

If $C = 0$, probably $d = 0$ too.

If $C > 0$, d can only get so large without depreciating more than C (this would be exploitive or extractive capital). So if C small & d large, m must be small.

Thus it's pretty well boxed in, when these parameters are tied together, and probably ~~could it be~~ $\frac{da}{di}$ could not be > 0.

($\frac{a}{i} = s\,m$ is basic to the constraint)

Thus, if $R = b\dfrac{1 - e^{-(i+d)m}}{i+d}$

That means

$$R = b\left[e^{-(i+d)} + \frac{1}{e^{2i} \cdot e^{2d}} + \dots \text{ etc.}\right], \text{ a decaying stream.}$$

When $\frac{b}{e^{nd}}$ is down to $(R-C)i$, $\underline{m}$ has arrived.

The most likely case for $\frac{du}{di} > 0$ is a mine.

C is low
d is high
m is as long as you like.

Thus, if mining and sustained use are rivals, higher $\underline{i}$ favors the mine, whose annual value actually rises, or may, if C is low.

This is a variant of the principle re ripening, that higher $\underline{i}$ favors patterns yielding earlier land income, (where land income is temporally interdependent,) while higher $\underline{E}$ have no such effect.

If $R = b\dfrac{1-e^{-(i+d)n}}{i+d}$ we may compare it with

cf.

$$\ln R = \ln b + \ln\left[1-e^{-(i+d)n}\right] - \ln(i+d)$$

$$\frac{d\ln R}{di} = \frac{n}{e^{(i+d)n}-1} - \frac{1}{i+d} \qquad (5)$$

$\dfrac{da}{di} > 0$ if

$$-\frac{d\ln R}{di} = \frac{1}{i+d} - \frac{n}{e^{(i+d)n}-1} < \frac{1}{i} - \frac{n}{e^{in}-1} \qquad (6)$$

(6) is true if $d > 0$.‡ But it rests on assuming $c = 0$, and if $c = 0$, $d = 0$ is almost certain.

If land rent is rising, $d < 0$, and even $c = 0$ won't make $\dfrac{da}{di} > 0$.

‡ This can be shown by showing $\dfrac{d}{di}\left[\dfrac{1}{i} - \dfrac{n}{e^{in}-1}\right] < 0$

62. Δi lowers S, leaving C unchanged.

Thus it ~~vary~~ makes land cheaper relative to bldgs., and causes substtg. land for cap.

But Δi leaves S·i unchanged, But it raises C-crf. again making capl. dearer relative to land. But taxes do the same.

63. $\frac{a}{i}$ is much more sensitive to Δi than is a.

Thus the common decline (of high & low C/R) is much greater than the differential decline of high C/R.

Say, like this:

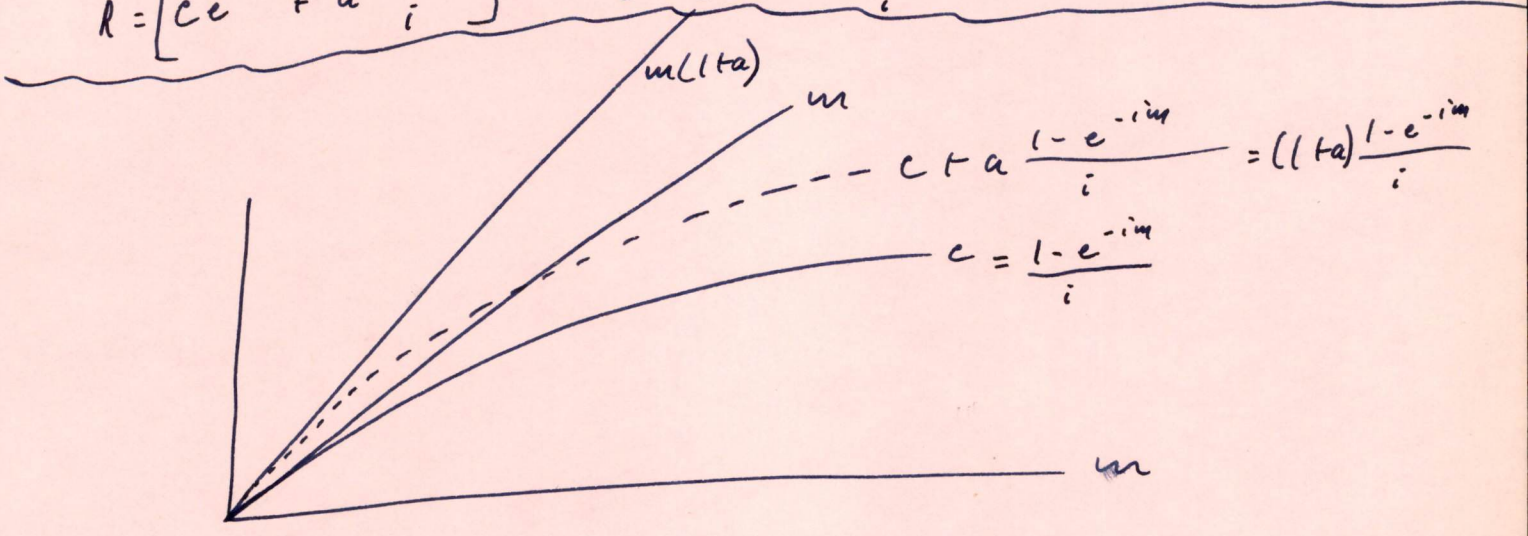
$-\Delta S$	C=0	C→R
From Δi	50%	55% (~~probably~~ 52 ÷ 2 %? (54%?)
From t	0	5

If they were equal at first, now the high C/R is 5% less -- and that is true if Δt or Δi.

GT.



