UCLA, 201C

$e = 2.7\underline{1828}\underline{1828}4\ldots$

$e^{\frac{1}{10}} = 1.10517$

---

$\log_e e = 1$

$\ln e^{\frac{1}{10}} = \frac{1}{10}$

$\ln(1+r) \approx r\left(1 - \frac{r}{2}\right)$

~~$\ln(1+r) \approx r - \frac{r^2}{2}$~~

so if $\ln e^{\frac{1}{10}} = \frac{1}{10}$

$e^{\frac{1}{10}}$ is $(1+r)$ & $\frac{1}{10} = r\left(1 - \frac{r}{2}\right)$

If $r = .10517$,

$r\left(1 - \frac{r}{2}\right) = .0996396$

.3677381

$$\left(1-\frac{1}{1000}\right)^{-\frac{1}{1000}} = \frac{1}{\left(1-\frac{1}{1000}\right)^{\frac{1}{1}}}$$

$$\left(1-\frac{1}{1000}\right)^{-1000} = \frac{1}{\left(1-\frac{1}{1000}\right)^{1000}}$$

HB
501
.H385

Hawtrey, Capital & Employment 1952, pp. 11-28

Re: Period of prodn = $\frac{1}{\text{Turnover}}$

HC
57
.H35, 1933

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When durable capl. is new, it depres. slowly. ∴ in transit to more longerous capl. structure, the early moves cut down replacement more than will happen later. Cf. Domar.

E.g. pending structures depres. slower than cars. So when we first begin to build them, ratio of their value to car value will be higher than after equilibrium.

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Edwin S. Mills, 'The Theory of Inventory Decisions', Ectra XXV, No. 2 (1957), 222-38.

(See if this is the same Ed. Mills. If so, send T(S.T.OC)

~~Ec. 201C~~

Readings ~~& supplemental list~~ from Lutz & Lutz, Theory of Investment of the Firm (~~Our little textbook that was to have been.~~)

1. Valuation.

Chap. 1, 'The Time Element in the Theory of Production and Costs.'

2. Replacement

R Chap. 2, 'Criteria of Profit Maximization.' Watch out for the fallacy on pp. 33-34, in their 'synchronized' (i.e. staggered) process. This error, slipping in early, ~~in the work~~, is repeated several times later and detracts greatly from the overall value of the work. Fortunately, they ~~reject the~~ soft-pedal the 'synchronized' process in ~~some~~ chapters, 8&9, saving them from utter ruin.

R Chap. 8 'Durable goods: Lifetime.'

Here they assume $C_0$ fixed and ask what determines life.

p. 107. The Infinite Chain, is most important. Here, as with their infinite chain of point-output cases (trees), pp. 32-33, they find the right answer. But this time they cannot bring themselves to junk it in their staggered case, although they mount a half-hearted effort (pp. 108-09, which wander aimlessly.)

pp. 109-14, Influence of Anticipated Obsolescence.

Note how this analysis can be transferred to appreciating suburban land, and bears on Ely's doctrine of 'tipping costs' — but leads to reverse conclusions.

Note their use of "goodwill" here for what in Chap. 2 they call "profit". (and what Ely or Ratcliff would call land rent.)

Chap. 9 "Durable Goods: the Choice of Technique."

"Technique" means the quantity of $C_0$, which in Ch. 8 was held fixed. ~~in Chap. 8.~~

As in Ch. 8, the truest case is the "infinite chain", pp. 120-22, and again they drop their erroneous 'synchronized' case.

3. Macro-economics & factor proportions.

Chap. 6, pp. 91-93, Measures of Capital-intensity

Chap. 9, ¶ 8, pp. 125-27, "Measurement of Capital-intensity.

Chap. 11, 'Capital Intensity and the Trade Cycle'
Ricardo effect, etc.

Chap. 12, 'Durable Goods: the Scale of Investment'

Criticizes acceleration principle

4. Public Finance

R

Chap. 19, pp. 224-36, 'Depreciation Methods'.

Buried deep in the muddy text, on p. 226 you find that only one depreciation method, the capitalization method, ~~holds~~ keeps a constant ratio of income-after-depreciation to book value. By great coincidence this is also the only method found by Conigny and Saffrey to be free of intertemporal bias when used to define taxable income. In seeking to understand why the I.R.S. has ~~does~~ not adopted this method long since, try the homicide expert's question, cui bono? (Never mind cherchez la femme.) Query: is the sinking fund method (p. 228) really equivalent to the capitalization method, as alleged (p. 231, n. 12)? ? doubt it, but haven't proved it. Can you beat me to it? Again, cui bono?

5. —

6. —

7. Credit rationing & allocation

Chap. 10, pp. 130-36, Choice of Technique (i.e. of $C_0$) with a Rising Supply-of-funds Curve.

Supplemental Assignment.

4. Public Finance

Babcock, Valuation of Real Estate

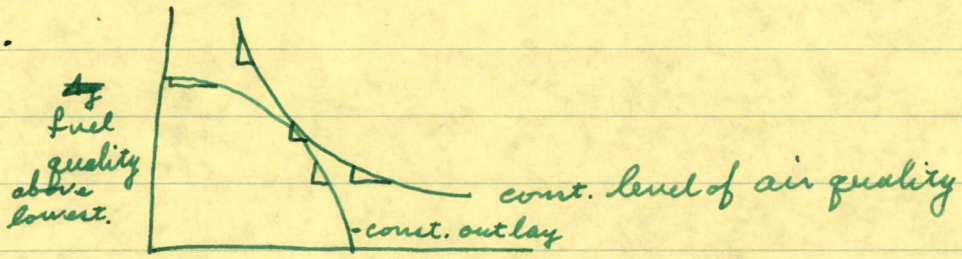
Chap. 21, "Depreciation, Obsolescence, and Building Life."

Ec. 701

4-15-64

1. Gas stations as e.g. of operating in stage III
Or of MR < 0

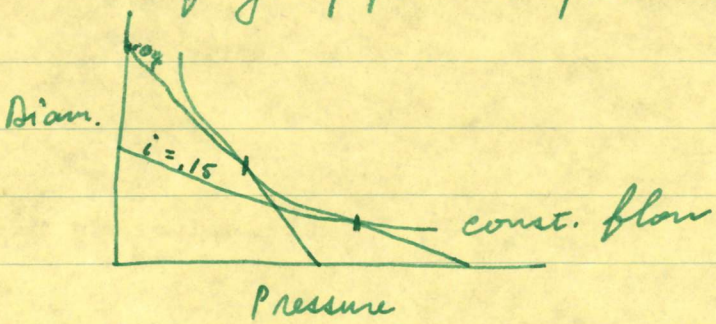
2.




Xptn. (bcs. of progressive moving-outwards of polluters)

Above simple scheme compresses much info. & common sense. Surprising how much it is needed by how many - e.g. ACC.

3. Trade-off between capl. costs & opg. costs is one of most common problems in pvt. & public policy. E.g. ~~double~~
Δ diam. of gas pipe or Δ pressure.




Convert cap. costs of pipe to yrly. basis by $c = C_o \frac{i}{1-(1+i)^{-t}}$

(Note implications for employment of labor)

4. Various "runs". No sharp line? Yes there is, bcs. replacement is discrete. LR & SR have meaning w. respect to replacement cycles.

"Hi cost" firm in SR includes those near end of replmt. cycle. Also those w. poor credit who habitually sub. opg. cost for capl. cost.

