When to Cut a Tree
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When to Cut a Tree

M. Gaffney, 1995

What is the difference between wealth maximization and short run profit maximization? How might the difference be reconciled?

Holding a tree uncut because its growth rate > the i.r. is an example of wealth maximization at the expense of short run profit maximization. Wealth maximization is maximizing the present value of your assets by investing them in ways that yield returns greater than or equal to the discount rate.

To reconcile the difference, add unrealized gains to your current revenues, and subtract depreciation. (The U.S. tax code lets you deduct and even overstate depreciation, but does not make you add unrealized gains to taxable income. This is the result of heavy lobbying, and a bias, and should not confuse you when defining real income, apart from tax considerations.)

In finding when to cut a tree ("Financial Maturity"), one thinks about the value of the site to grow the next tree. One also thinks about future uses of the capital tied up in the standing timber. A full solution takes both into account. This is a great example of optimizing -- defining a performance standard that combines two or more legitimate considerations. Often they are traded off. In this case they reinforce one another: one adds them together to get to total marginal cost of time. You balance them against current growth, which is the marginal product of time.

To bring the future of the forest site into the picture, a good way is simply to maximize PV in perpetuity. Maximizing PV automatically takes interest into consideration. Looking at a series of future cycles, rather than simply the first, automatically brings the future use of land into the picture. Here is how to do it.

As my students know by now, I use the following definition to unclutter the notation:

$$I = 1+i \qquad (1)$$

Following Einstein's advice, the purpose is to "make things as simple as possible, but not moreso." To keep it uncluttered, and focus on the most interesting points, I assume here two simplifying conditions.

a. All generations after the first are to have the same life (n) as the first.

b. Planting cost = 0. Timber replants itself on this land without human cost. Foresters call it "volunteer regeneration" and often actually let things happen this way, although it is lazy forest management and seldom the best use of land. Better, we can include future planting costs in the cost of harvesting the preceding stand. That just means a lower value for each "S", since "S" is net of harvest costs. Since these outlays come in the same year, the effect on solutions is the same as though we entered replanting cost separately.[1] We can easily enter planting cost separately later, when we shall have mastered the harder matters.

1. Faustmann's Solution

The procedure now is to define the present value, or DCF, of land <u>in perpetuity</u>, i.e. from an infinite series of future cycles of planting and cutting. As elsewhere, the symbol for the DCF of land in perpetuity is "T", from the Latin *Terra*. The symbol for the harvest value of timber is "S", for Stumpage, a term used in forestry.

$$T = \frac{S}{I^n} + \frac{S}{I^{2n}} + \ldots + \frac{S}{I^{\infty}} \qquad (2)$$

(2) is a geometrical progression, infinitely long, whose first term is S/I^n, and whose rate of progression is $1/I^n$. Plug those into the formula for the sum of a geometrical progression and we have:

$$T = S\left[\frac{1}{I^n} \cdot \frac{1}{1 - I^{-n}}\right] = \frac{S}{I^n - 1} \qquad (3)$$

To max T in Eqn. (3) set the derivative w.r.t. n = 0. There is a "poor man's calculus" to make it easier. The quotient is max when the % growth rates of top and bottom are the same. You

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[1]The main difference is that we will overstate the value of land that is currently bare, by omitting the first planting cost, K_o. We can correct for this, when needed, by subtracting K_o from (2). It does not affect the solutions shown there.

can see that intuitively: multiply both top and bottom by (1+x), and the (1+x)s cancel out. That is when:

$$\frac{S'}{S} = \frac{iI^n}{I^n - 1} = \frac{i}{1 - I^{-n}} \qquad (4)$$

Voilá! There you have it. That's it. See how easy? (4) is also mnemonic (easy to remember) because the right side is nothing but our old friend IPF (the Installment Plan Factor). Note that this solution for optimal harvest time requires that the tree's growth rate be greater than the interest rate [$(S'/S) > i$]. This in turn means that $S'>Si$. The current growth, S', must be enough to cover more than just interest on its own value. It must grow faster than money in the bank, that is.

Compare that with the solution of the simpler problem (the one-generation problem) where the solution is $S'/S=i$, or:

$$S' = Si \qquad (5)$$

(5) is the lazy economist's solution, it's so easy and, speciously, obvious. It's what you'll find in most texts. However, it is too lax and, therefore, too late. In (4), we are requiring a higher growth rate, meaning a shorter life for the tree before harvest. (Remember, the growth rate, S'/S, keeps falling as S rises.)

2. Faustmann's Solution and the Marginal Cost of Time

The solution to the tree harvesting timing question has an intuitive interpretation that makes it easier to remember. We will show it says simply that the marginal cost of time is $i \cdot (T+S)$. That is, tree growth must be fast enough to pay interest on both Stumpage and *Terra* Value (T).

Let us begin by looking at the simple-minded Marginal Solution, expressed in Eqn. (5). (5) is ordinary because its intuitive interpretation is so simple, so easy for economists to grasp without much thought. It says the marginal cost of time is just $i \cdot S$, interest on Stumpage alone. The trouble with that is this, it assumes the land is free. See the paper, "The Fallacy of Free Rent."