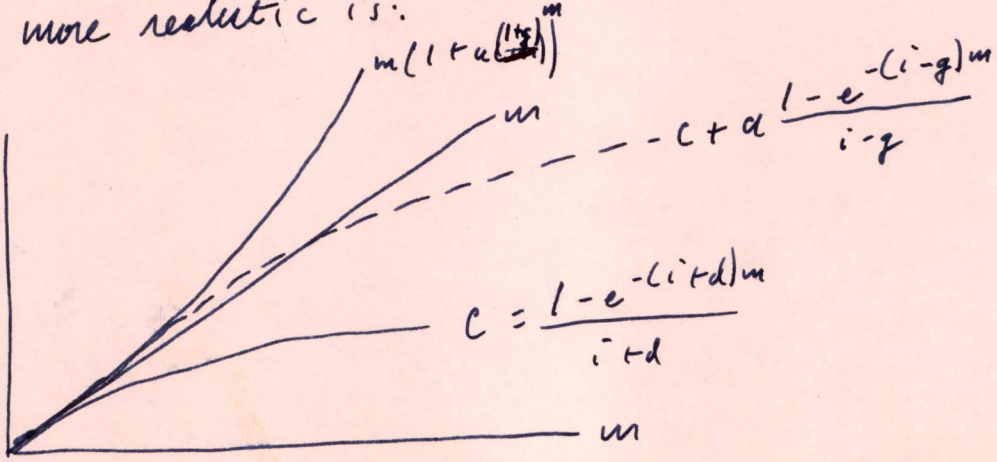
$$R = \left[C e^{im} + a \frac{e^{im} - 1}{i} \right] e^{-im} = C + a \frac{1 - e^{-im}}{i}$$



$$R = (1+a) \frac{1 - e^{-im}}{i}$$

$\frac{C}{R} = \frac{1}{1+a}$, where "1" is yrly. service imputable to C.

But more realistic is:



$$\frac{R}{C} = \frac{C + a\dfrac{1 - e^{-(i-g)m}}{i-g}}{C}$$

$$R = C + a\frac{1 - e^{-(i-g)m}}{i-g} = \frac{1 - e^{-(i+d)m}}{i+d} + a\frac{1 - e^{-(i-g)m}}{i-g}$$

$$\frac{C}{R} = \frac{1}{1 + a\dfrac{1 - e^{-(i-g)m}}{1 - e^{-(i+d)m}}\dfrac{i+d}{i-g}}$$

$\frac{C}{R}$ is decreasing $f(m)$.

So long life is likely to mean low $\frac{C}{R}$

In lesser measure, means low $Q = 1 - \left(1 - \frac{c}{R}\right)\frac{mi}{e^{mi}-1}$

In greater measure, means low $\frac{C}{TVR}$, or labor-intensity.

✓ 88. C_0 is not capl. cost. C_0 is labor cost.

C_0 is capl. investment, and invt is not a cost.

Capl. cost is $C_0(e^{im}-1) + a\frac{e^{im}-1}{i} - ma$

89. Hi $\underline{i}$ means less C_o, longer $\underline{m}$ (for depreic capl.)

This means less labor/land, in 2 ways: less labor, less often.
and a 3d way: less floor space for labor to work on.
Same for $\underline{t}$ on bldgs.

Thus, i & t are not friends of labor, re land. The capl. thy. that ignores land is thus very misleading. ✓

90. Hypoth:

Δi lowers s from timber by more than t does, because
Δi discounts full R, while t only applies to true base.

True, but we already know that. $\underline{\Delta i}$ is simply discounting future rents, along w. other revs.

$$R = e^{-im}\left[Ce^{im} + a\frac{e^{im}-1}{i}\right] = C + a\frac{1-e^{-im}}{i}$$

$$R - C = a\frac{1-e^{-im}}{i}$$

$$s = \frac{R-C}{1-e^{-im}} = \frac{a}{i}$$

If $a = 0$, t & Δi operate on same base.

Effect on rent shd. be same?

This proves nothing. But I've shown it elsewhere?

$$a = (R-C)\frac{i}{1-e^{-im}} = \frac{Ri}{1-e^{-im}} - \frac{Ci}{1-e^{-im}}$$

$$\frac{Ri}{1-e^{-im}} = \frac{Ci}{1-e^{-im}} + a, \quad \ldots \text{ leading to } a = a.$$

91. "Deferred low use", is one of high decay and long n.
t and Δi both encourage that. (But $\underline{t}$ may encourage it more, if object of early bldg. is to capture higher later rents.)
"Deferred realization" is the other kind, with land generating its own cash.

92. ~~Return to~~ 28, p. 22, cf. with correct statement of how prop. tax affects rents. #28 is wrong. Fails to key tax base to depreciation.

93. From 48, p. 53

$$\Sigma\left[(R-a)x\right] + \Sigma\left[ay\right] = \Sigma\left[(R-a)x + ay\right]$$

$$= \Sigma\left[Rx - a(x-y)\right]$$

So

$$S_c(1-e^{-i\underline{n}}) + C = \Sigma\left[R_n e^{-(i+\tau)n} - a_n e^{-(i+\tau)n} + a_n e^{-in}\right]$$

$$= \Sigma\left[R_n e^{-(i+\tau)n} - a_n\left(e^{-(i+\tau)n} - e^{-in}\right)\right]$$

$$= \Sigma\left[R_n e^{-(i+\tau)n} - a_n e^{-in}\left(e^{-\tau n} - 1\right)\right]$$

~~Stop this, leads no where.~~

$$= \Sigma\left(R_n e^{-(i+\tau)n}\right) + \Sigma\left(a_n e^{-in} - a_n e^{-(i+\tau)n}\right)$$

May give another insight into $\underline{a}$.

94. I said $\frac{c}{R} = \frac{c}{c+s}$

But $R < c+s$

$R = c+s(1-e^{-im})$

Thus I opened self to some confusion by oversimplifying.

95. There is no defined rent, but there is defined realizn., or ripeness.

Defined realizn. adds to capl., hence intensity.

Ai discourages defined of realizn. to does t.

96. ~~Agenda~~ From p. 5-9.