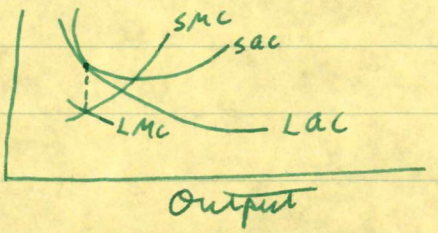
In LR, there are no hi cost firms bcs. there are no firms — they too are replaceable in LR, movable, etc.

In LR, then, there are only hi cost areas. E.g. Wisc. for corn.

Area that is low cost for premium outputs becomes hi cost for everything else — thus selects them out.

Moral: terms are relative. Use w. care.

5. Envelope cost curve.




a. Explain why LMC = SMC where LAC = SAC

b. " " env. doesn't go thru min. of SAC

c. If demand growing steadily, when do you abandon old plant?

6. S curve as sum of MC curves.

3-2-64

1. pp. 52-58; 69-76. Of fncl. mat.
Ch. 5, Liebh.

2. The operational formula

$$g' = gi + a = \frac{gi(1+i)^t - gi + gi - C(1+i)^t i}{(1+i)^t - 1}$$

$$= (g - C)\frac{i}{1-(1+i)^{-t}}$$, or cap. recovery factor, applied

not just to $g$, but $(g - C)$ ... i.e., your income net of costs of beginning again.

3. Interrmed. costs & recs.

Define $\underline{a}$ as: $a = \frac{\sum_0^t [R_n(1+i)^{t-n}]\, i}{(1+i)^t - 1}$

Either: a) Max. $a$

b) $g' + R_t \geq gi + a$, [where $g'$ and often $= 0$]

4. Usually most cost & rec. are interrmed.; and $g = 0$

Draw picture.

Zero-interest.

Max. $\frac{g}{t}$, when $g' = \frac{g}{t}$

$\frac{g \cdot c}{t}$ is same as $\frac{[g - C_0(1+i)^t]\, i}{(1+i)^t - 1}$ when $i = 0$

Intermediate costs & revenues —

move them to time $\underline{t}$, proceed as before. Or time 0 ...
concentrate them at the end points.

Depreciating assets.

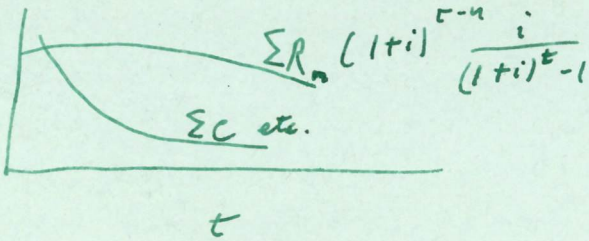
$g' = 0$ ; $g = 0$, two ... no salvage value.

MR of time is $R$

$R = a$

$$a = \Sigma\left[R_n (1+i)^{t-n}\right] \frac{i}{(1+i)^t - 1} \simeq \frac{1}{t}$$

Separate C & R, draw picture




Lens-shape. Note analogies to space problems.

In planning ahead, plan to max. $\underline{a}$, by maxg. distance between lines. Graph $\underline{a}$ separately.

In endg present cycles, end where $R_t \leq a$. Defender & challenger. That is when defender value = 0. Unless it can be moved -- then terminate even sooner, move to less imptnt "niche".

$R_t$ is mgnl. rev. If $R_t = a$, it won't change the overall time-ave., i.e. $a = \dfrac{\sum (R_n (1+i)^{t-n})\, i}{(1+i)^t - 1}$ ~~Mg.~~

Proving it wd. be fun.

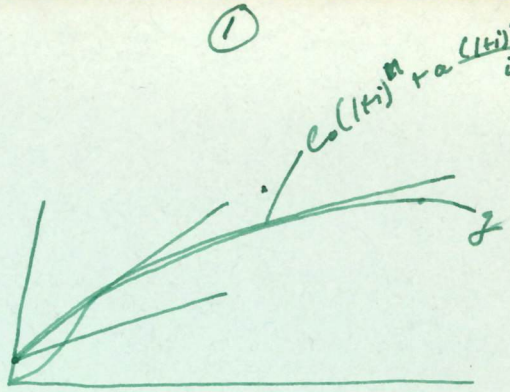
MNR, and i.q. concept.

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Demand curves.

1. Boulding






Max. i.

$i_B \equiv \left(\frac{g}{c}\right)^{\frac{1}{t}} - 1$ If $C = 0$, $i \to \infty$

Mkt. i.r. has no effect on soln.

Mgnl? ~~Ps~~ Cd. be so. if take $i_B$ as mgnl. cost of time.

But can't — no return on site.

Analogue ... max. A.P. — or any other partial solution

Ask for e.g.s. —

Output / man

" / ton of coal, per $ of bldg. cost

Benefit-cost ratio

"Efficiency", "productivity"

Watch for "fallacy of arrogation".

Trick is to meld all together.

Very impt. in ec. growth theorizing today. Boulding's path to factor capl. accumulation, etc.

Tie the elements together by basic =n:

$$g = C_0(1+i)^t + a\frac{(1+i)^t-1}{i} = (C+S)(1+i)^t - S$$

Solve for S (↑ as t ↑) & max; or solve for i and max. Depends on circumstances.

i probly best. Boulding just defined it too narrowly.

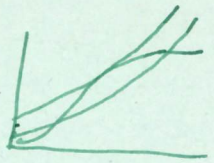
4. Max. discounted value. ~~(over)~~

$g' = g^i$ [cf. $g' = g^i + Si = i(g+S)$]

a. Get no return on land. (or feed, etc.)

b. Appear to get windfall in yr. 0, max $g$

$P_0 = \frac{g}{(1+i)^t}$



c. No change if $C_0 \uparrow\downarrow$

max. $\frac{g - C(1+i)^t}{(1+i)^t} = \frac{g}{(1+i)^t} - C$

d. Lutz' Laputan approach. Implicitly assumes site value = 0 by letting no. of overlapping inf. chains ↑ w/o adding to land cost.

5. Max. internal r. of r.

Mgnl. qual. breaks down?

Inf. return if $C = 0$.

But no return on site. [Pull out map.]

I.r. has no effect on solution.

Is not optimizing, but max $g$, a partial solution.

Analogous to max. a.P. of labor.

~~6. Exam — why Faust > others?~~

~~What if uneconomically understocked?~~

6. Cf. Boulding & Fauntman.

$$= g(1+i)^{-t}$$

Derive by max $g$ $\frac{g}{(1+i)^t}$ by derivation $= 0$.

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(1+i)^t g' - g\rho(1+i)^t}{(1+i)^{2t}} = \frac{g' - g\rho}{(1+i)^t} = 0$$

$g' = g\rho$

$$\frac{d}{dx} u \cdot v = u\frac{dv}{dx} + v\frac{du}{dx} = g$$

$$\frac{d\,u^x}{dx} = \ln u \cdot u^x$$

$$\frac{d}{dt}(1+i)^t = \ln(1+i)(1+i)^t = \rho(1+i)^t$$

1. Revise correct solution. Max. $a = \dfrac{[g_t - C_0(1+i)^t]\, i}{(1+i)^t - 1}$

Is same as max $g$ $S_e = \frac{a}{i}$.

