Ec. 117, Used Notes

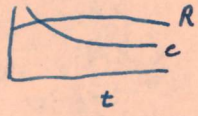
717. Final session, 1-15-63

Summary & Review.

1. Fncl. maturity

a. Max. time-ave. of net revs.

$$\text{Time-ave.} = \frac{R_m\, i}{(1+i)^m - 1} \quad ; \quad \text{more gnrly,} \quad \frac{\sum[R_n (1+i)^{m-n}]\, i}{(1+i)^m - 1}$$

Separate R_n & C_n, get 

MC of time = a; MR of time = usual MR curve, equals MC at max.

b. Correct fncl. mat. recy. for valuation. Max. a series of $\frac{\sum[R_n (1+i)^{m-n}]\, i}{(1+i)^m - 1}$

The i drops out, capn. factor becomes $\frac{1}{(1+i)^m - 1}$, for periodic incomes.

c. 2 elements of urgency, rent & interest, in case of appreciating assets.

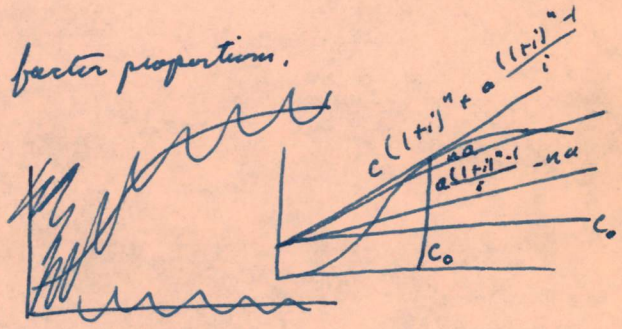
2. Durability, impt. of.

a. Sensitivity of valuation to i (or t) & factor proportions. $\frac{i}{A} \cdot \epsilon$ of value $\equiv \frac{\%\,\Delta V}{\%\,\Delta i}$

sensitivity of yields to ΔC: $\epsilon \equiv \frac{\%\,\Delta r}{\%\,\Delta C_0}$

b. Differential impact of taxes on valuation, & factor proportions.

c. Durability & factor propns. & emplmt.



3. Tax. capn.
   a. Series formula
   b. Bldgs partly escape, by adapting
   c. Land " " by reallocation.
4. Mnp

5. Value site ... burn down bldgs. (in contemplation only. Cite Essay on Resort Pig)

6. Uneven gross inc. over life is due to capl; land is const., gets const. imputed return

7. Positive effect of land spec.

$$r = \left[\frac{V_n + S_n}{V_o + S_o}\right]^{\frac{1}{n}} - 1$$

Helps bldrs. when their greater familiarity w. mkt., etc., gives them a big $(S_n - S_o)$.

8. l.q. ~~Tendency to equalize ... net~~.

Gross v. net

Cap. input is product of $Q^{x_i^n}$ ... same as interest cost. ~~... but not quite~~. Cap. input is indep. of freq., but labor input depends on freq. of capl-input, ~~varies w. freq. i.q.~~

Net i.q. reflects that; gross i.q. includes in C_o offsite purchases, which rep. employment of all inputs on other sites.

Annualized i.q.

Cap. recov. factor gives non-land input. $C_o \dfrac{i}{1-(1+i)^{-m}} \equiv c$

$\left[c - \dfrac{C_o}{m}\right.$ gives yrly. cap. input. $\left.\dfrac{C_o}{m}\right]$ is yrly input of C_o.]

c + current costs are total inputs, non-land.

Larger m means lower i.q.

9. ~~Hi~~ l.q. & emplmt.

Hi i.q. means hi emplmt.

10. l.r. & i.q.

Hi i.r. means hi i.q.

Dynamic i.q. .. only meaningful kind.

11. MC pricing

12. Supersession of utilities, Δg cost by decreasing cost.

13. " " , new by old. SMC & LMC

HARVARD UNIVERSITY GRADUATE SCHOOL OF BUSINESS ADMINISTRATION

Kinds of i.g.

1. Non-land inputs / Gross output — On a pvt. atomistic parcel. Gross pvt. i.g.

2. Imported inputs / Gross output ["Imported" from offsite]

3. On-site non-land inputs / ~~Gross output~~ value-added — The "value-added" i.g.

4. Offsite site-serving inputs (p.w.) plus non-land inputs / Gross output — The "social overhead" i.g. (gross)

5. Other possible combos of above.

Note re fixed capl., simple initial value is from offsite; all above that is on-site. So a, as a mere of capl. input, must be changed when we move to 'value-added' approach. I worked that through once: just subtract C_0 at $\underline{t}$, treat like site, to mere input.

"Value-added", like Keynes invmt. multiplier, is specious, but fallacious bec. cancels out middle terms, then strikes a ratio. Ratio wd. be much higher w. middle terms left in.

Probly each i.g. has its place. Gross i.g. reflects turnover; net or value-added i.g. fails to reflect it. Re mp of land, gross i.g. more revealing, I think. Cf. my anal. of eftv. intensity in R77 chap., gross i.g.s of comparable land are what shd. be equalized, not net. Work that thru! Re factor proportions, in 117a.

Cf. also social overhead i.q., or B/C, w. & w/o pvt. costs!

See Hammond, pp. 19-21

Tax Caps

$$T = P \cdot t$$

$$P = \frac{a}{i + t}$$

$$P = a\frac{1 - (1+i)^{-n}}{i} = \frac{a}{i}\ldots$$

$$P = \frac{a - Pt}{i}$$

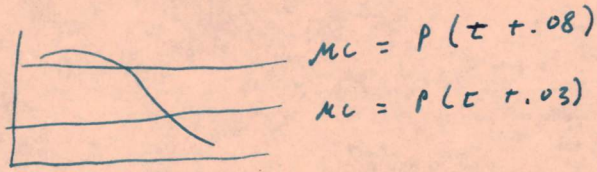
$$Pi + Pt = a$$

$$P = \frac{a}{i + t}$$

$$T = \frac{t\,\frac{a}{i+t}}{P} = \frac{1}{1 + \frac{i}{t}} \quad \text{So if } i = t,\ T = a \cdot \tfrac{1}{2}$$

$$T = \frac{.10}{.15} = \frac{2}{3}$$

$$T = P \cdot t = \frac{1}{i + t} \cdot t$$



after Δt, $\Delta T = P_1 t_1 - P_0 t_0$

$$-\Delta T = (P_1 - P_0)\, i$$

There will be an individ. for whom is no change: $P_1 t_1 - P_0 t_0 = i(P_1 - P_0)$

$$\text{or} \quad i = \frac{P_1 t_1 - P_0 t_0}{P_1 - P_0}$$

Higher i expands: lower i contracts.

117a, 9-19-62

Ass. R. Thomson, pp. 21-31
49-52
53-58
91-94
59-79

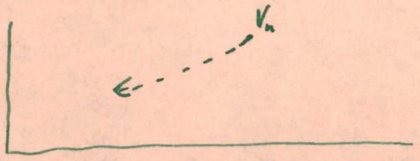
Get thru $P_0(1+i)^n$, and growth curve.

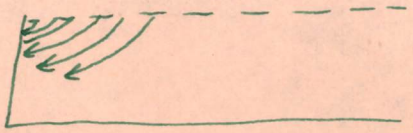
9-21-62. Announce Exam. for Fri, 9-28

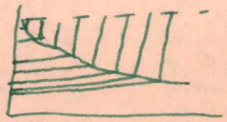
$V_n = V_0(1+i)^n$

Rate of exchange, intertemporal. Bring tables, give e.gs.
Give some problems

Discount: $V_0 = \frac{V_n}{(1+i)^n}$







Commensurability --- cost of xpts thru time. Like basing point or FOB values.

Some e.gs. — what do to future values.

Problems. -- give values of 1.05^{48}, let them figure reciprocals.

Objections to use of i.r.

Should risk be added to i.r.?

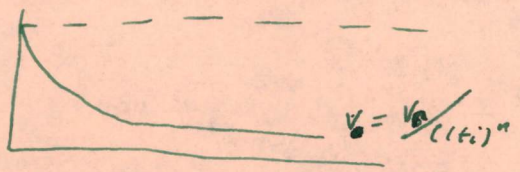
Macro-ec. arguments. Probly just allude to.

Summation. $S = \frac{1-r^n}{1-r}$

Valuation, comes from summation

117a, 9-24-62

1. Curve of exponential decay.



curve is a projection of y-intercepts

2. Problems.

a. What is V_0 of $\$100_2$ at 11%

$$V_0 = \frac{100}{1.11^2} = \frac{100}{1.23} = 81.3$$

b. at 5%, ~~$$~~ $42 in 10 years grows to $(1.6 \times 42 =) 67$.

How much is $\$78_{10}$ ~~worth~~ worth today? $\$78_{11}$?

$$\frac{67}{42} = 1.6$$

~~$78 \times 1.6 = 125$~~ $\frac{78}{1.6} = 4$ _.

~~$125 \times 1.05 = 125 + 6.25 = 131.25$~~

$$\frac{78}{1.6 \times 1.05} =$$

3. $V_0 = \frac{a}{1+i} + \frac{a}{(1+i)^2} + \dots + \frac{a}{(1+i)^n} = a\frac{1-(1+i)^{-n}}{i}$ Memorize.

9-26-62

4. Derive (3), above.

5. Adjust (3), has many forms. Here, show possibilities; later, go over one by one, brodez.

a. annualization

$$a = V_0 \frac{i}{1-(1+i)^{-n}}$$

$$a = \frac{V_n}{(1+i)^n} \frac{i\,(1+i)^n}{1-(1+i)^{-n}} = \frac{V_n \, i}{(1+i)^n - 1}$$

10-1-62 — Importance: is good to maximize.

b. Sinking fund; capital ~~recovery~~ recovery factor

$$a = V_n \frac{i}{(1+i)^n - 1}$$

$$c = V_0 \frac{i\,(1+i)^n}{(1+i)^n - 1} = V_0 \frac{i}{1-(1+i)^{-n}}$$

Ec. 117

10-1-62

Go over Thomsen

Skip Form. 4

Note Form. 6 $V_o = \dfrac{V_n}{(1+i)^n - 1}$

Show how ~~comes from~~ relates to basic form, so: $V_o = a\,\dfrac{1-(1+i)^{-n}}{i}$; $a = V_n \dfrac{i}{(1+i)^n -$

$\dfrac{a}{i} = \dfrac{V_n}{(1+i)^n - 1}$. But $\dfrac{a}{i} = V_o$, where $\underline{a}$ is perpetual.

Form. 7. Adds tax rate to i.r. Is gurly. imptnt

a. Parallel treatment

b. Bias vs. durability

c. Tax capn. $V_o = \dfrac{a}{i+t}$. Derive either by direct algebra, or

$V_o = \sum_0^\infty \dfrac{a}{(1+i+t)^n} = \dfrac{ra}{1-r} = \dfrac{(1+i+t)^{n-1}\,a}{\frac{1+i+t-1}{1+i+t}} = \dfrac{(1+i+t)^{1-\frac{1}{n}}\,a}{i+t} = \dfrac{a}{i+t}$

Thompson's 3 doctrines

Gross yield -- per year. Max, regardless of any cost.

Forest rent. Same, but deduct initial costs. Regardless of interest.

Soil rent.