Evaluate hypothesis: the rent of a forest is less reduced by t than by ai. Because the applic. of capl. is defined

~~Hyp. is false~~

$a = \frac{(g - Ce^{im})\, i}{e^{im} - 1}$

tax base is, ~~$V_n = Ce^{in} + a\frac{e^{in}-1}{i}$~~

~~or rather~~

$B_n = Ce^{(i+t)n} + a_t \frac{e^{(i+t)n} - 1}{i+t}$

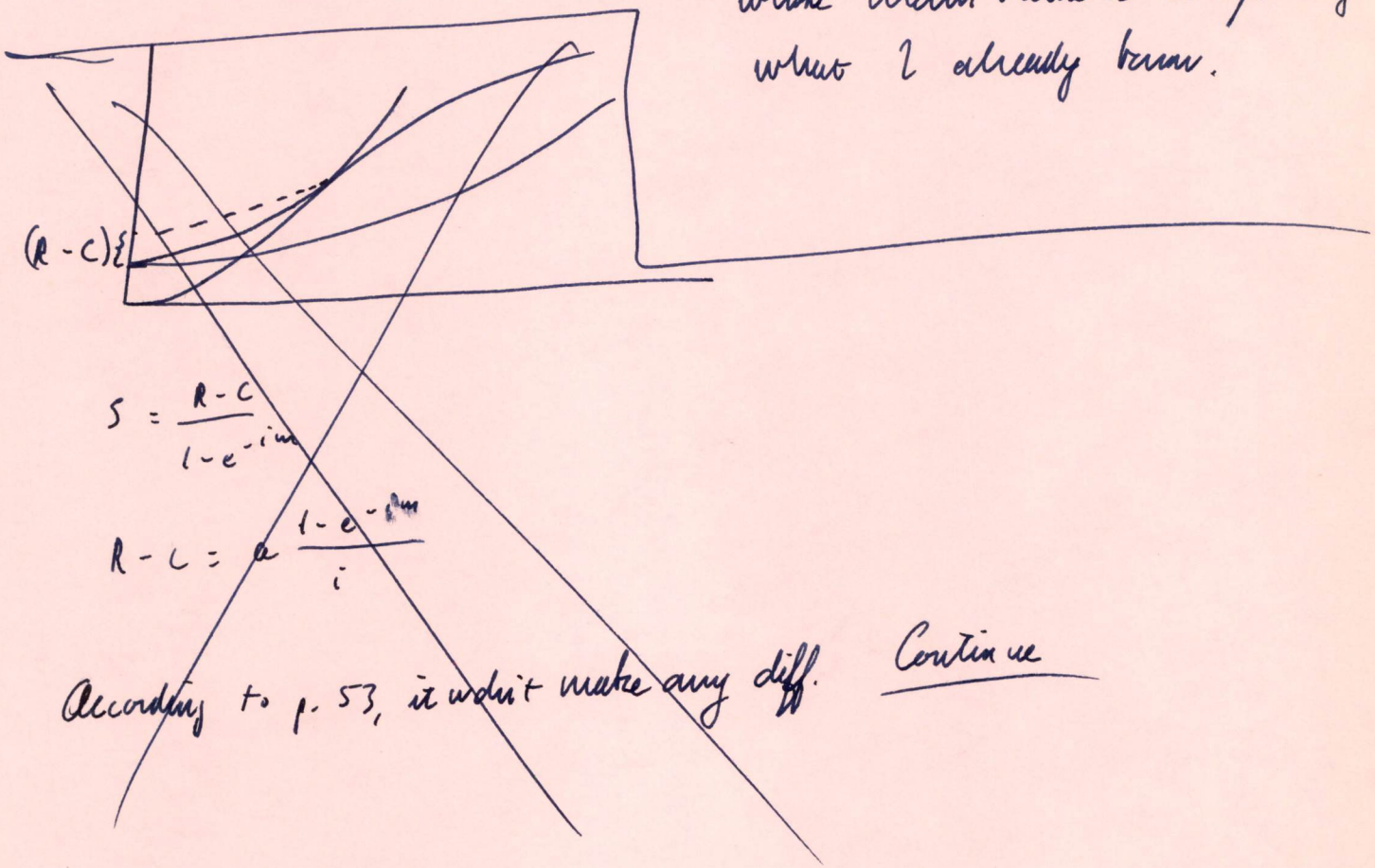
$PVT = \Sigma t B_n e^{-in}$ [or $\Sigma t B_n e^{-(i+t)n}$? I think the former]

This seems hard way to do it. Just solve for a, and see i & t are parallel.

$$NVT = t\sum\left[\frac{Ce^{(i+t)n}}{e^{in}} + a_t\frac{e^{(i+t)n}-1}{(i+t)e^{in}}\right]$$

$$= t\sum\left[Ce^{nt} + a_t\frac{e^{nt}-e^{-in}}{i+t}\right] \qquad (\text{Cf. p. 108})$$

Continue? Maybe later. The algebra wd. be long, and it should give me same answer I already have, by roundabout route, whose mean value is confirmatory what I already know.



$$S = \frac{R-C}{1-e^{-in}}$$

$$R - C = a\frac{1-e^{-in}}{i}$$

According to p. 53, it won't make any diff. Continue

97. i discounts future revs., regardless of how little capl. was stored up to achieve them. E.g. if ~~i high, b low~~ C low, b a high, value arises late. (If a low, i + t high, even later.)

Thus w. trees, i discounts full future value, like a tax based annually on disc. value of tree. But propty. tax is on lower value, the growth path. If rent is high, i.e. on good land, growth path is far under disc. ~~e.g.~~

This doesn't distinguish trees from houses? R_n from a house covers a_n as well as ~~i~~ cap. service (yield & depreciation)

Assume R_n doesn't rep. realizn. of any accrued rents, but only accum. interest from its share of Co. (Impossible / why not parallel treatment of Co & S?) Then its ~~wd. all have been~~ disc. value wd. have been fully in tax base all along: and $\underline{t}$ wd. hit it same as i. This is what my formula does assume — so does it apply to trees? No.

Bldgs. & trees are non-parallel. Bldgs. pay land rent yearly, it is not stored up. Each yr's R_n covers rent plus cap. recovery w. interest. Rent not stored up or plowed back, yearly. (Orchards are an intermediate case.)

But I'm not sure it makes a diff. here. Tax base, $B_n < T_n$, true. But that is the way rent shows up re trees -- and it does not show up in yrly. excess of revs. over cap. recovery. In either case, i discounts the rent which t misses.

But cf. 2 uses using diff. units of land time.