Derive by capitalizing; or by indep. max $g$ $\sum_{t=1}^{t=\infty} \dfrac{g_t - C(1+i)^t}{(1+i)^t}$

$$\frac{1 - (1+i)^{-t\infty}}{1 - (1+i)^{-t}} = \frac{1}{1 - (1+i)^{-t}}$$

Why do it? a) Considers whole of future time. Comprehensive. Balanced ... all inputs.

b) Consistent w. mgnl. annal.

$$g_t' = g\,i + a = i\,[g_t + S_e]$$

2. Important extended implications re factor propns. Macro & micro.

3. The operational form:

$$i = \frac{g'}{g - C}\,[1 - (1+i)^{-t}]$$

✓ Rate of growth = Cap recovery factor. Tabulated.

$$\frac{g'}{g-C} = \frac{i}{1-(1+i)^{-t}}$$

4. Const. annual costs, no effect.

Res. just balance $a$.

also taxes. [Except for differential capr.]

$$g' = g i + \frac{[g - c((1+i)^{\tau}]}{(1+i)^{\tau} - 1} i = \frac{g i (1+i)^{\tau} - g i + g i - c((1+i)^{\tau} i}{(1+i)^{\tau} - 1}$$

$$g' = \frac{(g - c)\, i}{1 - (1+i)^{-\tau}}$$

Ec. 701, 2-11-64

1. Compounding

$P + Pi = P(1+i)$, etc.

2. Discount



3. Valuation - summation

Series: $a + \frac{a}{1+i} + \frac{a}{(1+i)^2} + \cdots + \frac{a}{(1+i)^{n-1}}$

Sum of series: $\sum_{0}^{n} ar^n = a\frac{1-r^n}{1-r}$ (over)

Let $r = \frac{1}{1+i}$

$$\sum_{0}^{n} a(1+i)^{-n} = a\frac{1-(1+i)^{-n}}{1-\frac{1}{1+i}} = a\frac{1-(\quad)^{-n}}{\frac{i}{1+i}} = a\frac{1+i-(1+i)^{1-n}}{i}$$

$$= a\left[\frac{1-(1+i)^{1-n}}{i} + 1\right]$$

Now drop the 1, and the n+th term becomes the n-1,

and get $\sum_{1}^{n-1} r^n = a\frac{1-(1+i)^{-n}}{i}$

a basic valuation formula. Memorize.

~~Alt. forms.~~ $\frac{V_0}{a} = \frac{1-(1+i)^{-n}}{i}$

$$\sum_0^n r^n = 1 + r + r^2 + \dots + r^{n-1}$$

$$r\sum_0^n r^n = \quad r + r^2 + \dots + r^{n-1} + r^n$$

---

$$\sum - r\sum = 1 - r^n$$

$$\sum_0^n r^n = \frac{1 - r^n}{1 - r}$$

4. Alt. forms

a. $a = V_0 \dfrac{i}{1-(1+i)^{-n}}$ Annuitization

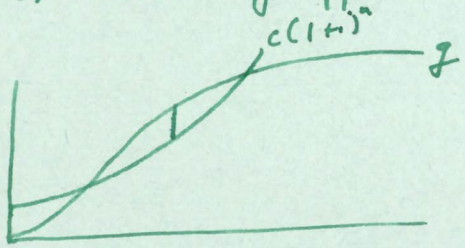
Capital recovery

b. Annualization of a future sum. (Sinking fund)

$$a = \frac{V_n}{(1+i)^n} \frac{i}{1-(1+i)^{-n}} = V_n \frac{i}{(1+i)^n - 1}$$

"Time-averaging".

5. Fncl. maturity application.




Max. annual equivalent of $g - C_0(1+i)^n$

$$a = g - C_0(1+i)^n \frac{i}{(1+i)^n - 1}$$

3-14-66

Supply of Capl.

Responsive to i.

Lower asset values, effect
~~Less~~ disinvesting at higher i.

Implications of i = 0.

Is there a social cost to ~~the~~ +the aggreg. supply? I.e., is it inelastic? Yes -- & can also migrate.

Overlooking saving, Δ inv. opps. → Δ inv., Δ capl. supply.

Δi also means Δ intensity of use. Discussing labor & land, we ruled this out. Case of capl., is much more imptnt. Δ turnover.

S.r. response may be much slower than long, bec. capl. frozen. Current Δi; impending liq. crisis. Hi i can't squeeze blood out of stones.

Supply also peculiarly affected by collateral values. So hi i may reduce supply, over some pd., if pulls floor from collateral.

Tendency to concn. Kochan's paper. Supply is different for diff. people. Insulated mkts, side by side. Will discuss more under dist. & time. Holland's paper. — add to reading list. ✓

Non-coordn. of factors. Rolph's tortured efforts to refute. Labor gets its discounted MP. E.g. sapling value; discovery value; value of a bldg.

Capl. is "residual" claimant, not land. Euler's theorem

is timeless. Introduce time, & capl. gets residual. The difference from interest is profit – close to common parlance.

Fn. of capl. – carry products thru time. Frozen up labor & land. Q. of capl. relative to output, depends on longevity of capl. E.g. if yr. inv. turns daily, it = daily output; if turns yearly, = 365 x daily output, & ave. unit takes 365 days to get thru.

Obvious, but many costs. are highly obscure en.

How measure? Defer to <u>Time</u>.

Why distinguish factors?

a. Different supply conditions
b. Man is msre. of all things. Tawney.
c. Capl. is residual
d. ~~Exis~~ Unemployed labor is bad.

Speaking tour

Johns.

Welf. ecs. MC = MP; effluent charge; measurement of benefits; role of taxes; Coase & distribn.

Interdisc. communicn. MDs

Urban renewal & taxes.

What kind of cities? Vert. vs. horiz. ~~etc~~. Externalities

disturbance. This seems to me to push the argument too far. Surely the issue of whether the marginal propensity to consume transitory income is zero, positive but less than that out of permanent income, or equal to that of permanent income is one that should be settled by empirical testing, not defined away. And prospective readers should be warned that Friedman and also Modigliani and Ando will probably not accept all the conclusions of this highly stimulating work.

In any case, the book is a first rate piece of scholarship, well worth the time invested in reading it for those who can read French. Until we are blessed (or cursed) with instantaneous translation machines, books like this will continue to be the strongest arguments for retaining Ph.D. language requirements. (The English summary at the end of the book is much too brief to convey much more than could be obtained from translating the Table of Contents.)

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*Economics of the Property Tax.* By DICK NETZER. Washington: Brookings Institution, 1966. Pp. xvii+326. $6.75(cloth); $2.50 (paper).

This book provides the most comprehensive treatment of the property tax since Jens Peter Jensen's *Property Taxation in the United States* was published in 1931. It is well written and organized. It can be profitably employed as required reading in both graduate and undergraduate public finance courses. It is also manageable reading for non-economists. Netzer has rendered tax students a valuable service.

The book treats the property tax empirically, theoretically, and normatively. Although Netzer does not compartmentalize these three phases of the subject (an impossible objective anyway), it is useful to review his effort in each of these areas.

The most valuable part of the book is the comprehensive empirical material. Netzer extensively uses the 1957 and 1962 Census of Governments and, especially, the surveys on property taxation that compare sales values to assessed values for samples of properties, broken down by geographic area and types of property (for example, urban versus farm, commercial, housing, land, etc.). These data, in turn, are related to wealth estimates by Goldsmith, GNP, and value-added data, government spending, and other tax yeilds. Frequently, estimating techniques are used to infer critical quantitative factors; whenever this is done, Netzer makes his methodology and assumptions clear, and indicates the possible shortcomings of his assumptions. He also draws upon numerous special empirical studies done by tax students, including unpublished doctoral dissertations and efforts of local agencies and study groups. All in all, Netzer's work sets excellent standards in taking diverse and often imperfect but voluminous data treating the behavior of a complicated tax system, and putting it in a form that gives the reader a comprehensive (but not superficial) view of the subject, yet with ample but not overwhelming detail. This performance alone is sufficient to warrant the book being a part of the personal library of every tax student.