Guilt-by-association: ties gross yield to mercantilism & Nazism. Forest fundamentalism. Forest rent to socialism.

General solutions to any problem. Time-averages, max. the difference.

Draw curves for timber; for buildings.

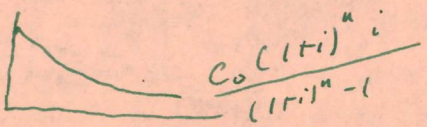
Then assign BB, & pass their "Present value rules", etc.

Separate rev. & costs, r & c

2 kinds improvements to land: depr., apprec.

Both begin w. heavy lumpy initial outlay, C_o

Time-ave:



$= C_o \frac{i}{1-(1+i)^{-n}}$

Spread costs over time. Pushes always toward longer life.

Revs., push toward shorter life

Ec. 117, 10-5-62

1. Explain getting S_e from a. Assumes a in perp. Otherwise cdn't capitalize.

Get same by summing: $\frac{V_n}{(1+i)^n} + \frac{V_{2n}}{(1+i)^{2n}} + \dots + \frac{V_\infty}{(1+i)^\infty} = V\frac{r}{1-r}$, $r = \frac{1}{(1+i)^n}$

$\sum_{x=1}^{x=\infty} \frac{V_x}{(1+i)^{xn}} = \frac{V_n}{(1+i)^n - 1} = V\frac{(1+i)^{-n}}{1-(1+i)^{-n}} = \frac{V}{(1+i)^n - 1}$

2. Explain use of a, mgnl. anal., looking forward

When approach terminal year, don't look back to sunk costs.

Look forward. Defender & challenger. Defender must earn return on its own ~~salvage~~ liqn. value plus future of site: $\frac{\Delta q}{\Delta t} \geq qi + a$

Future of site reps. what challenger can do. over time.

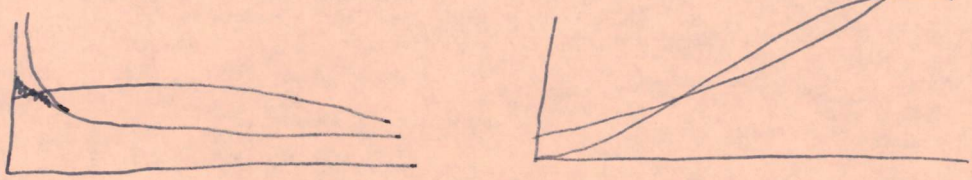
a surly preferable to S_e, bec. needn't look so far ahead.

But for symmetry, cd. also write, must earn return on liqn. value plus site value: $\Delta q \geq i[q + S_e]$; or, $\frac{\Delta q}{q + S_e} \geq i$

For depreeg improvements, liqn. value = 0, so $\Delta q \geq a \cong S_e i$

Note important role of land in replacement policy. Is usually the only reason to salvage old bldg. Urban renewal; slags bec: free xptn out; antic. bldg taxes; low land taxes.

3. Note lens-shaped area.



Max. revs., short life

Min. costs, long "

Max. net rev. medium. Ec's to golden mean.

~~3a. Note~~

4. Interdependency of valuation & replacement policy.

5. Residual valuation of old bldgs. (Necy. for calcg. depreen; for differential taxes)

The prior claim of land on joint income. Always an o.c.

5a. The steady nature of land claim. Actual income is periodic, usually. Land is of constant quality thru life of improvements; it is the improvement which vary. Land yields steady stream of services.

6. Diff. int.-elast.

Ec. 117, Mon. 10-8-62

Discuss BB, ch. 5

Their math. is good, & basic. But fail to ~~apply~~ allow for peculiar traits of site.

1. Where conflicts w. Thuneen, T. is right.

BB fails to take acct. of site factor: & gen. of age of invmt. at demolition, & reuse of site. I.e., overlooks ~~unique time~~ land, whose unique durability calls for separate treatment in all problems involv'g time.

BB leaves cut-off point up in the air, p. 153.

2. PV rule. "Adopt any project..." p. 152. Doesn't say how to max. or what to max. in plann'g the proj.; nor when to cut off.

Nor how decide between rival projects for same site. They later, p. 156, note some projects are mutually exclusive. Then, choose the higher one. But figures for only one go 'round, not a Se, just a V_0.

3. Adopt any project for which internal r. of r. > i.

They urge never use.

But how det. r. of r.? and how decide cutoff?

and how much impute to site? Can't know r. of r. otherwise.

4. Annual net bens. rule. Allege same as p.v. rule. "Adopt any project..." with $s > 0$. ~~$s = V_0 \frac{i}{1-(1+i)^{-n}}$~~, so if there is a $+V_0$ there is also a $+s$. Note that they cannot use simple capitalization, bec. they are dealing only w. one cycle. $V_0 = s \frac{1-(1+i)^{-n}}{i} < \frac{s}{i}$

The 2 rules are not same bec in cfg. 2 projects of different durability, n is different. One w. a longer n, and higher V_0, could have lower s.

If one includes site cost ~~of~~ with C_0, then must deduct at end, bec salvage site. Costs are then $C_0(1+i)^n + S_{e0}[(1+i)^n - 1] = C_0(1+i)^n + a\frac{(1+i)^n-1}{i}$

~~xxxxxxxx~~

And time-ann. of cost is then $\frac{C_0 i}{1-(1+i)^{-n}} + a$, or cap. recovery factor plus rent.

Ec. 117, Fri., 10-12-62

Blue Book, chap. 5

A. Their math. gurly. good. But none of their rules is correct, bcs. are off in outer space — forgetting that every investment constitutes use of a valuable site, with unique traits: 1) all projects on it are mutually exclusive (BB treats as an after thought;) 2) has a terminal salvage value which must be deducted from costs at t, if counted as a cost at 0. ($S_0(1+i)^t - S_t = S[(1+i)^t - 1]$); 3) has a future use, part of use of time.

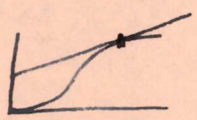
B. Neglect of site accompanied by neglect of cut-off date. p. 153.

C. Their 3 rules: 1, present value; 2, internal r of r; 3, annual net hens.

"Adopt any project ..." approach. Assumes perfect imperfection, mkts., and nothing mutually exclusive. Betrays 2 things about authors: free mkt. fundamentalists; not land econ. Only as after thought, mention mutual exclusion — then take highest. But not highest S_e, just V_0 ... they don't catch on to difference.

1. PV rule. $PV_0 = \sum \frac{R_n}{(1+i)^n}$

Weaknesses: don't tell us how to max., for any given project — i.e. when to cut off. By implication, cut off when PV a max. But that comes later than max. S_e. In fact, for depreciating asset, the older it gets, the higher is $\sum \frac{R_n}{(1+i)^n}$ .... no cut off point at all. Ridic.

With appreciating assets, means $i' = \frac{\Delta z}{z}$ 

But correct cutoff, $\Delta z = gi' + a$, comes sooner.

Max. p.v. rule becomes max. S_e rule when you consider subsequent uses of site: $S_e = \frac{R_n}{(1+i)^n} + \frac{R_{2n}}{(1+i)^{2n}} + \dots +$

Or when you max. annual =nt of periodic return to site, bcs. that is same as max g p.v. of site

But BB doesn't do that.

Re max. p.v. of appreciating asset.

a. Why is i of $C_0(1+i)^n = i$? That is on semi log paper only. Easier not to show that in general. Just show that at point of tangency $\Delta g = gi$, bcs. growth on cost curve = i x achieved value.

b. Most ecsts ~~ed~~ who hadn't thought about it wd. move to this Bcs. 1), Mgnl. solution appeals to them, $i = \frac{\Delta g}{g}$

2) Max. p.v.

2. Any proj. w. internal r. of r. $> i$.

What is intl. r. of r.? Keynes mec. Take simple timber case.

$$(1+x)^n = \frac{V_n}{V_0} \quad ; \quad x = \left(\frac{V_n}{V_0}\right)^{\frac{1}{n}} - 1$$

Weakness: again, no cutoff. What do if 2 rivals for same site? Max. x ... that doesn't help, leaves nothing for site.

W. ~~too~~ depreig asset, again, keep x by using longer ... every year adds something, takes away nothing. $V_0 = \frac{V_1}{1+x} + \frac{V_2}{(1+x)^2} + \dots + \frac{V_n}{(1+x)^n}$

Find x to satisfy $=n$. Every Δl yr. makes higher x.
E.g. suppose const. annuity, so $V_0 = a\frac{[-(1+x)^{-n}]}{x}$, (over) then $x = \frac{a}{V_0}[1-(1+x)^{-n}]$, $x \propto n$.
W. appreig asset, get cutoff, but too short.

$$g = C_0(1+i)^n + a\frac{(1+i)^n - 1}{i}$$

Assume — can't know x w/o knowing site value, solve:

$$(V_0 + S)(1+x)^n = V_n + S$$

$$x = \left(\frac{V_n + S}{V_0 + S}\right)^{\frac{1}{n}} - 1 \quad . \text{ If site value given externally}$$

Note that this lowers x a good deal. This is a valid method, sometimes.

3. Annual net revs. rule.

Allege same as max. pv., hcs $s = V_0\frac{i}{1-(1+i)^{-n}}$

But aren't thinking about life-span as an issue. If were, would see n can vary, so max. V_0 & max. s not same.

Check Blue Bk. again

$$V_o = a \frac{1 - (1+x)^{-n}}{x}$$

$$V_o x + a(1+x)^{-n} = a$$

$$(1+x)^{-n} = \frac{a - V_o x}{a} = 1 - \frac{V_o x}{a}$$

$$(1+x)^{n} = \frac{a}{a - V_o x}$$

$$x = \left[\frac{a}{a - V_o x} \right]^{\frac{1}{n}} - 1$$

$\frac{V_o}{a} = \frac{1 - (1+x)^{-n}}{x}$, look in tables for value of x.

Or, say $x = \frac{a}{V_o} \left[1 - (1+x)^{-n}\right]$, and x & n, from $x = 0$ when $n = 0$ to $x = \frac{a}{V_o}$ when $n = \infty$

$$g = C_o(1+i)^n + S_e\left[(1+i)^n - 1\right]$$

This is the way the factors are tied together, whatever we maximize.

$$g = \left[C_o + S_e\right](1+i)^n - S_e$$

$$g + S_e = C_o + S_e(1+i)^n$$

Solve for S_e, C_o (p.v.), or i; & max. any one. [Or $C_o + S_e$ jointly?]

$$S_e = \frac{g - C_o(1+i)^n}{(1+i)^n - 1}$$

$$C_o\,(p.v.) = \frac{g - S_e\left[(1+i)^n - 1\right]}{(1+i)^n} = \frac{g}{(1+i)^n} - S_e\left[1 - (1+i)^{-n}\right] = \frac{g}{(1+i)^n} - \text{p.v. of a}$$

$$= \frac{g + S_e}{(1+i)^n} - S_e$$

Downing points out that now, again, costs are not involved in P.V., same as my criticism of simpler maximization of p.v. Is there an answer? Yes. S_e reflects future costs. P.V. here is

$$i = \left[\frac{g + S_e}{C_o + S_e}\right]^{\frac{1}{n}} - 1$$

$$S_e + C_o = \frac{g + S_e}{(1+i)^n}$$

Not really a solution?