Program 2 has higher C_0 & shorter n.

$$R = C_0 + a\frac{1-e^{-in}}{i}$$

Both t & i lower a. But i also discounts it. The rival using more land-time wd. fall more than the other from si

$$S = \frac{R-C}{1+e^{-in}} = \frac{a}{i}, \text{ and it all cancels out.}$$

---

98. I end up not showing $\pm$ biased for forestry. But I show it's not biased vs. forestry, as long claimed. Bes. tax base is below disc. value of tree. In practice, immature trees undercassed, so there is that bias.

---

99. Agenda.

$$Q = 1 - (1 - \tfrac{e}{R})\frac{mi}{e^{mi}-1}$$

See if Δt is harder on high Q than si is.

Modify Q somehow to remove any friction involved in discj everything to PV. We need tax base as it is, to cf. apprecj & deprecj capl. e.g.

100.

$$a_t = \frac{i+t}{1-e^{-(i+t)m}}\left[\sum_1^m \left(R_n e^{-(i+t)n}\right) - C\right] \qquad (3)_{p.53}$$

Re timber, that becomes

$$a_t = \frac{i+t}{1-e^{-(i+t)m}} \cdot g e^{-(i+t)m} - C\frac{i+t}{1-e^{-(i+t)m}}$$

$$= \frac{(i+t)g}{e^{(i+t)m}-1} - C\frac{i+t}{1-e^{-(i+t)m}} \qquad (4)$$

In (3), ~~we~~ it's clearer what's happening. We seem to apply t to too large a base, R_m, thus discounting as though rent were part of tax base. But then we annualize with a c+f based on i+t, compensating.

I wish I saw intuitive meaning of (3) & (4) better. See p. 100

(1), p. 53, treats R_n (above a_n) as though it were all taxed yearly? But thus it applies only where rent is realized yearly & not stored up in capl. Can't apply to trees? Yes, it can.

~~The a_n is all~~

Maybe not. Can rewrite as on p. ~~93~~ 93, so:

$$s(1-e^{-im}) + C = \underset{(1)}{\Sigma\left(R_n e^{-(i+t)n}\right)} + \underset{(2)}{\Sigma\left(a_n e^{-in}\right)} - \underset{(3)}{a_n e^{-(i+t)n}}$$

(1) is too low, so add PV rents (2) less PV of rents as taxed (3)

(1) deducts too much tax -- ~~it taxes~~ the base there is always

$B_n = R_m e^{-(i+\tau)(m-n)}$, which is correct base plus $\sum_{m-n}^{m} a_n e^{-(i+\tau)(m-n)}$.

Then in year 0, base should be C, but $(1) = R_m e^{-(i+\tau)m} \left[> C, \right.$

$\left.$ so long as $a_t > 0 \right]$

Or, regroup again:

$$= e^{-(i+\tau)m} \left[R_m - a\left\{ \frac{e^{(i+\tau)m} - 1}{i+\tau} \right\} \right] +$$

Thus, that which we are discounting at $(i+\tau)$ is not the full R_m,

but only the excess of that R_m over $\sum a e^{(i+\tau)}$

To find a_t. a_t is the residual value that links C_0 and R_m and must satisfy:

$$Ce^{(i+t)m} + a\frac{e^{(i+t)m}-1}{i+t} = R_m \qquad (11)$$

solve for a

$$a = \left[R_m - Ce^{(i+t)m}\right]\frac{i+t}{e^{(i+t)m}-1} \qquad (12)$$

(11) says that tax is levied on growth path beginning from C_0 and rising along growth path where i & t are added to value each year, along with a_n embodied in tree.

(11) can thus be derived directly, intuitively. And (11) restates (3), etc. So all is consistent.

101.

$$Q = 1 - \left(1 - \frac{c}{R}\right)\frac{mi}{e^{mi} - 1}$$

Q is an increasing $f(i)$

If $Q = \frac{c}{R}$, and Δi lowered R, Q wd. rise with Δi.

In addition, Δi makes $\frac{mi}{e^{mi}-1}$ get smaller, raising Q.

This is to be expected. It operates 2 ways: lowers $\underline{a}$, and raises capl. input.

The lower $\underline{a}$ ~~tends to~~ operates to reduce capl. input, compensating. This only applies to trees. It means:

— higher i makes bldgs. more cap-intensive, more than it makes trees so.

— higher i defers cap. building until later -- reducing PV of Σa (& not changing PV of C_0)

Does $\underline{t}$ have same effects as $\underline{i}$? Well, in a way, but note that $\underline{t}$ increases Q in effect by having $\underline{t}$ B_n treated as an input. It doesn't exactly create false capl., bec. it lowers $\underline{a}$, too, than lowering statement of that input.

In effect, it results in overstating social cost of capl; understating cost of land.

102. Among trees, higher c/s means shorter cycles.
Thus the kind of capt.-intensity results from using more labor shorter cycles.
The kind assoc. w. using more land lengthens cycles.

---

103. $\ln \frac{1}{i} = \ln 1 - \ln i$

$$\frac{d}{di} \ln \frac{1}{i} = \frac{-1}{i}$$

$$\ln e^{-im} = -im$$

$$\frac{d}{di} \ln e^{-im} = -m$$

---

104. If future cycles yield higher returns than G_1, then

$$S = G_1 e^{-im} + G_2 e^{-2im} + \dots$$

$$= G_1 e^{-im} + G_1(1+g) e^{-2im} + \dots$$

$$= \frac{G e^{-im}}{1 - (1+g) e^{-im}} = \frac{G}{e^{im} - (1+g)}$$

$$S = G e^{-im} + S(1+g) e^{-im}$$

Will this change finding that t & ai have same effect on land use? Probably so, since it involves ripening.

It certainly changes basic eqn.

$$S(1-e^{-im}) + C = \sum_1^m \left((R_n - a_n) e^{-(i+t)n} \right) + a \frac{1-e^{-im}}{i}$$

Now the 1st & last terms don't cancel

$$S\left(1-(1+s)e^{-im}\right) + C = \sum_1^m \left[(R_n - a_n) e^{-(i+t)n} \right] + a \frac{1-e^{-im}}{i}$$

Might develop this. But it seems just to confirm or formalize known principle. ✓

105.