Netzer's theoretical orientation and hence economic analysis is highly and unfortunately conventional. There are two competing theories of tax shifting. One is the application of the Marshallian partial equilibrium method, which concludes that to the extent the price of a tax-afflicted item rises, the tax is borne by "consumers." Hence, the property tax on housing as well as on business real estate and personal property (inventory and equipment) are "consumption" taxes. Since low-income individuals consume a larger proportion of their income than do high-income individuals, the property tax tends to be "regressive." This is one of the reasons many students dislike the property tax.

The competing theory of tax shifting is explicitly based on an application of the Walrasian approach to price analysis. Although a partial tax causes the price of the taxed item to be higher (as the conventional theory asserts), it has this effect because it causes resources to avoid the taxed activity, which reduces supply. The resource displacement, however, necessarily means that more resources are in other activities, which increases output of other commodities and thus reduces their prices. Some consumers are worse off; others, better off. It is not meaningful, therefore, to say that consumers—in some aggregate sense—"bear" the tax. A more relevant question to ask is: The tax extracts dollars from the private sector. There are fewer dollars available for resource (capital and labor) earnings. Which resource owners experience the tax's "income effect" and

have fewer dollars and lower earning rates as a result of the tax?

If we apply this general-equilibrium price model to the property tax, we arrive at the following general conclusions: If the property tax's initial impact is relatively heavy on housing (as compared with commercial and manufacturing assets), capital resources are encouraged to enter non-housing activities. The prices of housing services are higher, but the prices of other products and services are lower. Over all, the net earnings of capital resources, including those to be created on the margins of new investment, are lower. Hence the rate of return on investment is lower. Owners of capital bear the tax. And, since ownership of capital is concentrated, the property tax is roughly progressive.

Thus Netzer, who appears to like progressive taxes, should give the property tax some positive points. Because the tax and its announcement effect reduce the rate of return, it may or may not have an impact on aggregate investment and hence on long-run growth, development, and the distribution of income as between workers and capital owners. It all depends upon the particular theory of investment, interest, or philosophy of history that one wishes to utilize. Although Netzer qualifies some of his assertions with the recognition that property taxes may affect interest (or capitalization) rates (which incidentally undermines his acceptance of the conventional wisdom that "consumers" bear property taxes), he addresses none of these theoretical issues head on. Indeed, Jensen's 1931 work merits a better score in the theoretical department.

The issue of whether property taxes affect rates of return and capitalization rates is central to the application of the tax capitalization theory to land or site value taxation. If land is not subjected to effective property tax rates which are as high as those imposed on capital improvements and capital goods, the opposite of tax capitalization will occur; that is, the property tax system will cause land values to *rise* if it also causes the over-all capitalization rate to fall. Since Netzer draws heavily on the classical tax capitalization doctrine in his treatment of land taxation, it is unfortunate that he did not devote attention to some fundamentals of interest or capitalization rate theory.

Netzer's normative evaluation of the property tax in part rests upon his theoretical orientation, that is, "consumers" bear most of it and it is thus "regressive."

Another major criticism of the property taxation—shared by Netzer and many tax students—is based on the observation of the great variance in effective property tax rates due to assessment practices that cause ratios of assessed to actual values to differ as between different tax payers. This result of property tax administration is well verified by evidence. But is it a valid basis for criticism? However much a lack of "horizontal" equity may be lamented, the relevant practical question is whether the property tax is any worse than other major taxes. There is a tendency for tax students to compare actual property (and sales) taxes with hypothetical personal income taxes which are devoid of the vagaries of administration or the unknown effects of many complicated structural features. (There is also a recent tendency to compare existing business taxes with hypothetical value-added taxes.) Far more knowledge of the operation of all types of taxes, as a function of administrative practices and detailed legal provisions, is necessary before it is possible to award final grades on horizontal equity to any class of tax.

Netzer concludes that the property tax is neither the best nor the worst "of all possible choices." The main reason for this conclusion seems to be that feasible alternatives yielding comparable revenue are not readily obtainable. Taxation of "unearned" land increments would yield inadequate revenues. More systematic use of explicit-user charges pose their own set of administrative and policy problems. Moreover, in Netzer's view, the property tax constitutes a rough proxy for user charges anyway. Perhaps Netzer is right on these general conclusions, but they should not be accepted without more careful scrutiny of the property tax system and alternatives, with an eye toward addressing what the best "mix" of financing devices should be. Nevertheless, Netzer's book will provide a good departure for a refocusing of effort centering on the problems of local and state government finance.

J. A. STOCKFISCH

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*A General Theory of Tax Structure Change during Economic Development.* By HARLEY H. HINRICHS. Cambridge, Mass.: Law School of Harvard University, 1966. Pp. xvi+154. $3.00.

This book, originally Hinrichs' doctoral dissertation, is ostensibly aimed at developing a

~~See Baumgarth 8:30~~



5-6-64

Imputations thru time. There is a residual imputee, not arbitrary. He who is committed irrevocably, has no o.c. Equity holder.

Is a fn. of capl. Other inputs have o.c., constantly.

Here is a true meaning for "residual". Ex post.

Makes Euler unnecy? No, Euler holds ex ante, & in absence of surprises. Also, holds for combos. of all inputs except capl., bcs. they are paid first. - the "discounted MP."

Factor inputs surely contributed at different points in time — held together by capital.

Surely, big dose of labor first, using negligible time of capl. & land. Then long pd. of latter 2.

During all this time, land has o.c., capl. doesn't. So impute to land the o.c., resid. to capl.

Some supply capl. w/o sharing residual or equity risks. They get lower i.r.? Maybe, but equity holder is really committing more capl. than appears, bcs. he must have reserves or collateral to absorb losses, that is committed by contingent liability. Counting that, he may be getting less on the entire commitment of capl. He also supplies more mgmt. This wd. acct. for gambling pref., lets it be consistent w. higher return on equities.

Temporal approach sheds new light on factor propns. They depend on durability or longevity

2 ways of looking at it: instantaneously, & thru time. They are equivalent. E.g. a forest. If each tree lasts 100 yrs, means labr only works on 1/100 of land each yr; & works in forest w. more ~~wood~~ trees per man, & higher ave. age.

If men work in factory, cap. contributes little if: a, Wages absorb most of product; b, Bldg. charges absorb most of product, but bldg. doesn't last long. — so the capl. charges are largely depreen., not interest.

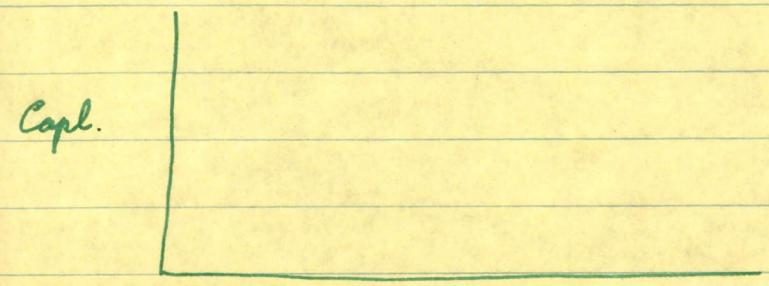
Prodn. is only capl.-intensive if interest absorbs most of product. Very durable. E.g. utilities?