Max'g p.v. should handle costs so: $p.v. = \dfrac{g - C(1+i)^n}{(1+i)^n} = \dfrac{g}{(1+i)^n} - C$

Properly, max. p.v. = $\left[g - C(1+i)^n\right]$

1. $-\Delta i$ causes investors to build in more durability. Thus Δs static i.q. — no, bcs. they also buy more land. w/o changing dynamic. Will reduce dynamic i.q., bcs they buy more land. Reduc'g i.q. is symbolic of qual reduction of ratio of flow/fund. [C.f. of a fund = $\frac{\text{flow, yrly}}{\text{fund}}$]

Land is intermediate between long & short capl. Bcs. capl. is divisible, land not. Cfg whole land, vs. whole bldg., land more i-elastic. Even a very durable bldg. But cf. land vs. extend'g life of bldg. beyond c. 18 yrs, when doubling i, from 4% to 8%, just halves present value, bldg. extension is more elastic. (No. of yrs depends on i_0. Doubling i from 1% to 2% halves p.v. at 70 yrs., etc.)

So lo-i man improves more "intensively", as to durability. But that's not really intensity, same Co. same or less space, longer lasting. Less yrly. or annualized output from given assets, bcs. slower turning. Here it is exply. implied to emphasize dynamic c higher % of int. & rent in final mix. Overall, $-\Delta i$ means, as to given assets, sub. land for capl. more of cap. inputs. Within cap., sub. long for short. Result, less cap. on more land, & the cap. turns slower. So, much less emplmt.

Re value-added, part of depreciation must be cancelled out, as an offsite purchase. Even so, v.-a. i.q. rises w. higher turnover or c.f., bcs more complements, on site. E.g. much of value of bldg is created onsite. Importing inputs allows higher i.q., ∴

2. Static or fund i.q. is of dubious meaning, bcs. l.v. not a fund. But value-added dynamic or flow i.q. shd correspond.

2½. The cap. recovery factor is not the cap. input alone, but the labor & capl. input. Except for cap. used to carry along early yrs. of site.

3. Flow i.q. is needed. 2 kinds

a. Gross i.q. $\frac{\text{Non-land inputs}}{\text{Gross output}} = \frac{\sum C_n(1+i)^{t-n}}{\sum R_n(1+i)^{t-n}}$

c. The capl. ÷ land ratio, or net inc. from each. Net out return of capl. from both output & input. Trouble is, return of capl. reqs. labor at Co, needs inclusion overall.

b. Net i.q. $\frac{\text{Non-land on-site flow inputs}}{\text{Value added on site}} = \frac{\text{Non-land inputs less offsite purchases}}{\text{Gross output less offsite purchases.}}$

Also needs adjustment for change in fund held onsite.

If found onsite wages, count that part of gross output? Is counted when liquidated, as R_n. If it wanes, tho, can't deduct that from gross or gross ind. net be gross? But value added on site is what we're seeking, so ok.

✓ When evaluating a whole cycle, $\frac{dU}{dn}$ is not a rev, bec U is finally liqd. & counted as rev. U' is rev. only when defining i.q. of one year. But latter is very misleadg, bec. i.q. of one yr just shows int. & rent inputs & resulting output. Doesn't show C_0 in output & input, so i.q. is too low.

Query: shd we value i.q., like rent, over long pd., or annually? Annually shd be ok if we consider growth of value of tree for sale to investor, rather than g. Then, output is

$i\left[a\left(\frac{(1+i)^n-1}{i}\right)+C_0(1+i)^n\right]$, i.e. int. on achieved value, and non-land input

is $i\left[a\frac{(1+i)^n-1}{i}+C_0(1+i)^n\right]-a = (a+C_0 i)(1+i)^n - 2a$ (Note proper i. leverage)

✓ (To that shd add $\frac{C_0}{t}$, as part of non-land input. Ui is just cap & land input, not labor.) For which, use cap. rec. factor × C_0.

so i.q. for tree growth $= \frac{\text{non-land input}}{\text{output}} = \frac{i\left[a\frac{(1+i)^n-1}{i}+C_0(1+i)^n\right]-a}{i\left[a\frac{(1+i)^n-1}{i}+C_0(1+i)^n\right]}$

$= 1 - \frac{a}{i\left[a\frac{(1+i)^n-1}{i}+C_0(1+i)^n\right]}$

This is not an evaluation of the whole cycle, however. For that, we need to spread output & non land inputs evenly over time, just as we have land. Annualize the value at maturity, m, for annualized output; annualize the accumulated non-land costs at maturity for annualized non-land costs.

~~Unrevised~~ ~~Maturity~~ value, $\frac{U_m}{g_m} = a\frac{(1+i)^m-1}{i}+C_0(1+i)^m = U_m$

annualized $U_m = U_m\frac{i}{(1+i)^m-1} = \boxed{a + C_0\frac{i}{1-(1+i)^{-m}}}$ = rent + cap. recovery

✓ Cum. non-land costs = output − land costs = $g_m - ma$?

✓ No need to annualize both num. & denom. i.q. $= \frac{g-ma}{g} = 1-\frac{ma}{g}$

If so, annualized non land cost $= a + C_0\frac{i}{1-(1+i)^{-m}} - ma\frac{i}{(1+i)^m-1}$

✓ i.q. $= 1-\frac{ma}{g}$ Same as $\frac{C_0(1+i)^m + a\frac{(1+i)^m-1}{i} - ma}{u}$

And i.q. $= 1 - \frac{ma\frac{i}{(1+i)^m-1}}{a+C_0\frac{i}{1-(1+i)^{-m}}}$ $\frac{ma}{a\frac{(1+i)^m-1}{i}+C_0(1+i)^m} = \frac{ma}{g}$

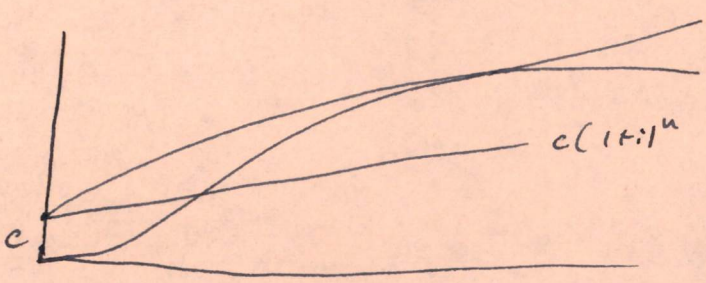
$\frac{a+C_0\frac{i}{1-(1+i)^{-m}} - ma\frac{i}{(1+i)^m-1}}{a+C_0\frac{i}{1-(1+i)^{-m}}}$

$1-\frac{ma}{g}$

Needs more work.

Int. is $C_0[(1+i)^t-1] + S[(1+i)^t-1] - tSi = (C+s)[(1+i)^n-1] - tSi$

$= C_0[(1+i)^t-1] + S[(1+i)^t - 1 - ti]$



As to annual output on site, it is int. on achieved value. Si. The "unsevered" value, U: But no, it is ill-ta, and more, like $\frac{C_0}{n}$

$U = (C+s)(1+i)^n - S$

annual growth of U, $\frac{dU}{dn}$, is $i(C+s)(1+i)^{n-1}$. Higher each year, absolutely. It is interest on achieved value. + S. ~~achieved value~~ unsevered value is the non-site input of each yr. — even tho S appears in it, that sum of past site contributions, not current. But is partial, ignores labor etc.

$(C+s)(1+i)^n$ is unsevered value + S. Each yr. U grows by int. on U + S. Output = $i(U+s)$. ~~Site~~ Nonland input = $U \cdot i$

$l.q. = \frac{U \cdot i}{(U+s)i} = \frac{U}{U+s} = \frac{(C+s)(1+i)^n - S}{(C+s)(1+i)^n} = 1 - \frac{S}{(C+s)(1+i)^n}$

This makes flow i.q. = static i.q. $\left(\frac{U}{U+s}\right)$; and it grows to a max. at harvest, net of funds. varying w. stage of growth cycle. It wd. not serve to cf. 2 rotations w. ea. other, overall. For that, time-ave. costs ÷ Time-ave. output better.

Whoa - this is gross i.q.? Not gross of imported funds - just includes labor.

Over whole cycle, non land input, ~~time~~ annualized, $= c = C_0 \frac{i}{1-(1+i)^{-n}}$

Output, annualized, $= g\frac{i}{(1+i)^n - 1}$. annualized i.q. $= \frac{C_0 \frac{i}{1-(1+i)^{-n}}}{g\frac{i}{(1+i)^n-1}}$

$= \frac{C_0}{g}(1+i)^n$. Good.... if we know what the cycle period is. But where that is indefinite, just take any reasonable & long period.

Value added over n is $g - C$

Non-land input ~~is~~ exerted on site is $C(1+i)^n - C$

$$V.a.i.q. = \frac{C(1+i)^n - C}{g - C}$$

But above assumes C is all from offsite. Actually most of C is labor exerted onsite. Only fertilizer, nursery stock, &c. ~~=~~ funds from offsite shd. be deducted. Denote them as f. ($f + l = C$, where l is labor)

Incomplete -- must add $S[(1+i)^t - 1] - ta$

$$V.a.i.q. \equiv \frac{C(1+i)^n - f}{g - f}$$

So now combine all thoughts in one solution. Annualize offsite funds & deduct result from bttm. p. X-3 for net i.q.

(Bence on primary & 2dry produc elast. now seems very imptnt. Who else has flirted so near vital topic? He points out how firm can put off disinj rets. to cap. inputs on fixed land by buying offsite inputs. That cd. also be described as sq turnover of his cap. Assign for (1 T.)

But he can also Δ c.f. by more C_o on site. Surely, tho, that is limited by ~~size &~~ smallness & someness of site — also, it involves exporting from one part of a site to another ... resumed on X-16.

1. Gross i.q.

a. Timber $$\frac{C_o(1+i)^t + S[(1+i)^t - 1] - ta}{g} = \frac{C_o(1+i)^t + S\left[(1+i)^t - 1 - \frac{ta}{i}\right]}{g} = \frac{g - ta}{g}$$

b. Land $$g.q. = \frac{\Sigma C_n (1+i)^{t-n} + S\left[(1+i)^t - 1 - \frac{ta}{i}\right]}{\Sigma R_n (1+i)^{t-n}} = \frac{\Sigma R_n (1+i)^{t-n} - ta}{\Sigma R_n (1+i)^{t-n}} = 1 - \frac{ta}{g}$$

2. Net i.q. (value-added)

a. Timber $$\frac{C_o(1+i)^t - f}{g - f}$$ $f \equiv$ funds imported from offsite

b. Land. $$N.i.q. = \frac{\Sigma C_n (1+i)^{t-n} - \Sigma f}{\Sigma R_n (1+i)^{t-n} - \Sigma f}$$

But does annualizn account for int on accum. $\underline{a}$? No. Non-land input is $a\frac{(1+i)^t-1}{i} - ta = a\left[\frac{(1+i)^t-1}{i} - t\right]$

Do result 2:

2. Net i.q.

a. Timber. n.i.q. $= \dfrac{g - ta - f}{g - f}$

b. Gen'l n.i.q. $= \dfrac{\sum R_n(1+i)^{t-n} - ta - \sum f}{\sum R_n(1+i)^{t-n} - \sum f}$

R here is + rev. only.

Note these are no longer same as $\frac{u}{u+s}$, the instantaneous n.i.q. or intra-cyclical n.i.q.