Cf. bldg. & tree, of hi C/R

The bldg. has much higher C/S

Indicating higher labor/land ratio. Tree gets hi C/R from low R, not high C. R is low bec. so much int. is deducted from gm. (& rent)

---

106. Compounding is adding int. Discounting is ∴ subtracting int., even tho the math. operation is to divide.

Same when you discount by tax rate.

---

101. Let $V_n = Ce^{(i+t)n} + a\dfrac{e^{(i+t)n}-1}{(i+t)}$ (13)

This just says growth path is along the taxed path. (I+ starts slower, and rises faster at end — but gets to same place, except for some shortening of life. Note how it erodes some of its own base, by thus lowering the growth path!)

Solve for a. (If assume no land tax, $S = a/i$. If land tax $= t$, $S = \dfrac{a}{i+t}$, where a is rent after taxes on capl., but before taxes on land.)

$$a = (i+t)\,\frac{V_n - Ce^{(i+t)n}}{e^{(i+t)n}-1} \qquad (14)$$

Now, effect of $\underline{t}$ on $\underline{a}$ is same as Δi.

Using approach of TTTI, p. 26

$\Theta = S - t \sum_{1}^{m} \frac{T_n}{?}$ ..... a tough approach. Already abandoned once, p. 95. And see p. 108.

Instead, figure Θ by new method, put into TTTI.

$$\Theta = \frac{a}{i} = \frac{i+t}{i} \; \frac{V_m - C \cancel{(i+t)} e^{(i+t)m}}{e^{(i+t)m} - 1} \qquad (15)$$

$$\frac{S-\Theta}{S} = 1 - \frac{\Theta}{S} = 1 - \frac{\Theta}{\frac{V_m - Ce^{mi}}{e^{mi}-1}} = 1 - \frac{i+t}{i}\left[\frac{V_m - Ce^{(i+t)m}}{V_m - Ce^{im}} \; \frac{e^{mi}-1}{e^{(i+t)m}-1}\right] \qquad (16)$$

16

1. p.r. is an Δg fn. of C.
2. Θ is a decrg. fn. of C, but its not as tho the tax was levied on C alone. The tax also raises denom., adding to its effect, w/o reducing the outstanding effect of C.
3. p.r is an Δg f(m). My TTTI, p. 27, is wrong here. Except it is still true that the anti-labor bias is stronger — note the leverage against C, lacking in the ratio on the right. But there is also leverage for $\underline{m}$, since -C is compounded.
4. By making C small, to avoid tax, one also reduces leverage of $\underline{m}$.

108.

Since timber value includes stored-up rent, and tax lowers rent, the tax on appreic cap. is partly capd. into lower values of immature timber. This greatly reduces tax yield, & esply. its PV.

It's also a big diff. of prop. tax on appreic & depreic. capl. If $C=0$, t on tree can rise to any level w/o eliminating value, just like land tax. Base is not fixed. [In (16), $\frac{e^{mi}-1}{e^{m(i+\tau)}-1} > 0$ (for finite m.)]

When we consider the possibility of lowering C, it wd. seem that propty. tax encourages:

a. Growing trees w. lower C, longer m
b. " " instead of bldg. bldgs. etc.

Range cattle are the other big land user where the value is largely stored-up land w/o much labor.

~~1.~~

In (16) I shifted qu. of how treat intermed. recs. — by just having V_m.

But if R_n is carried forward to m at i, rather than i+t, may cause bias.

From (13),

$$V_m = Ce^{(i+t)m} + a\frac{e^{(i+t)m}-1}{i+t} \qquad (13)$$

See master equn. for how it handles R_n.

$$\overset{\Theta}{S}_t(1-e^{-im}) = -C + \Sigma\left(R_n e^{-(i+t)n} - a_n e^{-(i+t)n} + a_n e^{-in}\right)$$

or just

$$C = \Sigma\left(R_n e^{-(i+t)n} - a_n e^{-(i+t)n}\right) \qquad (4)_{p.53A}$$

Here, R_n is disc. at i+t bcs. until received it is part of capl. value, & is taxed. That makes it like tree. But it is unlike tree bcs. its growth path is simple compound interest.

Yet what happens to (4) if there is just one R_m?

$$C = R_m e^{-(i+t)m} - a\frac{1-e^{-(i+t)m}}{i+t}$$

$$R_m = Ce^{(i+t)m} + a\frac{e^{(i+t)m}-1}{i+t}$$

The growth path puts Σa into R_m.

So if there are many R_n, Σa must get into ΣR_n — and there is no diff. of bldgs. & trees.

109A.

Re TTTT, p. 26

$$\theta = W - t \sum_{n=1}^{m} \left[\frac{(P+W)e^{in} - W}{e^{in}} \right] \frac{1}{1-e^{-im}}$$

$$\sum_{n=1}^{m} \left[(P+W) - We^{-in} \right] = m(P+W) - W \frac{1-e^{-iu}}{i}$$

$$\theta = W - t \left[\frac{(P+W)\, m}{1-e^{-im}} - \frac{W}{i} \right]$$

Showing I got from (23) to (24) by assuming the base was $(P+W)e^{iu} - W$.

I think I used. Should be $(P+\theta)e^{(i+t)u} - \theta$.

~~Best check this out further here.~~

this must be $(i+t)$ ~~for this~~

~~is found as~~ a ~~e^{i+t}~~ $i+t$

109B.

$$\theta = S - t \Sigma \left(\frac{(C+\theta)e^{(i+t)n} - \theta}{\cancel{e^{(i+t)n}}\ \cancel{e^{n(i+t)}}} \right) \frac{1}{1-e^{-im}}$$

where $\theta = \frac{a}{i+t}$

$$= S - t \left((C+\theta) \frac{e^{mt}-1}{t} - \theta \frac{1-e^{-mi}}{i} \right) \frac{1}{1-e^{-im}} \qquad (\text{cf. p. 95})$$

$$= S - (C+\theta) \frac{(e^{mt}-1)}{1-e^{-im}} - \frac{t\theta}{i}$$

~~108~~.

$$\theta\left(1 + \frac{t}{i} + \frac{e^{mt}-1}{1-e^{-im}}\right) = S - C\frac{e^{mt}-1}{1-e^{-im}}$$

No simple way out? Maybe I set this up wrong.

Continue if find time or see way.