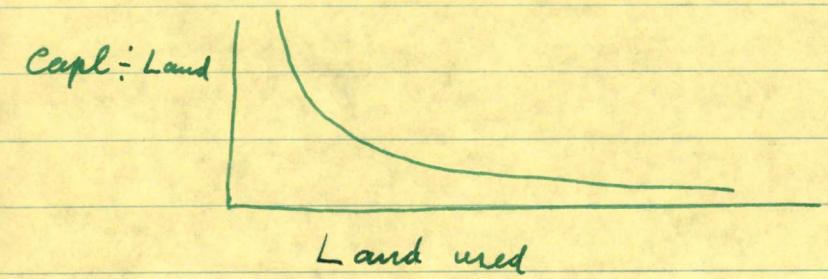
If cap. intensive, also usually land-intensive, unless durable on reprd. land.

What is called "capl. intensive" often isn't — e.g. fertilizer.

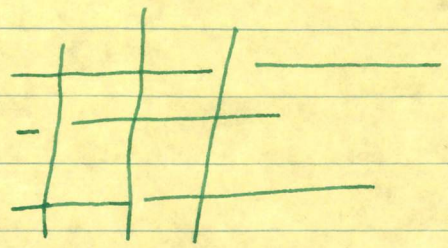
Wicksell's equilibrator. If surplus labor, lessen durability. The wealth of the world can be produced w. very different factor propns.

If automation threatens, it absorbs capl., forces up i, forces down wage, so capl. moves into shorter forms.

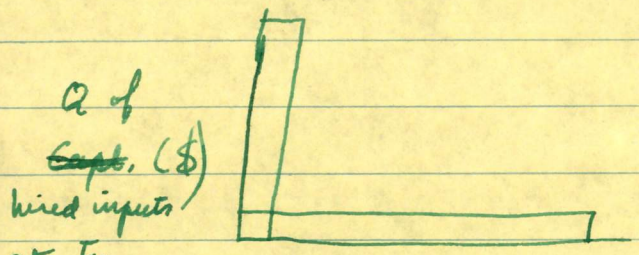


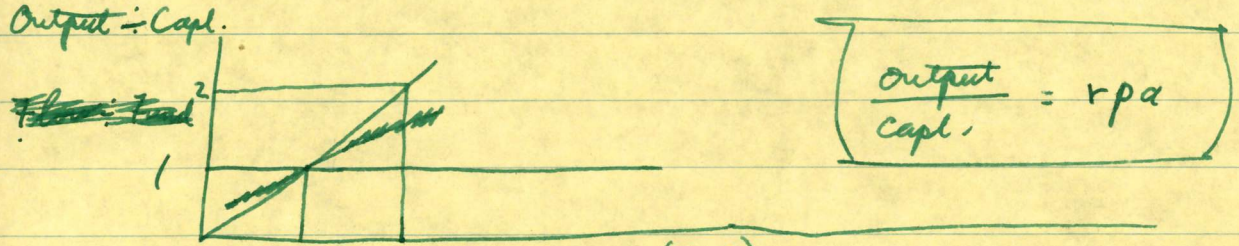




Capl ÷ Land is related to capl ÷ output





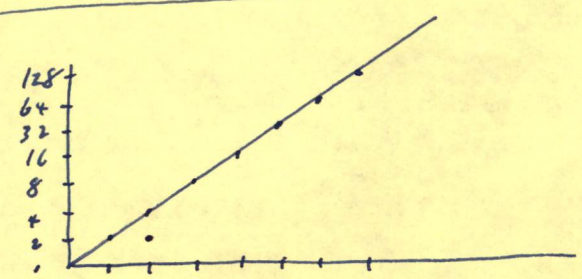



$$\frac{\text{Output}}{\text{Capl.}} = rpa$$

$$rpa = \frac{1}{\text{mean pd. of invmt.}}$$

2016, Tues., Nov. 7, '67

1. 


2, 3

4. $a = \left[g - C_0(1+i)^n + R_x(1+i)^{n-x}\right] \frac{i}{(1+i)^n - 1}$

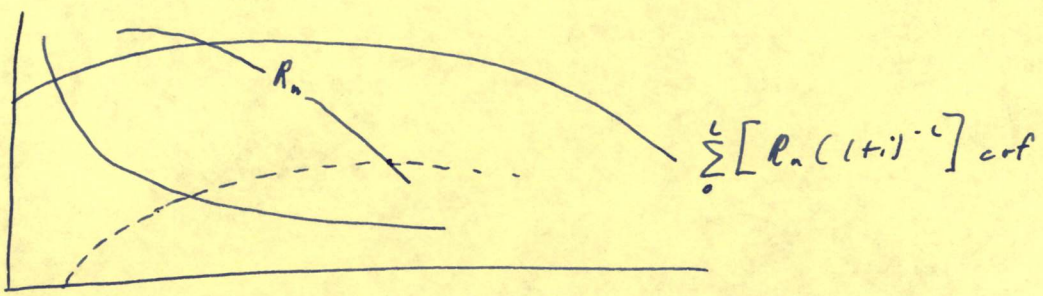
(Leads to more general formulation, $\Sigma$ ----)

5. ~~Function~~. Raises a; lowers g; raises g'. Indeterminate

6. ~~Max~~. a) $R_x = a$ [or $Si$]

b) $a = \Sigma$

14. $i\,V_n = (1+i)^n - 1$




1. $y \equiv \sum_0^L \left[R_n(1+i)^{-L}\right] \cdot crf$

$y$ is const. when $R_n = y$

2. $x \equiv y - C_0 \cdot crf$

$x$ is const. when $R_n = x$. By exactly same reasoning.

So if we terminate life at max. $\underline{a}$, it is also the time when

MC of time $(a)$ = MR of time $(R_n)$.

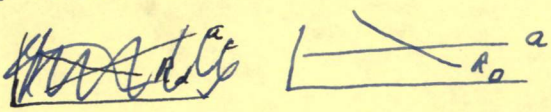
Replacement Pol.

Only applies to capl., not land. Land has no turnover. Vital distinction.

1. Depreig assets

a. Demolition & site renewal. Ratcliffe solution.

$$a = \Sigma\left[R_n (1+i)^{-n}\right] CRF$$

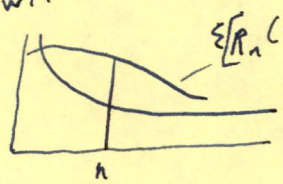



Problem: presupposes we have decided opt. $\underline{n}$ for challenger ~~& def.~~

Solution:

i. If can assume const. conditions, choose that $\underline{n}$ which maxs. $\underline{a}$.

Begin with cost curve. Note how it falls, spreading fixed cost over time. Why not go on forever? $\underline{a}$ falls. For 3 reasons: $R_n$ falls, and P.V. of any given $R_n$ falls bec. it has to be discounted further; and C+f falls. But $\Sigma$ grows.




~~ii. If $\underline{a}$ keeps rising, max. P.V.~~

If $R_n$ were const., $a = \frac{R_n \left[1-(1+i)^{-n}\right]}{i} \times CRF = R_n$

This is nice, bec.

a) as a mgnl. solution, demolition comes when $R_n$ from defender = $\underline{a}$ from new challenger. Show that.

b) as a solution in perpetuity, is same as max'g P.V.