Now we've defined i.q., how does Δi affect them? Probly raise gross & lower net. But I don't see how formulas contribute anything to solution.

A Big problem w. formulae: they are simplifications based on ass. we know $\underline{a}$. That wd. be ok, except part of purpose is to attent to size of $\underline{a}$ by using explicit non land costs. Even if we use $C(1+i)^t + a\frac{(1+i)^t-1}{i} - ta$ however, we can't use formula to show $\underline{a}$, we must assume it has bn. computed. That is forced on us by need to distinguish interest on a_0 to a_{t-1} from ta. It is just as well, anyway, bcs. emphasizes there are 2 steps involved in use of $\underline{a}$: 1, Compute as residual from future use; 2, apply as parameter to present use.

Results of higher i on i.q., ~~natl~~ gross & net, indiv. & society.

1. Individ.

a. Gross i.q. $= \dfrac{\Sigma R(1+i)^{t-u} - \Sigma a}{\Sigma R(1+i)^{t-u}}$

Use less land; less of durable cap.; less assets overall (assuming i comes to him as a cost, not due to his d opps.)

Will do more bus. w. same land, or same w. less. May do more bus. overall. Let's assume perfect & continuous mkts at all mgns & see. Say i goes up from 4% to 8%. He subs short for long assets

Bases of cfg: 1, Relative p.v.

2. Fully costs of holding (is rent one of these, for separable long future values?)

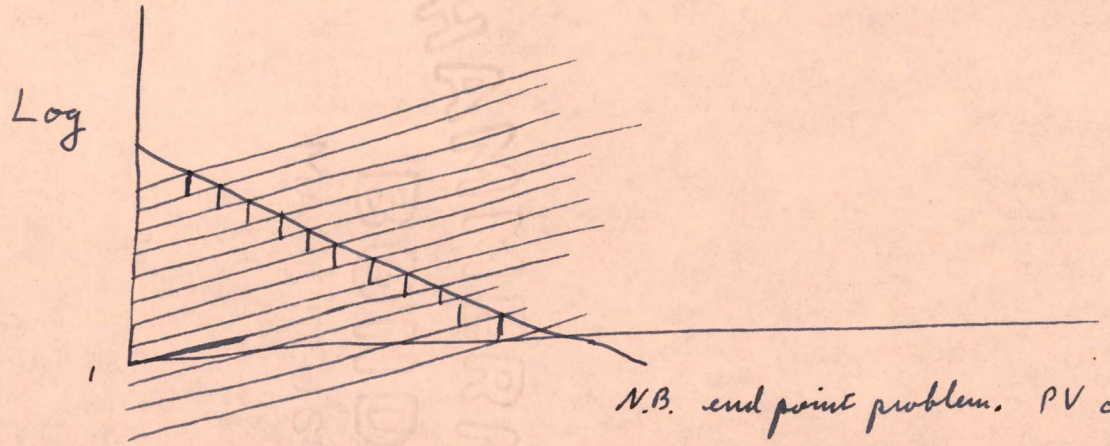
Problem: What is annual cost of holding V_{80}? It must more than double — now? Annual cost is $> i \times C_0(1+i)^n$. ~~It is $i \times U$, where $U = C(1+i)^n + S[(1+i)^n - 1]$~~

~~It~~, It is ~~rather~~ the growth of U, which is interest on $U+S$, or $i(U+S)$ or $i\left[C(1+i)^n + S(1+i)^n\right]$. It is $>$ int. on p.v. Altho p.v. $> U$, pv. $< U+S$. When U is very small, in early yrs of long wait, $i(U+S) >> iU$

My 1st scheme of imputation was inadequate, bec it treated rent wrong. Simply segregated it out. Error there shown by supposing $C_0 < p \cdot v_{80}$, implies you expand C_0 until reach equality. (or mgnl = ty?).

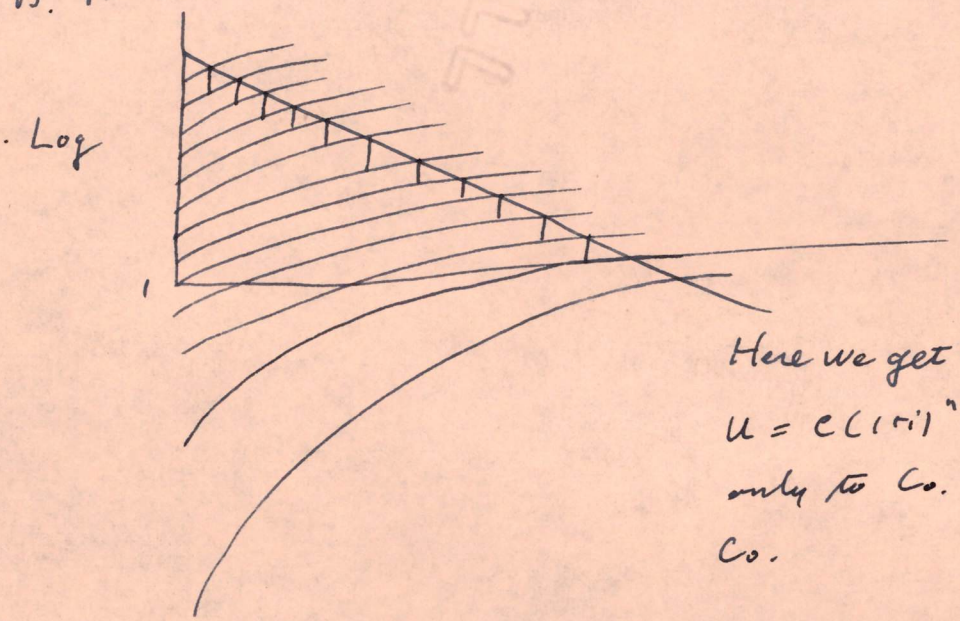
But ideally, $C_0 = p v_t$ only for last, or mgnl. year. Elsewhere ~~smaller~~ $C_0 < p v_n$. Even for mgnl. yr, must get rent

A. P.v. method, w/o site



N.B. end point problem. PV can never fall to zero. So must treat site cost separately, then cut off

B. P.v. method w. site

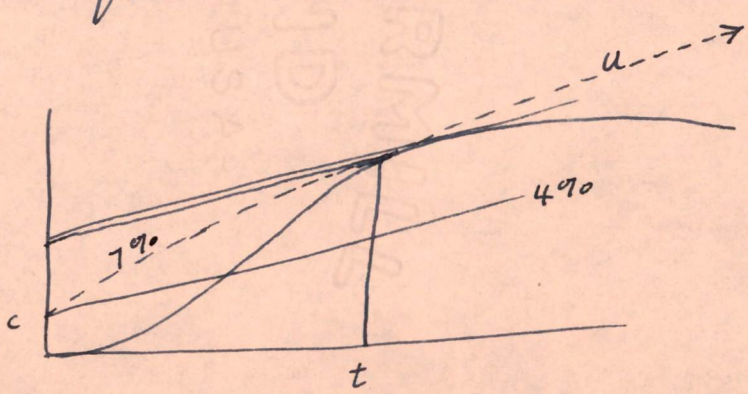


Here we get cutoff bcs p.v. can fall to zero? $U = C[(1+i)^n + S[(1+i)^n - 1]$. No, can fall only to Co. But method A, can't fall to Co.

How divide site among ~~n~~ yrs of light? In any year, evenly, ~~So remote~~ in absence of special conditions. Remote future values are hard put to bear their aliquot share. The 4% Δ of annual cost of holding the remote values is greater, bcs Co is so low. That is enough to know for now - can work out full system of acctg. later, under ARA grant next spring maybe.

Is signif. that, to show what we know must be true, that annual Δ to cost of holding remote value is much greater than for near value, we must call in site — even tho, on simple p.v. basis, point is obvious w/o site.

So i causes us to use less land; & less durable cap. But latter effect due to site factor, i.e. bec durable cap. uses site longer, & not bec it ties up cap. longer! At least, on annual cost basis, that seems to follow. Can it be?



$$u = (c+s)(1+i)^n - s$$

$$\text{True p.v., tpv,} = \frac{g - s[(1+i)^n - 1]}{(1+i)^n} = \frac{g}{(1+i)^n} - s\left[1 - (1+i)^{-n}\right]$$

$$= \frac{g+s}{(1+i)^n} - s$$

That is more i-elastic than simple p.v. Tpv can be 0 or neg.

Likewise, there must be leverage in annual costs of holding timber.

What is annual cost & rev. of holdg timber? gt is cost of holdg [timber + site], less cost of holdg site. $ui - a$!

Yearly Cost of holdg timber plus site is $i(u+s) = i(c+s)(1+i)^n$

" " " " " alone is $iu = i[(c+s)(1+i)^n - s]$

We must either subtract a, unaffected by i; or look to revenue side, & have ui - i drop durables bec their appreciation is lowered.

Costs & revs, annual.

Land: $MC \equiv V_i - \Delta V$

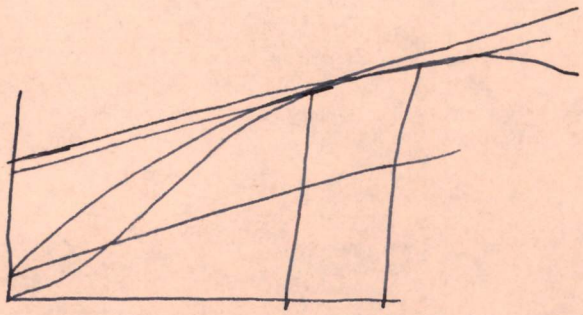
Eqm: $MR = V_i - \Delta V$

Timber: ΔV is only rev. So ΔV must cover $L = V_i$, on mgnl. acquisition.

To analyze w/o ref. to site, consider 50 yr bond. Cost, yrly, of holdg is $V_n i$. Why wd. you shift from long to short?

Or $V_n i - \Delta V$?

Assume you must hold them to maturity. Then, to find annual cost, we must evaluate full cycle, & annualize. Thus feel full weight of diff in i.



$C(1+i)^n + S[(1+i)^n - 1]$

$= (C+S)(1+i)^n - S \equiv U$

Yearly growth of U is $i(C+S)(1+i)^n$

Cost of holding U, yrly, is $\underline{?}$: $Ui + a$

Annual Cost $\stackrel{?}{=}$ $i(U+S) - a$

What we must do is find cost (yrly) of hold'g timber, assuming cost of site to remain constant at a (rather than rise w. i, like Si). Thus we get leverage.

$$U = \frac{g - S[(1+i)^n - 1]}{(1+i)^n}$$

a 2d possible approach is to say yrly hold'g cost is int less appreciation.

Consideration of long & short bonds, yrly cost, raises possibility that principle that $hi - i$ leads to short investm, fast turnover, depends on site cost! Bcs. at higher i, you shun short bonds as well as long.

$i\left[C(1+i)^n + \frac{a}{i}[(1+i)^n - 1]\right]$ $= iC(1+i)^n + a[(1+i)^n - 1]$

$= i\left[\left(C + \frac{a}{i}\right)(1+i)^n\right] - a$

This As more than a $\underline{i}$.

$i[(C+S)(1+i)^n - S]$

$i[C(1+i)^n + S[(1+i)^n - 1]]$

There remain many questions, but must leave them for now. E.g.