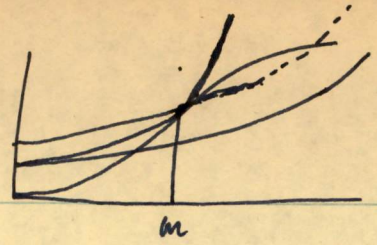
$$-C\frac{e^{mt}-1}{1-e^{-im}} + S = \frac{R_m e^{-im} - C}{1-e^{-im}} - \frac{Ce^{mt}}{1-e^{-im}} + \frac{C}{1-e^{-im}} = \frac{R_m e^{-im} - Ce^{mt}}{1-e^{-im}}$$

$$\theta\left[1 - e^{-im} + \frac{t}{i} - \frac{t}{i}e^{-im} + e^{mt} - 1\right] = R_m e^{-im} - Ce^{mt}$$

$$\theta\left[-e^{-im}\left(1 + \frac{t}{i}\right)\right.$$

$$\theta\left[\frac{t}{i}\left(1 - e^{-im}\right) + e^{mt} - e^{-im}\right]$$

Now $\frac{\theta}{s}$ and $(s-\theta)$ can be figured, and might come out not too badly.



110. A

Stumpage value, T, shd. generally be derived by looking forward, discounting harvest value rather than compounding history.

$$T_0 = C = g e^{-im} - S\left(1 - e^{-im}\right)$$

$$T_n = g_m e^{-i(m-n)} - S\left(1 - e^{-i(m-n)}\right) = \left(g_m + S\right) e^{-i(m-n)} - S$$

$$= g_m e^{-i(m-n)} - a\,\frac{1 - e^{-i(m-n)}}{i}$$

or

$$T_n + S = \left(g_m + S\right) e^{-i(m-n)}$$

i.e. T_n = g_m disc. less PV of all the rents,

i.e. $T_n = PV_g - PV_n \le a$

When $n = m - 1$

$$m - n = m - m + 1 = 1$$

$$T_n = g_m \frac{1}{1+i} - a\,\frac{\frac{i}{1+i}}{i} = \frac{g_m - a}{1+i}$$

$$T_{m-1}(1+i) + a = g_m$$

110B

So with a bldg.,

$$B_n = PV_n \sum_n^m r_n - PV \sum_n^m a_n$$

where r_n = cash flow or net rev.

a_n = rent of land

This is equally

$$B_n = PV_n \sum_n^m (r_n - a_n)$$

Since you subtract a_n each year from revs., there is a ~~two~~ buildup of values in the bldg. ~~inherent in the bldg.~~ whenever $r_n < a_n$, as with a tree.
Indeed, whenever $r_n < a_n + i B_n$

But whenever you sacrifice early revenues for later ones, the foregone revenues add to value of bldg -- in ordered, or whatever. They shd. not be specified as foregone rents, because revenue is foregone which might be interest too.

Rent and interest are both stored up in capl. -- [or in a l.v., if the payoff is in p.w. e.g.]

In terms of the original problem — do bldgs.

absorb unrealized rents like trees — the answer is yes.
During construction they always do.
~~"not in general"; but sometimes in specific cases, of defrosted~~
→ If rent is expected to rise, overbldg. is likely — sacrificing present & ~~realized~~ early lower rents, for a bldg. thought to realize on higher, later ones.

III. Cost of longevity comes in 2 forms

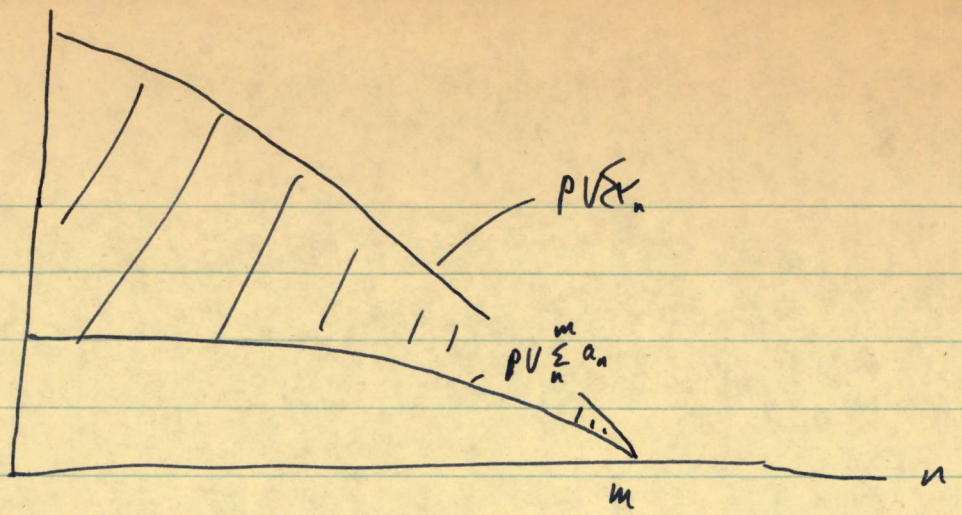
1. ΔC
2. Reduced revs. in early yrs. (& all years) This may be less recognizable — be careful not to overlook it.

E.g. a. Heavy, coarse materials.
b. Thicker walls reduce floor space
c. Eliminate all features that won't last

→ But this is like storing up rents in bldg. — & storing up int. Thus it is fitting that my formulae treat these as the same. An old bldg. is like a tree in that some early revs. were stored in it, to carry it to the present.

---

112. As bldg. ages, $PV \Sigma r_n$ falls, but so does $PV \Sigma a_n$ fall. Does this slow down depreen? It certainly does cfd. to just looking at $PV \Sigma r_n$. But land under bldg. maintains r_n, toward end of life, which speeds depreen. I.e., assuming m to be determined, then to have hi r in later yrs. means each passing yr. lowers value more. This is land's doing — if capl. alone yielded high revs., it wd. have much life left.



$B_n = PV\Sigma r - PV\Sigma a$ is shaded area. Σr falls off slower than $PV\Sigma r_n$ bec. $PV\Sigma a$ falls too.

Toward end of life, esply., the Σr curves are nearly $\parallel$.

So ~~drop of~~ this "slowing down of depreciation" does not mean rent of land is being stored up in bldg.