Show that. $PV = \frac{a}{i}$

ii. If assume rising land, get earlier demolition.

a) as a mgnl. solution, bec. mgnl. value of land in interim use rises above ave. value or annuity value.

b.) As max'g. solution, bes. PV is max'd. for shorter periods.

$$PV = \frac{\sum [R_n (1+i)^{L-n}]\ i}{(1+i)^L - (1+g)}$$

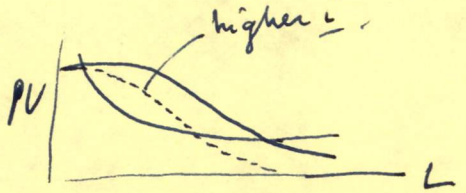
By subtracting g from denom. we Δ its % rate of growth. So at the previous max., now denom. is growing faster, so we're past the max. (Bes. what happens is, num. grows faster until max, then denom. grows faster.)

b. But demolition policy isn't really replacement policy.

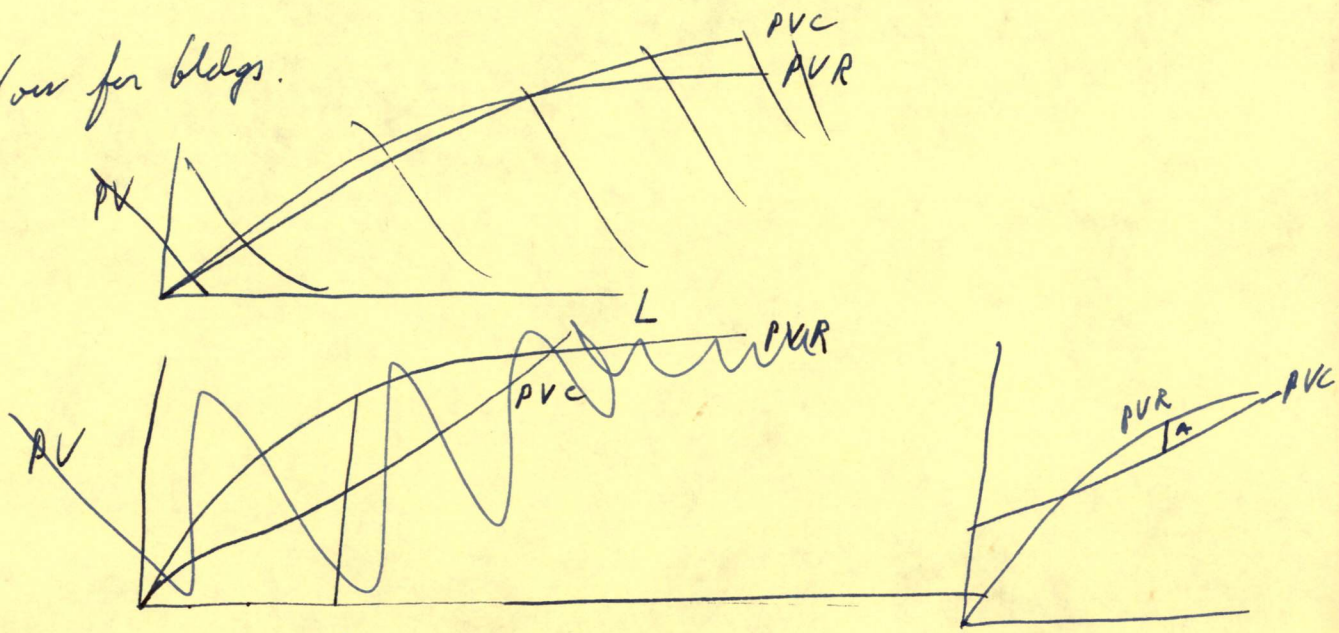
Value is recovered from bldg. long before it is scrapped.

Replacement decisions are made long before capt. recovery, at time of initial irreversible commitment. You build in the rate of recovery at time 0.

We already saw that for mines:




Now for bldgs.




Max. annuity value of A, i.e. $a = A \cdot crf$

This is another way of perceiving:




This is imperfect, but is better than trying to forcast in perp.

2. Appreciating assets

Replmt. decision here is not entirely built in, but made at maturity.

a. Fisher, faults.

Unexplained sudden jump

No effect of $\Delta C_0$

Fisher as mgnl. solution.

$g' = gi$

How prove? Geometrically, slope of g curve = $\lambda$ of int. curve.

Algebraically, max. $\frac{g}{(1+i)^n}$ comes when % growth rates are equal, i.e. when $\frac{g'}{g} = i$

No mgnl. return on land inst. or overhead. $C_0$ can't include land, bec. not recovered

b. Boarding.

Unaffected by any change of $i$.

Returns nothing to overhead.

$i_B$ can be $\infty$ if $C_1 = 0$; and solution arrives when $\frac{g'}{g} = \infty$

i.e. when $g = 0$

c. Faustman

1. Max. annuity from $A = g - C_0(1+i)^n$

- $\frac{da}{dn} = 0$, hammer & tongs approach

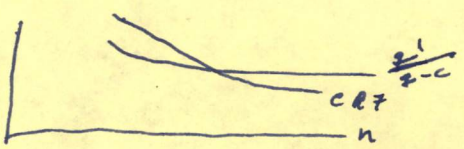
- equate % growth of num. & denom of $a = \frac{g - C(1+i)^n}{(1+i)^n - 1}\, i$

$$\frac{g' - iC(1+i)^n}{g - C(1+i)^n} = \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$g'(1+i)^n - g' - \cancel{iC(1+i)^{2n}} + iC(1+i)^n = ig(1+i)^n - \cancel{iC(1+i)^{2n}}$$

$$\frac{g'}{g - C} = \frac{i}{1 - (1+i)^{-n}} = CR7.$$

- This is operable.




How different the solution is from simple Fisher method depends on $n$. For high $n$ and $i$, $CR7 \to i$.

2. As a mgmt. solution, $g' = gi + a = gi + \frac{g - C(1+i)^n}{(1+i)^n - 1}\, i$

$$g'[(1+i)^n - 1] = gi(1+i)^n - \cancel{gi} + \cancel{gi} - Ci(1+i)^n$$

$$\frac{g'}{g - C} = \frac{i(1+i)^n}{(1+i)^n - 1} = CR7$$

3. If treat $S$ as part of $C_0$, get same solution

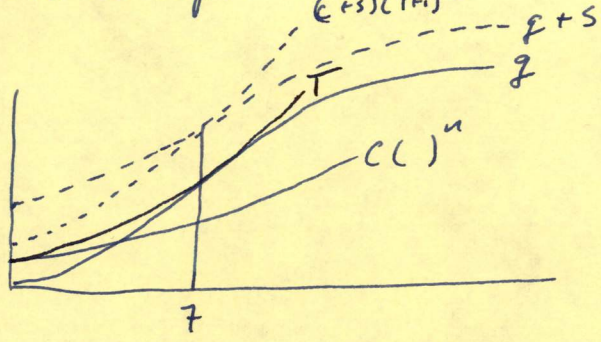
$$(C+S)(1+i)^n = g+S$$

$$S = \frac{g - C(1+i)^n}{(1+i)^n - 1}$$

4. If max. PV in perp., same solution

$$S = \frac{g - C(1+i)^n}{(1+i)^n - 1}$$

5. Geometric interpn. add $S$ to both $g$ and $C$ curves




at $t$, $g'$ is same as ~~$g'$~~ $(g+S)'$, bcs. $S$ is const.

