



DCFF Model with Rise of Site Value

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DCFF Model with Rise of Site Value

The Redeemable Bond model is a way of bringing the far future into evaluating any investment, instead of limiting ourselves to Generation #1. We have seen elsewhere another method of doing that, named for Faustmann. See the papers "Finding Optimal Life of Capital Affixed to Land," "Maximizing Value in Perpetuity," and "When to Cut a Tree." See also Appendix 2, hereto.

How does the Redeemable Bond model relate to Faustmann? In Faustmann we find the value of future generations endogenously, making them clones of Generation #1. The Redeemable Bond method takes the value of future Generations as given exogenously, expressed in Future Resale Value (F). It takes all future cycles after #1 and expresses their value at one point, the end of Generation #1. This is the model to use at ecotones, where land is rising in expectation of a different future use.

Unlike Faustmann, it cops out on how that future value is determined. It just takes it from that great outside force, "the market," which many modern economists treat as the new infallible source of spiritual, moral, and material guidance. However, that doesn't mean we must cop out. If we want, we can value the future use ourselves. The Redeemable Bond method just says that the job is done outside the analysis at hand.

Another difference is that Faustmann applies to PIPO investments, not DCF investments. However, that is only superficial: Faustmann can be adapted to DCF investments. See the paper "Finding Optimal Life of Capital Affixed to Land."

The analysis of most direct investing (i.e. actually making or owning real things, not just paper) involves the same math as Eqn. (1), which is why I started you out with it. Most direct investments consist of capital occupying and preempting space. When the capital is terminated, the space is released for another use. The space, therefore, is analogous to the Redemption Value, F, of the redeemable bond. The notation for space is T (Terra).

The other difference is that the original investment, P, is separated into two elements, K and T, for capital and land.

$$T + K = a \cdot \frac{1-I^{-n}}{i} + TI^{-n} \quad (4)$$

Why distinguish T and K? Notice that T appears twice in Eqn. (4), and K only once. This is because K is depreciable, it wears out and has no resale value. T does not depreciate, but often appreciates.¹ That is why T appears twice in Eqn. (4). You put

¹Land has infinite life, except for exhaustible resources, which are not treated here. They are a special case, not quite

it in at the start, like K, but you also get it back at the end, unlike K. You recover K in the cash flow, bit by bit, year by year.

Market forces work this out because buyers and sellers know the land outlasts the capital, and they price it accordingly,² so the part of the product imputed to land is just enough to cover rent, with nothing left over to recover principal. K, on the other hand, has to be priced low enough so its cash flow covers both recovery of principal, and interest on the unrecovered balance each year. We worked that out in the paper "Basic Math of Finance," pp. 14-16.

A. Valuation

1. Value of Land (T)

Solve Eqn. (4) for T:

$$T = \left[-K + a \frac{1-I^n}{i} \right] \cdot \frac{1}{1-I^n} \quad (5)^3$$

The expression in brackets is the DCF of Generation One (G_1), net of capital cost, K. The ratio to its right is the Land Value Multiplier (LVM). Here are some LVM values, to give you a feel for it, when $i=.05$.

Table 4: Land Value Multipliers when $i=.05$.

n:	1	2	4	8	16	32	64
LVM:	21.0	10.8	5.6	3.1	1.8	1.3	1.05

For example, when $n=16$, generations following the first one add 80% to the value of generation one.

What we have here is similar to Faustmann in its DCF form, as

like other land, and not quite like stores of man-made capital. See the paper on Reserve, an excerpt from *Extractive Resources and Taxation*, 1967.

²This is on the premise that the market works reasonably well, and forecasts correctly. In practice land is sometimes underpriced: then the buyer gets a free ride. More often it is overpriced, and the buyer gets in trouble. These dynamics are important, but are not dealt with here in the brief ten weeks available. Those interested in pursuing this matter see me for references.

³Yes, I know, this can be simplified a bit. That comes later.

developed in the paper, "When to Cut a Tree." There is also a skeletal demonstration of Faustmann in Appendix 2 to the present paper. A big difference is that Faustmann applies to PIPO, and Eqn. (5) applies to flows.

A second difference is that Faustmann, gives us a way to solve for optimal life, n . Here, in Eqn. (5), the terminal value of n is just assumed. " a " goes along at a level value and suddenly self-destructs. This (the part in brackets) is therefore often called the "One-hoss Shay" model.⁴ It's also called the "light-bulb" model, for the same reason.

Eqn. (5) can be adjusted (with added complexity) to show " a " declining, as is often true as capital ages. I have done so in a separate set of notes. See the paper, "Declining Cash Flow Models."