Clearly in (4), compared with (5), we have added something to the marginal cost of time. We can express that as the cost of

deferring future harvests, which is formally correct, but a little hard to visualize. We will therefore show next that what we have added is the annual value of the forest site, or ground rent. Foresters use the German name, Bodenrente, because this was originally worked out by a German forester (Martin Faustmann). We will refer to it simply as rent, and it is the same as $i \cdot T$.

We now ask, what is added to $S \cdot i$ in (5) to get the result in (4)? (I leave it to you to rearrange (4) to see what S' equals there.) That unknown extra something we will call "x", then find it.

$$\frac{Si}{1 - I^{-n}} = Si + x \qquad (6)$$

Solve (6) for x (an algebra exercise that I leave to you). You will find that:

$$x = i \cdot \frac{S}{I^n - 1} \qquad (7)$$

But the quotient in (7) is T, from (3), so x is interest on the site value, T.

$$x = T \cdot i \qquad (8)$$

Interest on site value is also rent. So "x" turns out to be rent.

So-o-o-o-o, what (6) and (8) boil down to is that tree growth has to pay interest on tree value ($S \cdot i$), plus interest on site value ($T \cdot i$). When the tree growth stops doing so it is time to cut and cash out, freeing both your capital and your ground for re-use.

For another algebraic way to segue from (4) to (8), see Appendix 1.

As to trees crowding other trees spatially, that affects the path of S and S' and is thus already accounted for in part in our model. There is another dimension to this problem, the relation of planting cost to growth rates, which we have yet to put in. That is another problem for another day.

3. Various False Solutions

Now we have answered "When to Cut a Tree" (and so easily, too!) it is fun to survey a few false solutions that have beguiled and bedazzled economists over the years, and still do. Different people have lined up behind at least four wrong maximizing solutions to the question, "When to cut the tree?" This is more than a classroom theory question: this is theory applied in the field. The parties also actually manage timber in the four different ways, and it makes years of difference. It's a good example of how theory drives behavior.

Here are four false alternatives:

A. Max S. Let each tree grow to its greatest biological capacity.

B. Max S/n. S/n is aka Mean Annual Increment (MAI). Let each tree grow until its mean yearly growth is a max.

C. Max $S \cdot I^{-n}$, that is the PV of Generation #1 (G_1).

D. Max IRR on planting cost. (Here, we need to express planting cost separately.)

Every maximizing "solution" or performance standard is also a marginal criterion. Let us see what marginal conditions correspond with each of the four maximizing solutions. Then we will show in brief the errors in each fallacy, viewed as a marginal solution.

A. Max S.

This occurs when the marginal product of time, S', falls to zero: $S'=0$. You have to be a pretty deep ecologist to go for this. It ignores all the economic costs of time, both interest and ground-rent. To be sure, there is a growing number of deep ecologists, whose values are, or should be, part of the market. On this matter, see Appendix 2.

B) Max S/n.

Maximizing S/n w.r.t. n occurs when $S'/S=1/n$. (You show this by applying the "poor man's calculus," letting the fractional rise of the top of the ratio S/n equal that of the bottom.)

We have seen in Eqn. (4) that in the correct solution, S'/S = IPF (the Installment Plan Factor). We know that IPF > 1/n (see the paper "Basic Mathematics of Finance," pp. 15-16.) You

can also show that IPF ---> 1/n when i ---> 0. That makes max S/n a zero-interest solution.

Hold on now, not so fast, how, for Heavens' sake, do we know that IPF ---> 1/n as i ---> 0? Looks to the naked eye as though it really approaches 0/0, which we were often taught is "indeterminate" - so we can stop thinking about it, right?

Wrong. There are two ways. First, if you are handy with calculus, apply l'Hôpital's Rule.[2] I'll leave that to you - it's in your calculus book.

Second, just press some keys on your calculator. Evaluate IPF for nice low values of i, like i = .0001. Round off your answers to two places - you'll find they are 1/n. Not "indeterminate" at all.

When i=0, S/n is the annual value of land, i.e. the rent. What this solution does, then, is to equate growth to the marginal cost of land-time, while treating the marginal cost of capital-time as zero. At the same time, however, it inflates the rent by assuming zero-interest. The second error tempers, without eliminating, the first. It's a subtle point that has eluded many. Don't be upset if you find it difficult, especially in this rush treatment designed for a short course. It's really a terribly important point, in understanding economics overall; but here we'll leave it at that.

It's interestingly different from the more common fallacy, (C). It's common today for economists to be aware of interest, and blind to rent. In Fallacy (B) it's the opposite: it's blind to interest, and overstates rent. It's an ideology that comes from the forestry profession, which viewed its mission as "full use" of forest lands, regardless of costs other than land.