$(g+S)'$ = growth of $(C+S)(1+i)^n$ or $i(C+S)(1+i)^n$

That is same as $i(g+S)$, bcs. at $t$ we see geometrically that ord. of $(C+S)(1+i)^n = g+S$

The $T$ curve has same $\lambda$ as $(C+S)(1+i)^n$ bc came $T = (C+S)(1+i)^n - S$

$T$ is inv. value.

6. Simul. require equal slopes & ordinates.

~~5. Lutz: falling in aggregation~~
~~(Save to next week)~~

6. Joint maximization of $i$ & $a$.

Max. $i$, subject to $i = \left(\frac{q+S}{c+S}\right)^{\frac{1}{n}} - 1$

It would be wrong to max. $i$ subject to $a = \frac{q - c(1+i)^n}{(1+i)^n - 1} i$

bcs. then $\frac{a}{S} < i_{max}$.

7. Zero-interest solutions. Max $\frac{q}{n}$ or $\frac{q-c}{n}$. Cf. Fauntmann. Max $\frac{q - c(1+i)^n}{(1+i)^n - 1} \frac{i}{}$

8. Are all solutions the same in compn? No. Show it with $i=0$.
They are same on mgnl. land, when $S$ & $a = 0$

9. Effect of changes in $c$, and $i$, and $S$.

10. What to max? Whatever is most scarce to the firm. Land & credit markets both very imperfect.

11. Factor propns. (Leads to macro; so does Lutz.)

12. What to do when data change in mid-rotation.

Drop historical costs; look to future. They were only relevant bec. were going to be repeated.

At margin, apply mgnl. form. All parameters are determined from future. $g' = i(g+S)$

13. ~~Which are~~ What is the sequence of rotations?

14. Thinnings -- transit to more general case.

max $\dfrac{\sum_0^t \left[R_n (1+i)^{t-n}\right] - \sum\left[C_n (1+i)^{t-n}\right]}{(1+i)^t - 1} = i$

[or divide num. & denom. by $(1+i)^t$, get c+f.]

15. Now suppose ~~$R_t = 0$, or is very small~~ $g' = 0$, & salvage value = 0

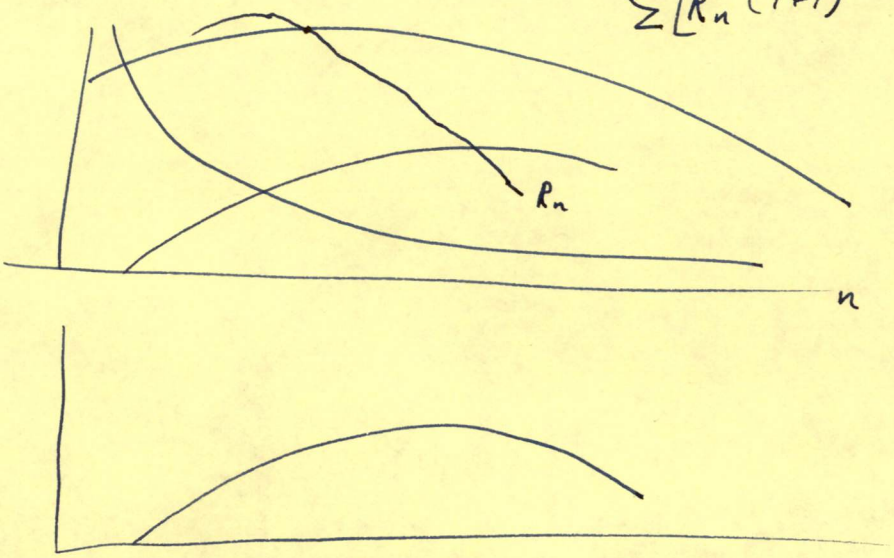
as mgnl. solution, we now have $R_n = a$

$$\frac{1}{(1+i)^n} + \frac{1+g}{(1+i)^{2n}} + \cdots +$$

$$S = \frac{1}{(1+i)^n} \cdot \frac{1}{1 - \frac{1+g}{(1+i)^n}} = \frac{1}{(1+i)^n - 1 - g}$$

---

$$\Sigma \left[ R_n (1+i)^{L-n} \right] \frac{i}{(1+i)^L - 1}$$




$\Sigma [R_n \quad ]$ is annuity. When $R_n$ = annuity, no change

Same with max. of lower curve.

201 C. Oct. 10

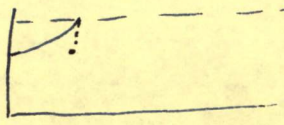
# 1. Valuation

1. $P + Pi = P(1+i)$

$P(1+i) + P(1+i)i = P(1+i)(1+i) = P(1+i)^2$

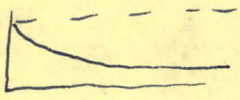
Rate of exch. between points of time. Inter temporal commensurability

2. Compounding & discount



$V_t = V_0(1+i)^t$

3. Discount is valuation. Usually involves summation.



$$V_0 = (1+i)^{-1} + (1+i)^{-2} + \dots + (1+i)^{-n} = \frac{1-(1+i)^{-n}}{1-(1+i)^{-1}}(1+i)^{-1}$$

$$= \frac{1-(1+i)^{-n}}{1+i-1} = \frac{1-(1+i)^{-n}}{i}$$

a. Derive sum of series rule.

N.B. a. Discontinuous. Math. less elegant, best ecky. more relevant.

b. Watch end points

c. Remember to mult. by value of annuity. -- here assumed = 1

4. Properties of fn.

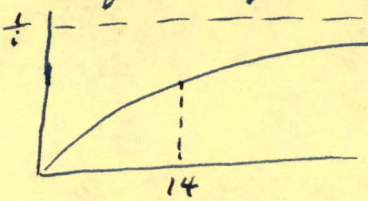
Before using tables, get good idea from Log 2.

a. $\frac{1}{i} \cong 20$

b. approaches 20 as asymptote.

c. When $(1+i)^n = 2$, $(1+i)^{-n} = \frac{1}{2}$, etc.

d. Half value of a perp. comes from 1st 14 yrs, at 5%.




Important implications for tax policy: assmt. freeze; acc. depreciation.

e. More if deprec'g; less if apprec'g. Defer.

f. When n is small, formula resembles multiplication by $\underline{n}$.

| $\underline{n}$ | | |
|---|---|---|
| 1 | $\dfrac{\dfrac{V}{1+i-1}}{\dfrac{1+i}{i}}$ | $= \dfrac{1}{1+i}$ |

as

| 2 | $\dfrac{\dfrac{1+2i+i^2-1}{(1+i)^2}}{i}$ | $= \dfrac{2+i}{\cancel{(1+i)^2}\ 1+2i+i^2}$ |
|---|---|---|

Will see this better when have formula in another form.

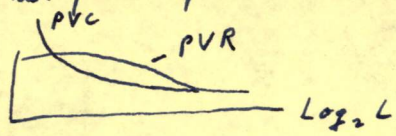
g. When n is large, ceases to be a fn. of $\underline{n}$.

Illustrate from Tables

Apply l'Hopital's Rule to show that when $i=0$, $V_0 = n$. (cover)

5. An application to mining, that points up relation of fn. to $\frac{1}{n}$

$$V_0 = \frac{1}{L}\,\frac{1-(1+i)^{-L}}{i}$$




Max. PV is often a solution to ec. problems.

But beware -- dodges the replacement problem.

6. ~~Other forms of the basic~~ Practical use

We rarely have perfectly steady annuities.

But it is lots of work to discount individual ~~values~~ years' values one by one, if life is long.