2. Value of Capital (K)

If you know T , and want to find K , solve Eqn. (5) for K .

$$K = a \cdot \frac{1-I^{-n}}{i} - T(1-I^{-n}) = (a/i - T) \cdot (1 - I^{-n}) \quad (6)$$

Eqn. (6) gives you the value of K net of land. It is the method to use once a building has been built. First you value the land from Eqn. (5), using values of a and K derived from the highest and best future, prospective use. Then plug that value of T into Eqn. (6) to value the standing, existing structure. This is the standard way that valuers separate land value from building value. It is called the "building-residual" method, because T is taken as given, and K (the building) gets what is left over.

Eqn. (6) has several uses:

One, valuing K tells you when the time has come to tear down the old building. It's pretty simple: when $K = 0$, the time has come. $K = 0$ when $a/i = T$.⁵ a is the cash flow from the current use (land-cum-building). T is the DCF derived from foreseeing the best future use. When $T > a/i$ it is past time to rebuild.

⁴The One-hoss Shay is from a classic poem, *The Deacon's Masterpiece*, by Oliver Wendell Holmes, about a "Wonderful One-hoss Shay" that lasted 100 years and a day, when it abruptly fell all apart. With such a mind for irony and economics, no wonder the poet's son and namesake became Chief Justice of the U.S. Supreme Court. It was something like Cinderella's coach at midnight (except the coach turned back into a pumpkin).

⁵ $K = 0$ also when $n = 0$, but that is redundant because it is $a/i = T$ that makes $n = 0$.

To do that you must first tear down and haul away. Remember, though, the idea is to rebuild. If you just tear down and haul away you've only destroyed value. Obvious, yes; but look around the city and you'll see it is not obvious to all owners. They lose a lot by taking the down-time of land so lightly; and so does our whole society. Once they put their assets into an airplane, ship, or power plant, they sweat buckets to minimize its downtime. The traditional American attitude toward land is more cavalier. It's an attitude that should change, but is mainly changing for the worse.

Two, valuing K this way is theoretically meaningful because a structure, once built, has little or no salvage value. I.e., it has no "opportunity cost" separate from its present form and site. The site, on the other hand, never loses its opportunity cost. You see them tearing down old buildings all the time, to salvage the land beneath them. "They ain't making any more of it," so such salvage is basically the only source of land for the future, as it has been for the past.

Three, land and building may have separate owners. In such cases, land income takes legal priority over other income.

Four, buildings are depreciable for income tax purposes; land is not. You have to separate them to state your income correctly. Also, improvement assessments levied to pay for public works are levied against land, not buildings.

Five, you need fire insurance for the building value, not the land value. (You need title insurance for the land.)

B. Rate of Return (ROR) as Affected by Capital Depreciation and Land Appreciation⁶

Eqn. (4) shows how the parts are tied together. Like all equations it can be solved for any element, so you can reason from what you know to what you want to find out. Let's solve it for the ROR, or i .

To adapt Eqn. (2) to the case of direct investments we break P into two parts, T (land) and K (building, or other depreciable capital). Then we break T itself into land price at two different times. T at time zero is Z; and T at the future time n is F. I do this to avoid extensive use of subscripts, which clutter things up. Z is T_0 , and F is T_n . $\Delta T = F - Z$.

⁶For an extended analysis of this case, showing the effects of different income tax provisions, see the paper on Reserve, "Land Gains, Fast Write-off, and Incentives to Build." My students in Public Finance master this material without swooning; so can you.

$$\text{ROR} = a \cdot \frac{(1-I^n)}{(Z+K) - FI^n} \quad (7)$$

1. How Z depresses ROR

You put land into the beginning of the investment cycle, and recover it unchanged at the end. It is like a catalyst in a chemical reaction, participative but untouched. This does not mean its presence is free. Occupying a time-slot in space costs you. The cost is the money tied up in land over n years. The effect is to lower the ROR. Table 4 gives some examples.

Table 4: ROR on Cash Flow Followed by Sale of Land, with Stable Land Price. (Parameters: $n=10$; $a = .15$; $K = 1$; $F = Z$)

Z	Z+K	ROR (%)
0.0	1.0	8.14
.25	1.25	5.90
.5	1.5	4.60
1.0	2.0	3.17
4.0	5.0	1.10

2. Rent or Buy?

The need to carry land at a high value over a normal investment cycle lowers the ROR: lowers it a lot. It lowers ROR by adding to the base of total assets on which a is the return. This reduction is only partly compensated for by recovering land after n years -- 10 years in Table 4.

You might avoid this by renting the land. In that case, however, you add annual rent as an expense, deducted before arriving at cash flow, a . You might do better or worse, depending on particulars. It depends on whether land is overpriced relative to its rent, something that varies with circumstances. Your financial success depends in part on your judgment call, whether to rent or buy. Your judgment call in turn depends in part on your technical ability to reckon ROR correctly.