1. Should the i in $(1+i)^n$ be steady, or Δ also?
2. Shouldn't we better begin from g_t and work back?
3. Etc. ad inf.
4. a 3d way of analyzing factors affecting effect of i on assets is horizontally, how much is i.r. per period of life. E.g. is 12% per yr = 1% per mo., 25% per 2 yrs, etc.

5. Can we subtract na, as on top of p. X-5, in getting yrly cost

Surely, recall from Fnd Mat. that higher i mean shorter rotations, hence greater c.f., higher gross i.q. ∧ (Net i.q. perhaps lower, if Co all-comm effects) Effect on maturity tempered by reduc'g l.v., or $\underline{a}$: tempered, but not offset. As to i.q., smaller $\underline{a}$ accentuates ~~drop~~ rise of i.q. w. Δi.

If $\underline{\text{individ.}}$ gets higher $\underline{i}$, S_e exogenous & const., gets double speedup, bec. $\underline{a}$, instead of falling, appears higher, as $S_e i$. Now i.q. is ambiguous — more land at his $S i$, less i.q. than if more at mkt $\underline{i}$

✓ For a complete picture of factor progns, let's use more than i.q. Get all inputs & combos of inputs, over cycle, as % of total input.

Thus, i.q. $= \dfrac{\sum C_n(1+i)^{t-n} + S[(1+i)^t - 1 - ti]}{\sum R_n(1+i)^{t-n}}$ But num. includes labor & int., both.

are several i.q.s: Labor, capl., & land, 3 shd. add to $= \sum R_n(1+i)^{t-n}$

Labor i.q. $\cong \dfrac{\sum C_n}{\sum R_n(1+i)^{t-n}}$

Cap. i.q. $\cong \dfrac{\sum[C_n(1+i)^{t-n} - C_n] + S[(1+i)^t - 1 - ti]}{\sum R_n(1+i)^{t-n}}$

Land i.q. $= \dfrac{at}{\sum R_n(1+i)^{t-n}}$

And various combos.

An 'input' is something embodied in the product. Not land, but services thereof. Not ~~persons~~ men, but their services, which we call "labor". (Metonymy is a great enemy of clarity in vocabulary)

As to capital, the capl. used in t_0 for C_0 is not an input. It is used to hire labor, is traded for labor (standard man-days). Labor is the input. (or funds, tracable back to services.) The cap. input is the service of capl., like other services a fn. of time x quantity. $C_0[(1+i)^t - 1] + \left[a\frac{(1+i)^t-1}{i} - ta\right]$ Note that land is still treated differently. If there are other interved. inputs, C_n, carry's them to t in also a capl. input, $\sum(C_n[(1+i)^{t-n}-1])$

A fallacy wd. be to say capl. & land were same: capl. input is $C_0[(1+i)^t-1]$ and land input is $S[(1+i)^t-1]$. Truth is, latter is input of land plus some cap. Land input is S if t or at, i.e. $\sum_0^t a_n$. Why. Land input doesn't grow over life — capl. does.

Effect of Δi on the capl. input depend on how much of other inputs it is combined with. If last one yr., 1 capl. is combined w. 20 labor -- & maybe 1 land. Labor &c. C_0 input can absorb Δi. But not if last 50 yrs. Same wd. go for Δa.... in fact, there is way to Δ c.f. w/o Δi, maybe best argument for land tax.

$\bar{i}$ once expressed that by merging int. & depreen. as costs of holdg capl. $MC = Vi + \Delta V$; & including lign. in output, part of MR, even MVR. And that may be legit. in treating whole bldgs. as units.

Now express by sayg w. land, there is no other input to absorb the Δi? And/Or that w. land you cannot escape Δi; w. capl., you adjust c.f., avoid full impact, so for that reason Δi doesn't cut back yr holdgs of cap. so much. That is 2 reasons: a, Most capl. is under 18 yrs durability, so C_0 can absorb the loss; b, Longer invests. can be shortened; c, Rent falls, tempering blow (for aggr. $-\Delta i$, not indiv.); d, Δ c.f. also permits keeping labor busy on smaller land; e, Can move capl. into more imported fends, subs. for loss of land.

Cap. is a fund; an input is a flow. Cap. is the receiver, that funds flows — (the reservoir? No.)

Cap. differs from land in that capl. can only be put in in conjunction w. the flows it congeals. Likewise is taken out w. them, embodied in products. Is their surrogate. a tie-in, inescapable — both ways. Complete comply, a mutual dependence. So there is a composite input, & int. is just one cost, wages the other. So if i rises, cost of composite input L.V. is indep. of flows. doesn't rise in propn. $c = C_o \frac{i}{1-(1+i)^{-t}}$ .... not of capl., but of capl. tied to labor. Certes, can reduce propn. of cap/labor by + Δ c.f., but doesn't lower C_o, & further reduces sensitivity to Δi. Combo. of elast. to alter propn, but absolute impossibility of escaping basic tie-in, makes C_o insensitive to Δi. Recycling of cap. means, too, Δ o.f. replaces C_o quicker, maintaining static mean level of cap. just combines w. more C_o or labor

Δ land input per yr doesn't mean hiring more labor nearby. No tie-in of that kind.

So land has its tie-in over time, capl. has its w. flow of inputs at C_o. Combo. ops. to make land input more i-elastic.

This is obvious in the p.v. approach. Annual cost approach, I came to grief by beginning to realize that C_o is not part of capl. input. (P.v. approach is essentially means of subtracting cap. input from future lign. value, getting value of labor input now.

$V_n - V_o[(1+i)^n - 1] = V_o(1+i)^n - V_o(1+i)^n + V_o = V_o$)

But C_o is tied to cap. input. Annual cost of carrying cap. is just iK, but deprecn. is tied to it on cost side, & lign. on rev. side, diluting i.

$MR \equiv$ lign. + apprecn = deprecn. + int. $\equiv MC$

Even w. timber, is an element of in & out, so maybe i. has less leverage there than w. land? Has more on p.v., but maybe offset. over whole life, by C_o? Work that out. Only cost not Ki, but $C_o \frac{i}{1-(1+i)^{-t}}$ + a, or something. Regardless of whether asset is apprec'g or deprec'g, $\underline{c}$ is only composite cost, & is less i-elast. than Si, & than $Si - \Delta S$

ΔS shd not be cfd. to $-\Delta V$ of a fund, from lign. To call S a fund is false funding.

Nice thing about $\underline{c}$, asset need not be deprec'g, applies to apprec'g ones too. So emphasis is where it belongs, not on apprecn but on human origin — a diff. of kind rather than degree. There must be ult. lign, tho, so is still necy to estab. that. But is no problem of confusing trees w. land, e.g.

On rev. side, a gen'l Δi will ΔMR, if $MR \equiv U'$.
It will take it out of either C_o or a. If C_o, clearly $\Delta U'$; ~~this~~ if a, why a also part of U'. So . . .

—

As to p.v. approach, l.v. is more than a fg i-sensitive, even if one appreciates, bec $S_e = \frac{g}{(1+i)^n - 1}$

And all land uses where R succeeds C are like S_e.

—

All capl. can be regarded as growing -- to a maturity date of complete liqn. "Depreciation" is partial liqn.

—

Surely, $c = Vi + |\Delta V|$, except c is const. over life, while $Vi + |\Delta V|$ tends to get smaller w. age.

———

By p.v. approach, trees appear more i-elast. than land? Not the land on which trees are grown, but const. inc. land. But by cap. recon. approach, yrly cost of holding timber is $C_o \frac{i}{1-(1+i)^{-t}}$, Ci is min. possible value, & i downstairs offsets i upstairs. Reconcile. When you annualize costs, must also annualize revs, & they are also i-elastic. Thus Δi ~~reduces~~ $\rightarrow \Delta c$, less than $\rightarrow \Delta Si$; but also lowers $\frac{R_t\, i}{(1+i)^t - 1}$, and so may have sharper overall result than on Si, (where S is unaffected.)
But as to where S is timberland, higher yrly cost must be worked out somehow, probly by charging int. on $[S(1+i)^n - S]$, or something similar.

Cap. is a fund from storing up flows. It grows in value over life, & is liqd. None of those is true of l.v., except maybe the 2cl.

If invr. holds tree over part-life, he lays it by sale, so yrly cost is cap. recov. over yrs he holds it. Is it parallel if he buys empty land, that appreciates? One diff. is, the tree is on a site & must earn a return on that as well as C_0. How about tree on mgnl land?

Answer is, if you buy & sell land fast, treat it as an inventory, you can deal in it ok at higher i ... your yrly cost is Purchase $\frac{i}{1-(1+i)^{-n}}$ and by buying low & selling dear you can overcome handicap of hi i. It won't be easy, bcs you'll still need big capl. re labor. But r.e. dealer escapes full force of hi i. Man who gets full brunt is buyer for use.

But that implies radically new approach to my alloc & time-indivisibility theory. Land has a life, based on resale. My critic was right who said can avoid impact by frequent sale? No, not generally. Land must remain under durable improvements to the end, so if we do amortize land purchase in antic. of resale, it must be over $\underline{\underline{T}}$ yrs, same as the most durable inner core of the house, the lowest, longest stratum. So at best, $S_0 = a\frac{1-(1+i)^{-T}}{i} + \frac{S_T}{(1+i)^T}$ still much more sensitive to i than the bldg. On annual basis, cost of hold'g land is $a = S_0 \frac{i}{1-(1+i)^{-T}}$ p/$, same as lowest, longest stratum - all others must use the appropriate $\underline{n}$

Note that means, re cap. recovery, of bldg., must apply to each stratum seperately. Suppose critic forces us to apply to whole bldg. life, claims bldg. turns over as slow as land could? Cap. recov. is approp. charge assuming you get no return of cap. until $\underline{\underline{T}}$, as w. tree.

If you do get early returns, they either reduce capl., or are part of $\underline{c}$ ...?
If we treat aggreg., return of cap. is part of MR. On land, return of capl. comes entirely at end of cycle, $\underline{t}$; w. cap., comes nearer beginning. *

If you Δ c.f. on a site by importg funds, what does that do to other sites? Δ demand for raw mtrls. Some sites must export them. Cash grain, calves, minerals, ... primary products. Those sites also import inputs tho ... fertilizer, mining machines. Some of them become hi i.q. 2dry producers sometimes add value drawn physically from their land ... feeders, e.g. ... but urban ones hardly do.