So break up problem into 3 or 4 reaches. ~~Does~~ This often suits nature of inv. decisions anyway, which have discrete elements -- e.g. a succession of land uses.

$$V_0 = \frac{1-(1+i)^{-n}}{i}$$

$$\frac{d}{di}\left[1-(1+i)^{-n}\right] = --n(1+i)^{-n-1} = n(1+i)^{-n-1}$$

But $i = 0$

$\therefore V_0 = n$

Also, the formula is useful for reducing an irregular stream of future revenues and costs to a single annuity number. But this leads us to next step.

7. Other forms of basic eqn.

Converts between lump sum & annuity

a. Recip. is Cap. Rec. $a = \frac{i}{1-(1+i)^{-n}}$




Is useful to convert a PV to an annuity, over life. E.g. water rts. case on Kaweah, valuation of sleeping ripns. $\Sigma[R_n(1+i)^{-n}]$ CRF

approaches i as asymptote. Gets there surprisingly fast.

If $n=1$, $a = 1+i$

Doesn't distinguish income from cap. recovery. That requires separate treatment, & is vital to inc. tax matters.

Conversion to annuity is useful for commensurability of alternative uses of land, ~~[illegible]~~ of different lives. PVs are not comparable, but annuities are -- and you don't have to forecast succeeding uses in perpetuity.

b. Amount of an annuity -- amt to which it compounds.

Mult. PV by $(1+i)^n$ ; get $\frac{(1+i)^n - 1}{i}$

Is value reached by a sinking fund

c. Annuity from future lump sum, recip. of (b)

$$a = V_n \frac{i}{(1+i)^n - 1}$$

a "time-average". Apply l'Hopital again.

Also derive from CR7.

Also cd. be derived independently as $\Sigma$.

8 Value of rising annuity, at g %

$$V_0 = \frac{1}{1+i} + \frac{1+g}{(1+i)^2} + \cdots + \frac{(1+g)^\infty}{(1+i)^\infty} = \frac{1}{1+i} \cdot \frac{1}{1 - \frac{1+g}{1+i}} = \frac{1}{i-g}$$

$$V_0 i = 1 + V_0 g$$

9. Value of periodic income.

$$V_0 = \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{2n}} + \cdots + \frac{1}{(1+i)^\infty} = \frac{1}{1 - (1+i)^{-n}} \cdot \frac{1}{(1+i)^n}$$

$$= \frac{1}{(1+i)^n - 1}$$

Oct, 24. Ecs. 201C

1. Auditors, Trapping Oath
2. Use 8th floor lib.; bus. lib
3. Beware 2d ed. of Bldg.
4. Relation to reality. Begin w. simple, regular cases. Add complicating elements as we proceed. Creep before you crawl.
5. Signif. of cap'g an annuity. Assume it lasts forever.
   This is right approach in replacement problems.
6. To p. 6 of notes.

201C

~~Bob: Send title; request readings~~

~~Fri, 2 PM~~

1. Clean up remnants of point case

a. When tax shifting straightens the ~~convex~~ r curve, is the $\bar{i}$ curve a mirror image of the ~~old~~ original $[(1+i)^n(1-t)+t]$ curve?

The new curve is $[(1+i_b)^n - t] \div (1-t)$

New curve is such that $(1+\bar{i})^n(1-t)+t = (1+i_b)^n$

$i_b$ is const.; $\bar{i}$ is variable.




It's a mirror image because slope of ray to upper curve must be raised above $(1+i_b)^n$ by same amount as slope of ray to lower curve is depressed below it.

b. Bias to future. Some investments especially favored: growth stocks; exploration ~~and/or~~ -- you get expensing, in fact; tree farms; land speculation; ~~fighting~~ appropriation of ~~a~~ right-of-use or seniority resource -- air line franchises, TV channels, suburban zoning favors, patents, water rights, etc.;

Re growth stocks, pay no heed to official income figures, bcs. unrealized appreciation is omitted. "We had losses this year, ~~but~~ bcs. of reorgn., or conversion to new model." That just means they wrote off capl. cost as current cost; and/or failed to count appreciation as income. A stocks mkt. analyst could go far on that alone.

c. Two concepts of shifting – inter temporal and to consumers.

Third concept needs to be developed: shifting to landowners. $(C_0+S)(1+i)^m = (g \text{ [illegible]})(1-t) + S + t \cdot C_0$ Then hold $i$ const, lower S. The lower S then results in longer holding periods. That is next project. Maybe you'll beat me to it. It's just a matter of getting the bugs out.

In any case, inter temporal bias remains.

d. Neutral tax. Tax appreciation each yr.

$$1 = -\frac{i \cdot t}{(1+r)} - \frac{it(1+i)}{(1+r)^2} - \dots - \frac{it(1+i)^{m-1}}{(1+r)^m} + \frac{(1+i)^m}{(1+r)^m}$$

$$1-\left(\frac{1+i}{1+r}\right)^m = -it\,\frac{1}{1+r}\,\frac{1-\left(\frac{1+i}{1+r}\right)^m}{1-\frac{1+i}{1+r}}$$

$$1 = \frac{-it}{1+r-1-i}$$

$$r-i = -it$$

$$r = i - it = i(1-t)$$

e. But the neutral tax is known to shorten life? Not ~~in~~ in a closed ec. Lower $t$.

But there are no closed ecs. Capl. migrates. Note English devaluation. Even labor does -- the 'brain drain', the unavoidable result of England's taxn. of salary income and

creates a brain drain, in sense of reducing the brain gain. We need to wait for a net outflow to recognize a problem.

exemption of land income. In U.S., ~~it's~~ our tax policy also

So we should analyze in terms of open economy.

Now, ~~not~~ neutral tax is only half neutral. It drives out capl. ~~As the~~ Then it rations scarce capl. at the higher rate of interest appropriate. Is inter-temporally neutral.

Only land tax can be completely neutral.

2 Full salvage case

Neutral tax is now obvious.

a. Before tax

$$1 = \cancel{i_1 + i_2 + \cdots +} \frac{i}{1+i} + \frac{i}{(1+i)^2} + \cdots + \frac{i}{(1+i)^n} + \frac{1}{(1+i)^n}$$

$$1 = i\,\frac{1-(1+i)^{-n}}{i} + (1+i)^{-n} = 1$$

b. After tax

$$1 = i(1-t)\,\frac{1-(1+r)^{-L}}{r} + \frac{(1-t)}{(1+r)^L} + \frac{t(1+r)^{-L}}{} \cancel{+ \frac{(1-t)\,i}{r}\left[1-(1+r)^{-L}+(1+r)^{-L}\right]}$$

$$1-(1+r)^{-L} = \frac{i}{r}(1-t)\left[1-(1+r)^{-L}\right]$$

$$r = i(1-t)$$

c. Verbally, is even more obvious.

3 Depreciating asset case. Apply same principles.

a. $1 = \frac{i}{1-(1+i)^{-L}}(1-t)\frac{1-(1+r)^{-L}}{r} + t[PVW]$

b. $PVW = a\left[\frac{(1+i)^{-L}}{1+r} + \frac{(1+i)^{1-L}}{(1+r)^2} + \cdots + \frac{(1+i)^{L-1-L}}{(1+r)^L}\right]$

$= a\frac{(1+i)^{-L} - (1+r)^{-L}}{r-i}$

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c. $\frac{1}{a} = (1-t)\frac{1-(1+r)^{-L}}{r} + t\frac{(1+i)^{-L} - (1+r)^{-L}}{r-i}$