But wait, there may be another plus for buying. Maybe the land will rise during your time-slot, or "period of ownership," as the tax folks say. That would improve your ROR. Let's see how much.

3. Rising Land Value

We need some math to overcome the complexities, but let's begin with a simple case that almost explains itself.

a) Simple example: ΔT offsets depreciation

Here, the appreciation of land value, ΔT , is just enough to offset the depreciation of the building or other capital (K) that yields a cash flow for n years. Your salvage value thus equals your initial investment, so your ROR is simply the cash flow divided by the initial point-input of $Z+K$. Look at Eqn. (7) and see how much cancels out when $F = Z+K$. You are left with:

$$\text{ROR} = a/(Z+K) \quad (8)$$

It is as though K didn't wear out at all, but lasted forever, like land. Eqn. (7) becomes like the case of a redeemable bond you can buy for its redemption value of \$1,000. This makes $\text{YTM} = \text{current yield} = a/P$. (See the 3rd paragraph after Eqn. (2), p.2.) In the bond market such a condition is not unusual, and would in fact seem "normal" and expected. In the case of a direct investment it is a coincidence caused when $\Delta T = K$.

It's amazing, though, how many owners in a rising market come to expect this, and forget that capital (K) wears out. They fail to provide for replacing it, assuming that ΔT will take care of that, and more besides. They consume ΔT by letting it substitute for replacing their K as it depreciates. I have listened to some who have come to regard this as not only natural, but as some kind of natural right, or entitlement. They feel outraged and cheated when it doesn't work out that way. The human mind plays funny tricks - you need to know about these quirks as well as the math.

Coincidence or not, that simple finding in Eqn. (8) is a handy landmark to keep in view as you sail in these strange new seas. You need a stable pole star for orientation. It is not in human nature to trust mathematical formulae until we have worked with them long enough to develop understanding and confidence. Nor is it easy to keep in mind highly generalized formulae, nor to adapt them quickly to changing particulars, even though they are designed for such adaptation. But here is something you can understand and accept readily. Use it as a rule of thumb for situations that approximate it. Use it as a benchmark to check the credibility of your answers for situations that are obviously better or worse.

b) Example where $F = 2Z$

Eqn. (8) makes it clear that you get a higher ROR when Z is low -- i.e. you buy in cheap. If you are really lucky you could start from a base with Z at zero. Then, $\text{ROR} = a/K$. Life is more than fair to those who get in on the ground floor. That is why it is less than fair to newcomers. One may do even better when ΔT is more than K, obviously, and we will be seeing how much better. However, mark this well: the enjoyment of ΔT by one investor makes it tougher on the next one.

The gratuitous gain enjoyed by Groundfloor O'Grady is an added burden to his successor Newcomer Nellie who had the bad business judgment to be born 30 years later. O'Grady's juicy resale value, F , is Nellie's initial land cost, Z . Nellie gets on where O'Grady got off. O'Grady got a free ride, but it's Nellie who pays. In a rising market, each generation holds the next one up for ransom. Even though the elevator may keep going up, Nellie's ROR is to be based on a higher ante -- a number, remember, that you divide by in figuring ROR. The higher Z will offset much of, all, or more than the gain, (varying with particulars).

So the same ΔT that gladdens the hearts and raises the animal spirits of one generation serves to depress those of the next. Here we have the makings of a business cycle. This point is not pursued further here, but is worth a lot of thought by people venturing into the future world of finance.

There are two opposing forces at work here on the ROR. The burden of carrying Z lowers the ROR; the promise of ΔT acts to raise ROR.

Table 5 illustrates the relative weight of forces acting on the ROR. It is an exercise in Sensitivity Analysis. It's a bit subtle, so prepare to slow down and study it a while, or you'll miss its points.

In the Table, T doubles over an investment cycle of 10 years. The higher the base (Z), the higher is the absolute value of ΔT . But there is a loss in the ROR, caused by having a higher base, as illustrated in Table (4). In Table 5 the loss is partly offset by the gain from ΔT , resulting in a slight drop in ROR as Z and ΔT are both higher. (ΔT is tied to Z , the initial value of T , because of the assumption that T doubles.)

Row 6 (ROR_1) shows what the ROR would be if $\Delta T = 0$, and therefore $F = Z$.

Read the "Morals" under the Table to help interpret the findings.