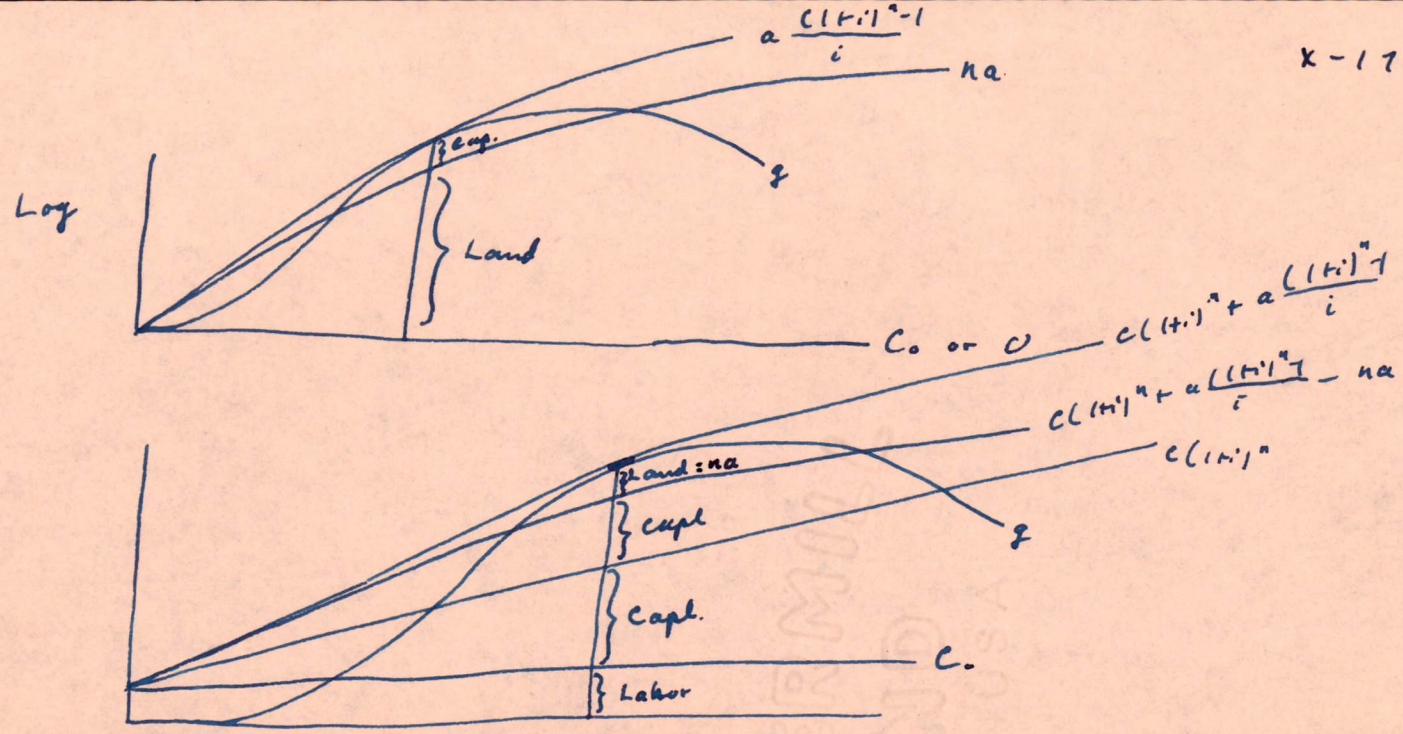
Still, Δ c.f. by importg funds is suspect, in aggreg., bcs requires extensive use of other land. all fams can't be dry lot feeders – someone must grow calves & grain. But we cd. Δ c.f. at all levels: a, By Δg onsite labor; b, by importg more funds: grain fm more fertilizer, power, machines; calf grower more feed, grass-seed, medicine, wire, etc.

So qualy, higher fund c.f. isn't at expense of others, but supplies more raw mtrls for others, & mkts. Just so it is balanced.

As to forestalling dim. rets by Δ f.c.f., individ. can do that better than society, bcs individ. moves into another sector, takes over intensive fms. from others. Society can go quite far tho, by all Δg f.c.f. in step.

If C_o & $\Sigma C_n = 0$, l.q. = $\dfrac{S[(1+i)^t - 1] - ta}{g}$ which $\propto t$

So long life makes a stratum cap.-intensive faster than land-intensive, even if start out w. no C_o. In fact, maybe bcs. you start w. none. $C_o(1+i)^t$ grows slowly w. $\underline{t}$ at first. But cap. input is not $C(1+i)^t$ but $(C+S)[(1+i)^t - 1] - ta$



Here is good way to show breakdown or imputation.
The longer, the more cap. intense, bcs land component is capl.

True on any stratum, but not $\sum$ strata bcs. they fall off.

Surely, by 'intensity' or i.q. I will mean net of funds. Funds add to net i.q. by interest, & complement's other inputs; but surely, v.a. i.q. more mngful.

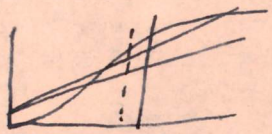
'Static' i.q. gives only cap ÷ land, no info. on turnover, labor input, etc. Nor on whole cycles. Hence very poor.

Funds in C_o, (T_o), surely will be more simply there compet. w. labor.

so, alternative to $MC \equiv V_i + |\Delta V|$ is now $MC = c = C_0 \frac{i}{1-(1+i)^{-n}}$

Henceforth use t, terminal, for d, m, or l.

1. Have seen, timber case, i.g. = $\frac{C_0(1+i)^t}{g_l} \cong \frac{g - ta}{g}$



Δi meant higher i.g. bcs.

a. Higher rply price on cap.

b. Lower a

c. Spread C_0 over fewer land-years. Emphasize last. Higher i.g. comes from higher c.f., not necly. from more cap./acre.

i.g. = $\frac{g - ta}{g}$. as t grows, i.g. falls

Need to annualize to give emphasis

Try this one again Friday.

1½. Applies only to all cases, bcs costs come earlier than recs.

Mgnl. cost = $C(1+i)^n - C$
ℓ becomes smaller as n gets smaller, (ℓ is higher!)

2. Was first approxn., oversimplified. (over)

Non-land
~~Cap~~. input actually is higher: $C_0(1+i)^t + a\frac{(1+i)^t - 1}{i} - ta = g_l - ta$

Conclusion same, tho: higher i means higher i.g.

3. Big effect of higher i is higher c.f., hence higher i.g.

As to static i.g., or $\frac{C_0 \frac{i}{1-(1+i)^{-t}} - C_0}{R_\ell}$ or, gnly, $\frac{\sum \frac{C_n(1+i)^{t-n} i}{(1+i)^t - 1} - C_0}{\sum R}$

not necly. higher. Higher i might cause more, or less, cap./acre, static.

Bcs. cuts back some more than land, & some less.

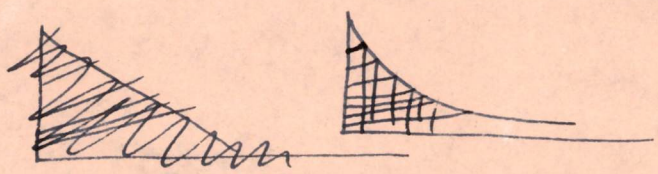
4. Land is indivisible. Higher i ~~to~~ reduces value of whole purchase.

Cap., before invmt., can determine durability, selectively chop off later yrs

Regard bldg. as n bldgs.

Grow in value like trees,

$C_0(1+i)^\ell + \frac{a}{\ell}\frac{(1+i)^\ell - 1}{i}$

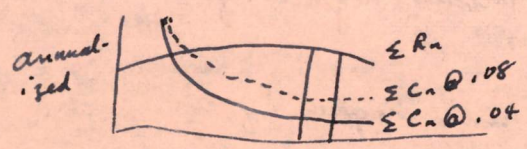


Last ones' present value is very i-sensitive

Shorter t_1 means combine C_0 w. less land-yrs; (also w. fewer capl. yrs)
Subs. labor for land.

$$l.q. = \frac{\sum C_n (1+i)^{l-n}}{\sum R_n (1+i)^{l-n}}$$

Seems parallel, but costs come earlier, so $\Delta i \rightarrow \Delta\, i.q.$
Δi also causes l to come sooner. so from given C_0, get fewer revs.



annualized $\sum R_n$ little affected by Δi, on whole.

$$= \frac{\sum \left[R_n (1+i)^{l-n}\right] i}{(1+i)^l - 1}$$

ann. R_0 made larger; ann R_l made smaller.
Balance out.

5. E.g. from tables, using simple discount.

~~Δi from~~ 4% & 8%, 18 yrs is watershed: sub. land for elements over 18 yrs; sub. elements under 18 yrs for land.

6. Elasticity of rate of return to ~~decline~~ rise of $\frac{L_t}{C_o}$, declines w. time.

E.g., one yr. wait, $r = \left(\frac{L_1}{C_o}\right)^{\frac{1}{1}} - 1$, say $.05 = \left(\frac{105}{100}\right)^{\frac{1}{1}} - 1$

Can raise to .06, or by 20%, by a 1% drop in C_o.

$$\epsilon = \frac{\% \Delta r}{\% \Delta C_o} = \frac{20\%}{1\%} = 20$$

E.g., 50 yr. wait, $.05 = \left(\frac{100}{\text{[illegible]}}\right)^{\frac{1}{50}} - 1$. Raise r to .06, requires dropping C_o to 5[illegible] $\epsilon = \frac{20\%}{\text{[illegible]}\%} = .45$, cf. 20 above.

7. So higher i causes cutback on both land & capl. Will show in a moment, probly cuts back more on capl. But prime effect is Δ c.t. of capl.

That means applying C_o more often. C_o is labor (& offsite funds). Big Δ in labor per acre. Substit. labor for land & capl. Means to full emplmt. Frequent renewal of sites.

Will try to identify bottlenecks preventing.

8. As to cap./land ratio, Si shd raise it. Consider annual MC.

$MC = Vi \pm \Delta V$

$= m.\ MR = Vi - \Delta V$

or $i = \frac{MR + \Delta V}{V}$

9. Add taxes, show their effect on allocn.

$MR = MC = V(i + \tau) - \Delta V$

Taxes & depreciation swamp i.

If no taxes, & appreciation, i counts for everything.

Mon., 10-21-62, Ec. 117

Assign Bunce ch. 2 on elast. of prodn.

Loose end: growth of U, Unearned value. $C(1+i)^n + S[(1+i)^n - 1]$. Shd = $\frac{g_t - S[(1+i)^{t-n} - 1]}{(1+i)^{t-n}}$

Effect of i on intensity of land use. | when parameters remain constant.

Preview. Are 2 meanings of intensity. Δi will Δ gross intensity, not clear about net.

Signif.
1. ~~Ig~~ Surely assumed lower i means higher intensity, but will show not so
2. ~~Ig~~ Δi shd. Δ N.I., if overall. Contrary to Kellner
3. ~~Ig~~ Is reason to think mkt. tends drive all to = intensity — but what is it?
4. ~~Ig~~ Diffl. i misallocates durable ress, if indivisible in time

5. ~~Ig~~ $\Delta i \rightarrow \Delta$ cap., not by Δ sav'g, but more intensive use of existing stock, or faster turnover or c.f.

6. ~~Ig~~ Help plan gr. own invmts at diff. i.

Intensity

1. Is a quotient. Msrd in same coin. Acre not homogeneous. So more land by value, or rent.

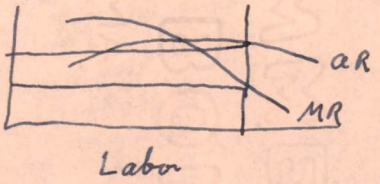
Then immedly depart from elemental idea that cities land use more intensive. Most intensive land use is on marg land, where land contributes least share of total.

2. Proper treatment of capl. input is a problem, for 2 reasons:
   a) Some comes from offsite. Extreme e.g. of hiway or r.r. track
   b) Even if all created onsite, it ~~both depre~~ yields up both its subsistence, & an income or increase over time. Part of the increase is due to the site — e.g. in case of timber or cattle. Part is an increase on the cap. accum. from past contributions of the site — so is problem of separating land from non-land inputs. The increase is part of output — but when? Etc.

3. Begin w. simple case, everything annual, no imported funds.

$$I.q. = \frac{\text{Non-land inputs}}{\text{output}}$$

Note some relations.