~~113.~~

113. The tendency of hi i, and t, to shorten life is partly offset by lower a. This applies to both appreic's capl. and any capl. which defers realizn. of a by sacrificing early for late revs.

Bec. lower a reduces the capl. tax base. (Looking backwards.)

(Looking forwards, the lower a means a higher residual bldg value, but that is something else — I think.)

→ It is conceivable that, owing to leverage, t might reduce a by more than t.g. thus slowing harvest.

Not likely, w. tree, but maybe w. house? Work it out.

Use FMT.

114. Paradox

Assume $C = 0$

$$B_n = a_r \frac{e^{n(i+t)} - 1}{i+t} \qquad (1)$$

and

$$B_n = \Theta\left[e^{n(i+t)} - 1\right] \qquad (2)$$

$$\Theta = \frac{a_r}{i} \quad \text{(assuming no land tax)} \qquad (3)$$

Which is wrong?

If I assume (1) is right, ~~then~~ & (3) is right, then

$$B_n = \Theta \frac{i}{i+t}\left[e^{n(i+t)} - 1\right] \qquad (4)$$

With Δi, I don't run into this problem, so this may be a difference of i & t. If Δi, Θ is lower. If Δt, Θ not lowered by higher cap. rate, so Θ is higher & is reduced by coeff. $\frac{i}{i+t}$ to convert it to $\frac{a}{i+t}$. Implying $\Theta \frac{i}{i+t} = \frac{a}{i+t}$

or $\Theta i = a$

114 A.

~~$\theta = R_m e^{-} - C$~~

$$\theta \doteq R_m e^{-(i+t)m} - C + \theta_m e^{-im}$$

No, not right. But check right formula & be sure $r+$ is divis by θ_m only at $\underline{i}$.

$$a = \frac{i+t}{1-e^{-(i+t)m}} \left[R_m e^{-(i+t)m} - C \right]$$

$$\theta = \frac{a}{i}$$

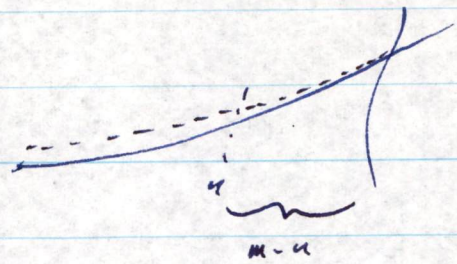
$$\theta = \frac{i+t}{i} \left(R_m e^{-(i+t)m} - C \right) + \theta_m e^{-(i+t)m}$$

Offhand, θ_m is here wrongly divis. by $(i+t)$. Yet this may be part of compensating for the $\frac{i+t}{i}$ in there. Wish I had more direct understanding.

115.

~~The difference of~~

$a\frac{e^{im}-1}{i}$



$a\frac{e^{in}-1}{i} \overset{?}{=} a\frac{e^{im}-1}{i}e^{-i(m-n)} = a\frac{e^{in}-e^{i(n-m)}}{i} \rightarrow a\frac{e^{in}-1}{i}$

Growth path of π is $a\frac{e^{mi}-1}{i}$ $\quad (C=0)$

so $g_m = a\frac{e^{mi}-1}{i}$

$V_n = a\frac{e^{mi}e^{-(m-n)i}-1}{i} = a\frac{e^{ni}-1}{i}$

If we know g_m, i, a, how find V_n? If we know a & i, that's enough. But how about discounting from g_m?

$\frac{V_n}{g_m} = \frac{a\frac{e^{ni}-1}{i}}{a\frac{e^{mi}-1}{i}} = \frac{e^{ni}-1}{e^{mi}-1}$

$V_n = g_m\frac{e^{ni}-1}{e^{mi}-1} = \frac{g}{e^{mi}-1}\left(e^{ni}-1\right) = S\left(e^{ni}-1\right)$

~~See FM for better way.~~

$V_0 = g_m - a\frac{e^{im}-1}{i}$

e^{mi}

~~$V_n = Ce^{mi} + (c+s)e^{ni} - s$~~

~~$g_m = (c+s)e^{mi} - s$~~

$$\frac{V_n}{g_m} = \frac{(c+s)e^{ni} - s}{(c+s)e^{mi} - s}$$

~~Still use THT~~

$$\frac{g}{e^{mi}} = V_0 + a\frac{1-e^{-mi}}{i}$$

$$V_0 = \frac{g}{e^{mi}} - a\frac{1-e^{-im}}{i} = \frac{g - a\frac{e^{im}-1}{i}}{e^{im}}$$

$$\frac{g}{e^{(m-n)i}} = V_n + a\frac{1-e^{-(m-n)i}}{i}$$

$$V_n = \frac{g}{e^{(m-n)i}} - a\frac{1-e^{-(m-n)i}}{i}$$

If $f = Ce^{mi} + a\frac{e^{mi}-1}{i}$, then $V_n = Ce^{ni} + a\frac{e^{ni} - e^{-(m-n)i} - 1 + e^{-(m-n)i}}{i}$

Check ✓

With a tax,

$$V_n = \frac{g}{e^{(m-n)(i+t)}} \stackrel{?}{-} a\,\frac{1-e^{-(m-n)(i+t)}}{i+t}$$

2d term is PV_n of future rents, Σa, from n to m. But why discount them with t? To offset the overstatement of taxes in 1st term. Above is same as (4), p. 53A:

$$V_0 = \Sigma\left[(R_n - a_n)\,e^{-(i+t)n}\right] \qquad (4)$$

$$= \Sigma\left(R_n e^{-(i+t)n}\right) - \Sigma\left(a_n e^{-(i+t)n}\right)$$

What is being discounted with t is the excess of revs. over rents.

Re another problem, it seems that $\underline{a}$ is more fundamental here than θ.

SCANNED

AS

FOUND

i = .07

0 missing?