Table 5: ROR When T Doubles in 10 Years, Various Values of Z & F
Parameters: $n = 10$; $K = 1$; $a = .15$; $F = 2Z$ (except in Row 6)

		1	2	3	4	5	6
1	$Z =$	0	.25	.50	1.0	2.0	5.0
2	$Z+K =$	1.0	1.25	1.50	2.0	3.0	6.0
3	$F =$	0	.5	1.0	2.0	4.0	10.0
4	$\frac{a}{(Z+K)}$.15	.12	.10	.075	.05	.025
5	$\frac{Z}{Z+K}$	0.0	.20	.33	.50	.67	.83
6 ($F=Z$)	ROR_1 (%)	8.14	5.90	4.60	3.17	1.95	0.90
7	$\frac{2Z}{(Z+K)}$	0	.40	.67	1.00	1.33	1.67
8 ($F=2Z$)	ROR_2 (%)	8.14	7.82	7.66	7.50	7.37	7.27
9	$\frac{ROR_2 - ROR_1}{ROR_1}$	1.00	1.32	1.67	2.37	3.78	8.08

Morals:

i. Row 6 is there to show the effect of Z on ROR when $F = Z$, so $\Delta T = 0$. Row 5 is the ratio of future value to present value used to find ROR_1 in Row 6. Moving from left to right, higher values of Z are posited - but remember, the value of T remains constant here, equal to Z, throughout each time-slot being analyzed. The higher they get, the lower goes ROR_1 , even though F remains equal to Z. It's the higher level of Z that depresses ROR_1 .

ii. Row 8 shows how a positive ΔT acts to compensate, and offset the tendency of T to depress ROR. In Row 8, ROR_2 holds nearly steady, moving from left to right. (If we showed the effect of tripling T, ROR would rise slightly instead of falling slightly as it does in Row 8.)

iii. Row 9 is the ratio of Row 8 to Row 6. It shows how much the ROR is boosted up over the actual ROR on K by the factor of ΔT . The boost is, as you might expect, greater where Row 1 is greater. Row 1 is Z, and since $K=1$ it is also the ratio to K. In a rising land market, investors will favor land-using investments, relative to what they should be doing.

iv. The effect of ΔT is to make ROR insensitive to the amount of land (Z) being used with a given amount of capital (K). This insensitivity is shown in Row 8. It reflects

the fact that land appreciation offsets the cost of carrying land over time, making land very cheap to hold during the course of the investment cycle.

c) Example where T rises by various multiples from given base

To round out our understanding of this matter, I now add a third dimension to the table, letting T rise by multiples other than 2. (2 is the fixed multiplier in Table 5). Accordingly, Table 6 shows the effect on ROR of having T rise by multiples from .5 to 12, over 15 years.

Table 6: ROR when T rises by various multiples over 15 years
Parameters: $n = 15$; $K = 2$; $a = .20$; $Z = 1$; $Z+K = 3$

F/Z	ROR on pure Z	ROR on Z+K	ROR_1/ROR_2
(1)	(2)	(3)	(4)=(2)/(3)
.5	-4.52	1.77	-2.55
1.0	0.00	3.11	0.00
2.0	4.73	5.14	0.92
2.25	5.56	5.56	1.00
3.0	7.60	6.67	1.14
6.0	12.69	9.85	1.29
12.0	18.02	13.69	1.32

Morals from Table 6:

i. Column (2) shows the ROR on bare land (Z), assuming there is no K at all, and no a . At the higher values of F (above 2.25) the investor does better not to use the land at all, even though this entails idling the land and foregoing the positive net cash flow it could yield. This strange effect results because at discount rates higher than 5.56% the present value of the positive cash flow is negative. Putting it another way, the ROR on K alone, at $Z = 0$, is only 5.56%, so if pure land speculation yields more, the individual chooses to buy more land instead of building on what she already owns.

ii. The combined ROR in column (3) is dominated by F, the resale value of land. With Z assumed equal to one, F is the multiple of F/Z. The investor will pay close attention to the likely future of the value of land under the capital she has built.

The condition that bare land speculation yields more than

productive building is a disequilibrium condition, i.e. one that tends to work itself out in the perfect markets premised these days by ordinary theorists. As they put it, arbitrage anticipates all future windfalls. That just means that the Groundfloor O'Gradys of this world keep raising the entrance fees for the Newcomer Nellies. However, real life is likely to be a series of journeys between equilibrium conditions which elude us like the rainbow's end, the will o' the wisp, and the Holy Grail. So don't think this "disequilibrium condition" never happens. It is all around us all the time, and has been since Columbus landed and introduced the Old World concept of fee simple land tenure. Still, let's ask how equilibrium is approached.

Faced with high expected returns from land speculation, the investor seeks to buy more land rather than build on what he has. This hikes the ratio of land to capital in his operation, and this, in turn, raises the marginal return expected from adding capital.

When many people buy more land their bidding raises the price. This reduces $\Delta T/T$ and also reduces a/T . The combination of those two effects with the first one finally may produce a new equilibrium where the positive cash flow from using land is not neglected. It's amazing, though, how long it can take: 1492-1995 is 503 years, and we're still disequibrated. So don't hold your breath waiting for equilibrium.