Non-land inputs = $Q \times MR$

Output = $Q \times aR$

$$I.q. = \frac{Q \cdot MR}{Q \cdot aR} = \frac{MR}{aR}$$

Low i.q. characterizes intensive land use, cheap inputs. Hi i.q. characterizes cheap input land.

But note paradoxes. I.q. depends on cost of inputs as well as Q. U.S. farm may be higher i.q. using one hour/acre than Jap. using 4.
Also note common fallacy, judging effic. by aR.

4. Now import some funds. E.g. fertilizer, exhausted aply.

Now have 2 i.q., net & gross.

$$G.i.q. = \frac{\text{Non-land inputs}}{\text{output}}$$

$$N.i.q. = \frac{\text{Non-land inputs} - f}{\text{output} - f} = \frac{\text{Inputs exerted on site}}{\text{Value-added on site}}.$$

G.i.q. > n.i.q. How much higher depends on e.f. of capl (save that which is replenished from onsite.)

Relative importance of land as a distrib. share often understated where turnover is hi, by use of G.i.q. Re distrn., n.i.q. is more relevant measure.
Share of cap. overstated by interpreting return of fund as increase on fund.
But this isn't means of determining distr. share --- land portion may be too little.

4, (cont.)

Bunce shows we can increase output from land much more by going into 2dry elasticity of prodn. Don't add cap., but turn cap. Note that lets us add labor, w. same land & cap. -- another position relation of i & empl.

5. Now apply some inputs over time. E.g. a ~~house~~ barn

assume all mtrls. found onsite.

Now more input over whole cycle, annualize. [Can also more annual i.g. intra mural to cycle, might be imputed to decision at time-ingres]

d ≡ demolition ~~d ≡ yr. of demolition~~ $l \equiv$ yr. of liquidation $\equiv m \equiv$ yr. of maturity

$$\text{Non-land input} = \sum\left[C_n(1+i)^{d-n}\right]\frac{i}{(1+i)^d-1}$$

Troublein, this is not the entire non-land input. Must add $\left[a\frac{(1+i)^d-1}{i} - da\right]$

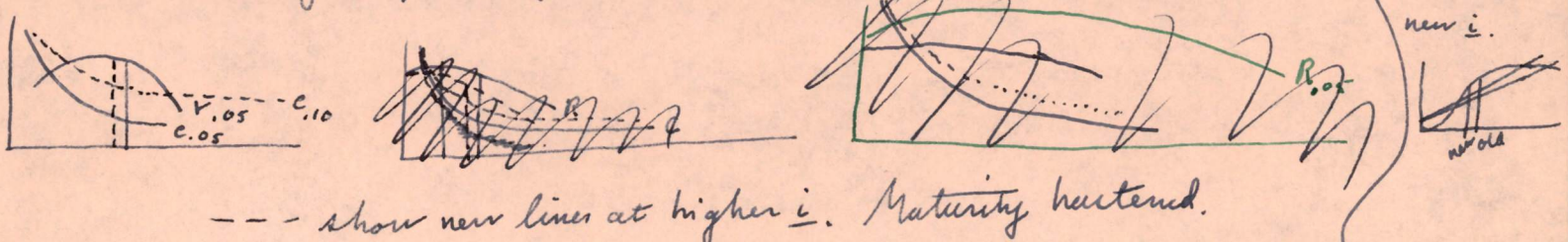
$$\text{Output} = \sum\left[R_n(1+i)^{d-n}\right]\frac{i}{(1+i)^d-1}$$

Easier just to subtract da from output, in numerator. As d rises, intensity falls.

I.g., annualizg term drops out, I.g. $= \dfrac{\sum\left[C_n(1+i)^{m-n}\right]}{\sum\left[R_n(1+i)^{m-n}\right]}$

In timber case, i.g. $= \dfrac{C_0(1+i)^l}{s}$ Higher i means cut sooner, higher i.g., even judged by old i, l moves at new i.

Visually, useful to go back to graph of time-anc. C & R



--- show new lines at higher i. Maturity hastened.

What happens to $c = C_0\dfrac{i}{1-(1+i)^{-m}}$ when i rises? c rises — only a little

Easier to trace when $c = C_0\dfrac{i(1+i)^l}{(1+i)^l-1}$

when m is small, but moves as $m \to \infty$, $c = C_0 i$. Tables confirm.

There must be an intermediate age where Δi leaves annual value unchanged.

$r = \dfrac{\sum[R_n(1+i)^{l-n}]i}{(1+i)^l-1}$ What happens to $r = R_m\dfrac{i}{(1+i)^m-1}$ when i rises? Gets smaller, l moves as m gets longer. But first few years of curve will be higher? No, but as l gets longer, l-n gets longer for each yr. so curve shdn't fall much. But certainly won't rise as much as C, & probly will fall some.

So maturity comes sooner. Cap. turn faster. Site rent lower; capd. rent even more.

So what has Δi done to i.q.? Gross & net are same, no offsite funds. Raised i.q.

In terms of i.q. = $\frac{\sum [C_n (1+i)^{m-n}]}{\sum [R_n (1+i)^{m-n}]}$, note C_n likely to be C_o raised to $(1+i)^m$; R_n more weighted to later yrs, say R_m, raised to no power at all.

Higher intensity doesn't come from higher C_o. Rather: a) Higher annual price ~~value~~ on cap. input (repg. better alternative) b) Higher c.f. of cap. input --- or, put the other way, less land-time per unit of $\frac{C_o}{cap.}$. So both int & depreciation are higher. More frequent applic. of labor. c) Lower $\frac{a}{}$.

Note paradox of land rent. It is the headsmen or demolitioner: it is lower, yet demolition comes sooner. Decline of l.v. is a tempering factor. If it failed to fall – e.g. if are analyzg hi-$\underline{i}$ individ. — then get double push to faster c.f.

6. "Fraud", you cry. Playing w. words. Changed defs. on us. Hi-i still means less C_o per acre. You just have rubber yardsticks for cap & land, or $\underline{i}$ and $\frac{a}{}$ change.

Partly true. But C_o per acre isn't everything. & that might fall, too.

... But first, distinguish 2 cases, individ & social.

7. Indiv. vs. aggreg. effects.

a. Individ.

Due to hi i, uses sharply less land. Worth less to him, but costs as much as ever.

Worth less to him bec: a, Annual value smaller b, Capzes at higher i. But external or objective cost is same. And multg same S by higher i, has to cut back.

Δi & intensity of land use, cont.

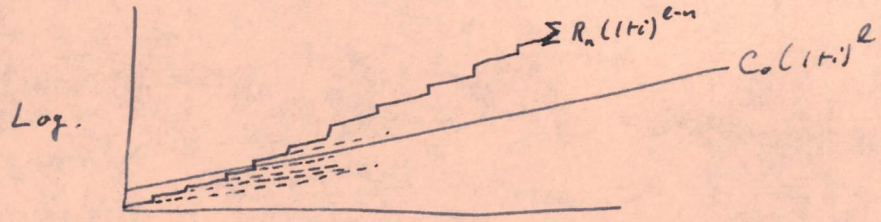
Does he use less cap., too? Any reduction in C_o compd for by faster recovery. I.e. you can reduce int. burden on cap. not just by using less, but using it a shorter time. That also means combining it w. less land-time.

To simplify, remove one area of choice by assuming const. output/acre. Maintain output/acre w. less cap./acre by faster return of cap.

b. Aggreg.

Before plunging ahead, outline all the things & combns. of them I intend to prove, re i & intensity. Keep it brief! Use proofs already worked out.

$$\text{l.q.} = \frac{\sum C_n (1+i)^{t-n}}{\sum R_n (1+i)^{t-n}} = \frac{C_o (1+i)^t}{\sum R_n (1+i)^{t-n}} \cong \frac{C_o (1+i)^t}{R \frac{(1+i)^t - 1}{i}} = \frac{C_o}{R} \frac{i}{1-(1+i)^{-t}} = \frac{c}{R}$$



Now if Δi, $\frac{c}{R}$ rises bcs.

a) Faster cap. recovery

b) More int.

Might temper by lower C_o, but that is at cost of revs., no gain in intensity.

We can't go on forever in this vein. So show 2 more easy proofs.

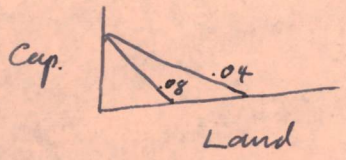
1. annual cost of holding land = Si

" " " cap = Ci – deprecn. (See what can do here w. $c = a + Ci$, where a is sink's found deprn)

$MR = MC = Si$

Etc.

2. Indiff. curves



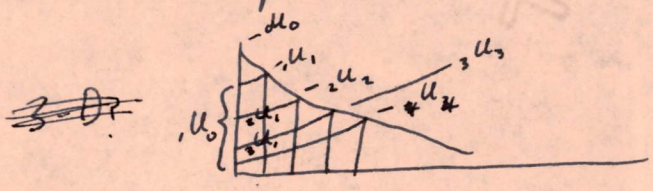
2 ideas

1. $C_o(1+i)^{\ell} - C_o$ is the interest cost

$\frac{C_o(1+i)^{\ell} - C_o}{C_o(1+i)^{\ell}}$ is int. as % of total cost, & $= 1 - (1+i)^{-\ell}$

When ℓ is hi, int. → 100% of total costs. Hi i naturally pushes you away from invmts where i is hi % of costs.

2. Re depreciation



Annual cost of hold'g capl. is interest on U_1, strictly, on $\frac{U_0+U_1}{2}$ plus % a.

" rev. " " " " recovery from $_1U_0$ to $_1U_1$ = $_1U_1 - {}_1U_0$

Liquidation = $_0U_0 - {}_1U_0 \equiv L_1$ is not a cost (input) nor revenue (output) but a transfer. Input was exerted, & output produced, earlier, when C_0 created.

But as to rate of rev., that is easily adjusted? In fact, we begin by figuring what we need to justify hold'g capl, then determine lifn. value from that. I.e. we can assume remainder grows at any rate we want to. Can assume, if wish, in yr. 1, most of gross is lifn., & remainder has grown fast ... thus, hi $\underline{i}$ → hi lifn !!??

Define return of capl. as R_n, net of C_n. I.e. the whole revs., less current costs. That depletes fund to $_1U_0$, but then it grows to $_1U_1$. Thus, return on capl. is $R_n - C_n - ({}_0U_0 - {}_1U_1) = {}_1U_1 - {}_1U_0$. Now simplify.

L_n is current net revs., year n, or $R_n - C_n$ (lifn.)

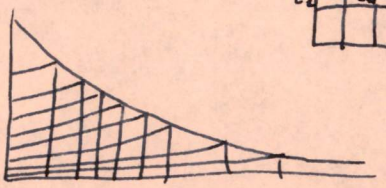
D_n is $U_{n-1} - U_n$, or observed depreen.