116.911

TABLE 1

CASH FLOW ANALYSIS OF THE LAND DEVELOPMENT BY A PUBLIC CORPORATION
OF A THEORETICAL CORRIDOR CITY FOR POPULATION OF 150,000 PEOPLE OVER 20 YEARS

ITEM	YEAR ($'000)																							
	−5	−4	−3	−2	−1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Accumulated Investment b/fwd	–	11770	12594	13476	14419	18171	37971	39723	45165	45496	46236	49412	47773	40916	42743	39240	32393	30331	12571	226				
Plus Current Year's Expenditure	11000	–	–	–	2563	22452	6127	10558	8098	8160	14642	10131	13319	19643	14109	18934	24686	22209	23800	9821	7139	4176	4066	1751
Less Current Year's Revenue	–	–	–	–	–	5136	6974	8071	10743	10445	14699	14895	22853	20612	20179	27900	28732	40791	36160	37690	8957	5199	5199	2222
Net Annual Investment	11000	11770	12594	13476	16982	35487	37124	42210	42520	43211	46179	44648	38239	39947	36673	30274	28347	11749	211					
Plus Interest	770	824	882	943	1189	2484	2599	2955	2976	3025	3233	3125	2677	2796	2567	2119	1984	822	15					
Accumulated Investment c/fwd	11770	12594	13476	14419	18171	37971	39723	45165	45496	46236	49412	47773	40916	42743	39240	32393	30331	12571	226					
Accumulated Profit																				27643	29461	30484	31617	32088

NOTES: The table should be read in conjunction with the associated paper.

Development costs, land revenues and staging of commercial facilities have been calculated using Canberra data as the basic guideline.

All calculations are based on constant prices at current levels.

120

TABLE 2

THEORETICAL CORRIDOR CITY

POPULATION GROWTH AND RESIDENTIAL BLOCK SERVICING

Year	Population (at end of year)	Residential Blocks	
		Annual Block Starts	Annual Block Sales
–1	–	800	–
1	800	1,200	800
2	2,500	1,400	1,200
3	5,500	1,600	1,400
4	9,000	1,800	1,600
5	14,000	2,000	1,800
6	20,000	2,200	2,000
7	27,000	2,400	2,200
8	34,000	2,600	2,400
9	42,000	2,800	2,600
10	50,000	3,100	2,800
11	60,000	3,500	3,100
12	72,000	4,000	3,500
13	85,000	4,200	4,000
14	98,000	4,400	4,200
15	112,000	–	4,400
16	124,000	–	–
17	134,000	–	–
18	142,000	–	–
19	147,000	–	–
20	150,000	–	–
		38,000	38,000

$$\ln(1+r) \cong r - \frac{r^2}{2} = r\left(1 - \frac{r}{2}\right)$$

117. This is remarkably accurate. Can you figure out why?

Marc

P.S. I found this by calculating

$$\frac{\ln(1+r)}{r}$$

for various r, and it is $\cong 1 - \frac{r}{2}$

You are rediscovering the triumphs of antiquity, or at least of the 17th century.

Proof one: (setting aside)

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ for $|x| < 1$ by geometric series formula

set $r = -x$

$\frac{1}{1+r} = 1 - r + r^2 - r^3 + \cdots \quad (|r| < 1$

Integrate both sides $\ln(1+r) = r - \frac{r^2}{2} + \frac{r^3}{3} - \frac{r^4}{4} + \cdots + C \quad |r| < 1$

set $r = 0$, find $C = 0$

$\therefore \ln(1+r) = r - \frac{r^2}{2} + \frac{r^3}{3} - \frac{r^4}{4} + \cdots \quad |r| < 1$

Furthermore, in this particular series for positive r the error is less than the first term not used.

2nd Proof: Calculate the Maclaurin series for $\ln(1+r)$. You get the same thing.

118. $$\theta(1-e^{-mi}) = -C_0 + \sum_1^m \left(R_n e^{-(i+t)n}\right) - a\frac{1-e^{-(i+t)m}}{i+t} + a\frac{1-e^{-im}}{i}$$

If $\theta = \frac{a}{i}$ last term cancels out.

But if there is a land tax, $\theta = \frac{a}{i+\tau}$

and in addition, θ_m is discounted at $(i+t)$

Then it is next to last term that drops out, leaving.

$$C_0 = \Sigma\left(R_n e^{-(i+t)n}\right) - a\frac{1-e^{-im}}{i}$$

and

$$a = \frac{i}{1-e^{-im}}\left[\Sigma\left(R_n e^{-(i+t)n}\right) - C\right]$$

and

$$\theta = \frac{a}{i+\tau} = \frac{i}{i+t}\,\frac{\left[\Sigma\left(R_n e^{-(i+t)n}\right) - C\right]}{1-e^{-im}}$$

Agenda. Repeat all tax impact conclusions by normalizing models.
Use Davis paper models.

Begin w. property tax.

120.

Are a's plowed into capl. when it is a bldg?

If not, why does Q rise with a? (See p. 77)

121. a tax on capl., t, is burden on bldgs. & all capl. than an equal Δi, if land appreic.

~~B~~

If there is intertemporal dependence of rents, there is capl. I.e., to defer rent is to plow it into capl., e.g. a tree. This is assessed & taxed, so Δi has same effect as Δt on PV.

But if higher future inc. comes w/o sacrifice, there is no intertemporal dependence, & no capl. to tax. So Δi reduces PV of future inc., while Δt does not.

$$\Sigma = 1\left[e^{-i} + e^{-2i} + \cdots + e^{-mi}\right]$$

$$\Sigma = 1[$$

$$(C+S)e^{mi} - S = R_m$$

$$Ce^{mi} + S(e^{mi} - 1) = R_m$$

$$C = \frac{R_m - S(e^{mi}-1)}{e^{mi}} = Re^{-mi} - S(1 - e^{-mi}) = PV\cancel{R} - PV\Sigma a$$

$$= PV\Sigma(R_n - a_n)$$

$$C = \frac{R_1 - a_1}{e^{i}} + \frac{R_2 - a_2}{e^{2i}} + \cdots + \frac{R_m - a_m}{e^{mi}}$$

This is how the annuity looks when you allow that R_n reps. stored-up fund as well as int. It leads right into my propty. tax eqn. No allocating among years is needed — you lump all together.

$$Ce^{mi} = 7V_m\Sigma(R_n - a_n)$$

A bldg. that appreci. or retains value in old age as a result of higher local replmt. costs has stored up rents like a tree.

Old-law tenement privileges, e.g.

and higher local bldg. costs due to new codes, union rules, materials, wages, etc.