C) Max SI^{-n}

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[2] l'Hôpital's Rule is really Bernoulli's. l'Hôpital was Bernoulli's rich patron, whom Bernoulli had to humor, just as the Egyptian Astronomer Royal left us with the constellation "Coma Berenices" (Bernice's hair), honoring a beautiful feature of his patron's Queen. The things scientists will do to get support! l'Hôpital got his peculiar name from an ancestor who was a member of an order of Knights in The Crusades, the Knights of the Hospital. These Knights were given the island of Malta in 1520 (remember Bogie in *The Maltese Falcon*?). They were major players in European power games for centuries.

You max SI^{-n} when $S'/S=i$, so $S'=S \cdot i$. We have already seen in Equations (5)-(8) that this equates current growth with the current cost of capital, while ignoring the current cost of land.

D) Max IRR

You max IRR when $S'/S=i^*$, and $i^*>i$. You find i^* by taking the nth root of S/P less One, where P=Planting Cost. We have no explicit P in our equations, but we can still show the error in (D) from this very fact. It is possible to grow timber with P=0, by volunteer regeneration. In such a case, $i^*=\infty$, which is absurd. I'll leave it at that.

Figure 1 (hand-drawn) shows the basic juxtapositions of IPF, i, and 1/n as functions of n. IPF is always higher than either i or 1/n. For low n, IPF ---> 1/n. For high n, IPF ---> i.

You may use Figure 1 to illustrate the marginal forms of Faustmann, and false solutions (A), (B), and (C). Draw another curve, of gentle negative slope, labelled S'/S. Draw it so it intersects all three of the curves shown, and finally crosses the abscissa. The intersection with IPF is Faustmann's. The intersection with i is (C). The intersection with 1/n is (B). The intersection with zero is (A).

Figure 8: Marginal Costs of Time, Expressed as Percentages

Appendix #1, "When to Cut a Tree"

This is an alternative algebraic route to showing that we maximize T w.r.t. n when S′, the growth of stumpage value, just covers the sum of interest and rent.

From Faustmann, Eqn. (4), the optimal marginal condition is:

$$S' = Si \cdot 1/[1-I^{-n}] \qquad (9)$$

The purpose of (9), which is a simple revision of (4), is to factor out Si, in order to identify the other element on which Faustmann requires the tree to earn interest. We know there is another element, because S′ in (9) is greater than Si. But we cannot immediately see what that element is, because the information comes to us in the denominator of a ratio, rather than as something we add.

Method: add and subtract I^{-n} to and from the numerator, letting us then factor out 1.

$$S' = Si \frac{1+I^{-n}-I^{-n}}{1-I^{-n}} = Si[1+1/[I^n-1] = i[S+S/(I^n-1)] \qquad (10)$$

But $S/(I^n-1)$, the last term, is Land Value, T.

So Faustmann boils down to this:

$$S'=i(S+T) \qquad (11)$$

The left side of (11), S′, is the marginal product of time -- the yearly growth of S.

The right side of (11), i(S+T), is the marginal cost of time -- the foregone gain of tying up the Stumpage and the Land for another year. Each of these has an opportunity cost. The Stumpage could be liquidated for cash and reinvested. The Land could be sold to another investor and the cash reinvested. From a social viewpoint the land could not be liquidated, but it still has an opportunity cost: it could be cleared and put to use growing the next crop. Its annual rent would be a, and a=Si.

Appendix 2: Forest Values other than Harvest Values

Let's not fall into the Scrooge fallacy however, one in which the value of all forests is solely producing timber for pulp or lumber. So far we have made that assumption, to keep things simple and focus on one aspect of management. As you know, however, forests also provide recreational and esthetic and ecological values. In modern times, these values have risen in public consciousness. Many of them are common property values that are real, but underrepresented at the cash register. Good, holistic forest management finds ways to take them into account. I make no pretense that the present model does so. Such accounting is another big topic in itself, one not addressed here.

Max S is no doubt right for the Generals Sherman and Grant Trees in Sequoia National Park. For them, in fact, "S" becomes irrelevant, since they are maintained and managed to meet another kind of demand, as they should be.

The great mass of forest land provides commercial timber values primarily. In between the Grove of the Giants, and the uninteresting flat rainy gloomy sites where most timber grows, are lands where the battles rage between loggers and protectors of the Spotted Owl. These battles occur because there is a lot to be said on both sides of the case, and there are lands where the calls are close. The conflicts are really interesting to economists, who can and do contribute a lot to finding optimal trade-offs.

The best solutions are not trade-offs at all, but win-win solutions like replanting cutovers immediately, and managing vigorous young stands to maximize wildlife habitat. There are a lot of careers out there for modern forest managers who can see all sides of these complex issues where traditional economics is still important, but isn't everything.

Meantime, Max S is extremely costly in wasted capital and wasted land. In terms of our present model, it is indefensible. In a more holistic model that includes other than harvest values, our model is still an important element, to explain the extremely high cost of maintaining old-growth forests, and the desirability of finding alternative ways to meet ecological-esthetic-recreational demands.