G, growth, is $L - D$

If $\frac{G}{U}$ is just enough, & i rises, can adjust G up by lowering D, by lowering U_0, by lowering C_0. Where cap. lasts one year, can

ΔG greatly $\frac{\text{}}{\text{}}$ as % by small % decline of C_0. Call it the elasticity of yields — the % Δ in rate of return from a 1% drop in price of C_0.

$$\epsilon \cong \frac{\%\,\Delta\left(\frac{G}{C}\right)}{\%\,-\Delta C} = \frac{\%\,\Delta\left[\left(\frac{L}{C}\right)^{\frac{1}{n}}-1\right]}{\%\,-\Delta C}$$



E.g. if $C_0 = 95$, $L_1 = 100$, $G = 5$

Now, to ΔG by 20%, $-\Delta C$ by 1%

and if $C_{25} = 20$, $L_{25} = 100$

Lower C_0 by 5%, to 19, and $\Delta\ r = \left(\frac{L}{C}\right)^{\frac{1}{n}} - 1$ negligibly.

Elasticity is the key concept here, & most promising approach. Work out a little skeleton table.

But G is return on whole of U. $\frac{G}{U}$ is current rate of return on whole.

Save for later struggle. For now, heuristic method within easier reach.

Define $r \equiv$ internal rate of return.

$${}_nC_0(1+r)^n \stackrel{?}{=} L_n$$

$$L_n = {}_nC_0(1+i)^n + S(1+i)^n - S = ({}_nC_0 + S)(1+i)^n - S$$

$$r = \left[\frac{L_n + S}{{}_nC_0 + S}\right]^{\frac{1}{n}} - 1$$

(Note how i's S tends lessen r)

$$\epsilon \equiv \frac{\%\,\Delta r}{\%\,\Delta C}$$

Effect of Δi on value of $r = \frac{\sum R_n (1+i)^{l-n} i}{(1+i)^l - 1}$

4% & 8% [4% / 8%]

45-383

$R_0 = 0$

$R_n = 1$

$\frac{\sum R_n (1+i)^{l-n} i}{(1+i)^l - 1}$

l	$\frac{i}{1-(1+i)^{-l}}$	$(1+i)^{-l}$	$\frac{i}{(1+i)^l - 1}$	~~$R_0(1+i)^{l}$~~	$R_1(1+i)^{l-1}$	$R_2(1+i)^{l-2}$	$R_3(1+i)^{l-3}$	$R_4(1+i)^{l-4}$	$R_5(1+i)^{l-5}$	$\sum_0^l R_n(1+i)^{l-n}$	
1	1.04 / 1.08	.96 / .93	.998 / 1.004		1 / 1					1 / 1	1 / 1
2	.53 / .56	.92 / .86	.49 / .48		1.04 / 1.08	1 / 1				2.04 / 2.08	1 / 1
3	.36 / .38	.89 / .79	.32 / .30		1.08 / 1.17	1.04 / 1.08	1 / 1			3.12 / 3.25	1 / 1
4	.28 / .30	.85 / .73	.24 / .22		1.12 / 1.26	1.08 / 1.17	1.04 / 1.08	1 / 1		4.24 / 4.51	1 / 1
5	.22 / .25	.82 / .68	.18 / .17		1.17 / 1.36	1.12 / 1.26	1.08 / 1.17	1.04 / 1.08	1 / 1	5.41 / 5.87	1 / 1
10	.12 / .15	.									
20	.07 / .10										
50	.047 / .082										

Conclusion: When i doubles, annuity value of C_0 rises, and the % rise Δs with l. Ann. val. of $\sum R_n (1+i)^{l-n}$ depends on time distribn. If R_n is const., remains unchanged. If R_n rises, i.e., later years get more weight, annuity value falls as Δi. And v.v.

Most likely relation, R_n falls over time, and so ann. val. rises slightly w. Δi. But much less than ann. val. of C_0.

So regard ann. val. of R_n as unchanged? No, bec then ind. never reach l. R_n must diminish w. time. Can have a few early yrs of low or no R_n, however, so for a few yrs ann. eq. rises even as R_n are diminishing.

$$\text{l.q.} = \frac{\text{Non-land inputs}}{\text{Output}} \quad \text{or} \quad \frac{\text{Output} - \text{land input}}{\text{Output}} \quad \text{(if we have good measure of land input)}$$

Ways of measg. &/or defining intensity over time.

1. P.V. method

$$\text{l.q.} = \frac{\text{p.v. of non-land inputs}}{\text{p.v. of output}} = \frac{\sum C_n (1+i)^{-n} + a\,\frac{1-(1+i)^{-d}}{i} - \frac{da}{(1+i)^d}}{\sum R_n (1+i)^{-n}} \; ?$$

2. Cumulative or terminal value method (as of demolition date)

$$\text{l.q.} = \frac{\sum C_n (1+i)^{d-n} + a\,\frac{(1+i)^d - 1}{i} - da}{\sum R_n (1+i)^{d-n}} \quad \left(= \frac{\sum R_n (1+i)^{d-n} - da}{\sum R_n (1+i)^{d-n}} \right)$$

3. Annualized value method.

a. Annualize terminal values, annualizg factors cancel, same as 2 & 1.

b. Current method.

i) simple & free $\quad \text{l.q.} = \frac{Ui}{Ui + a} = \frac{\frac{dU}{dn} - a}{\frac{dU}{dn}}$

Off to a wrong start – doesn't count $\frac{C_0}{n}$ as a qlty. input.

$$U = C_0 (1+i)^n + a\,\frac{(1+i)^n - 1}{i}$$

$$\frac{dU}{dn} = iC_0 (1+i)^n + a(1+i)^n \cong i\,(C+s)(1+i)^n$$

ii) Sum of p.v.s method, for depreciz asset treated as sum of appreciz elements

$$\text{l.q.} = \frac{\sum [U] \times i = \sum \frac{dU}{dn} - a}{\sum \frac{dU}{dn} = \sum (U) \times i + a}$$

4. Land-time method, or "horizontal" matching

Take a given C_0, can combine w. varying ~~units of land time~~ no. of land-years, depending on durability

$$\text{l.q.} \cong \frac{C_0}{C_0 + da}$$

C_0 doesn't rep. capl. input, tho. but labor & imputed funds. C_i reps. capl. input, & greater durability is that, too, explg. upon compounding. So durability, or -Δ c.f., means subbing capl. & land for labor & funds. Reduced i.q. comes from spreading the labor & funds thin – not the cap. input. Seems to imply that cap. recovery factor is not the cap. input.

What is the "capl. input"? Ci, or C+Ci? $C_0 \frac{i}{1-(1+i)^{-m}} - \frac{C_0}{m}$

C reps labor, initially, largely, (disregarding offsite funds)

Δ C.f. is best regarded not as Δg cap. input, but ~~sub~~ Δg labor input (keeping const. land & capl. input) (probly requires some Δ land input also. — Δ c.f. is subst. cap. for labor

Can use various bases of reference, describe same thing in different ways.

Take a cycle of cap. as base, & Δ c.f. completes cycle w. less land-time, same labor.

Take fixed land & labor, Δ c.f. not possible. Emphasizing effect on factor propns.

Take labor as fixed, Δ c.f. combines it w. less land-time & less capl. Subs. labor for both capl. & land-time

So Δ c.f. is key to empl.

Δ C.f. of capl. doesn't Δ the capl. input. But Δs labor & offsite fund input, which must pass thru form of capl. Thus comprehends all other inputs except services.

Pile cap. up hi, in Times, i.e. Δ durability & given cap. complements less land space? No? Keep Q of C_o const. on a site, but -Δ c.f., & cap. complements more land-time. Conflict? No: you -Δ c.f. by reduc'g intensity of site use.

[Q. of cap. liqd. yrly – is not an input, but a bottleneck thru which labor & imported fund inputs must pass.]

When you use cap. to complement more land-time, it is more land-years per conversion, not per C

-Δ c.f. of fixed C_o on fixed site, ~~also~~ static i.g. of $\frac{C_o}{S_o + C_o}$ is not changed. Neither is $\frac{C_i}{i(S+C)}$, unless i has risen (which reduces S). But, to build in durability means sacrific'g current values – probly some space coverage. Reduces need for land. So, makes C_o ~~little~~ less comply to land; & much less comply to labor & foreign funds. C_o is more comply to land only in sense of driving away labor to use other land. C_o requires more land-yrs, but less land space. Doubles up, so to speak -- double C_o on same land, last longer.

Tree case shows that cap. input is not liqn., but U_i. Liqn. is disinvmt. of cap. values previously built up; not input, & not output either. Output is appreen + product of current inputs exerted on site this year.

Major remaining problem is allocating a currency ΣU.

Agenda

1. Effect of $\pm i$ on intensity

2. At least 2 kinds of intensity, net & gross. 2 kinds of net: net of imported funds; net of onsite cap. turnover (which means net of labor inputs largely)
   Also a 3rd kind, in physical units, unaffected by change in their prices. See 8

3. 2 assumptions re Δi:
   a. Overall
   b. One individ vs. others at old i

4. Demonstrate conclusions several ways
   a. annual costs, differential effects
   b. Present values
   c. Indiff. curves
   d. Horizontal life span. Cf. rates of interest per life span. e.g. $\frac{1}{2}$% per mo.
   e. Earlier arrival of fund. maturity, detd by 1) max. annualized rent.
      2) Mgnl. = n of cost & revs. of time.

5. Demonstrate for:
   a. appreig
   b. Depreig
      1. conventional
      2. New accty system
   c. annual basis, for either kind

6. Spell out role of land in determining turnover.
   Tempers effect of Δi
   Augments i
   If there is no rent (bonds, trees on mgnl land) does Δi cause shift to shorter maturities?

7. Note double effect of Δi on l.v.: lower $\underline{a}$; & higher capn. diviser.

8. Intensity in physical terms. Or does that mean anything over time?

What made Burns on Zoby start. seem so unjust?

HAMMERMILL BOND MADE IN U.S.A.

10-29-62

Effect of Δi on factor purchases, or i.q.s.

1. P.v. approach.

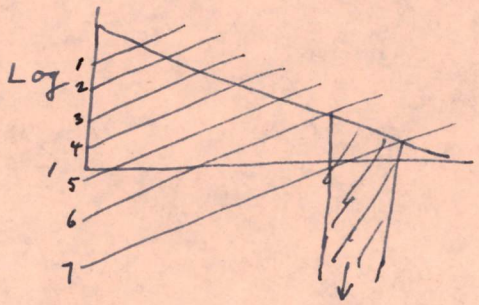
a. As between land & bldgs etc.: land on whole more sensitive to $\underline{i}$.

Refer to sheet.

b. But bldgs may be modified. Yrs. of life are divisible. Chop off later years. After c. 20 yrs, become more i-elastic than l.v.

1. Give e.g. from tables, using simple p.v.

~~2. But must~~ Show valuation concept, 1st approx.

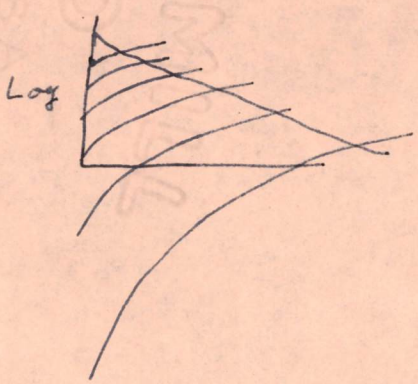


Add to C_0 the ΔC_0 from 6 to 7, add shaded column of revs. Cf. MC vs. p.v. of MR_n.

2. Leverage of Δi even greater when use proper method:

$$\text{True p.v.} = \frac{R_n - \frac{a}{m}\frac{(1+i)^{m-n}-1}{i}}{(1+i)^{m-n}}$$

$${}_nV_0 = \frac{R_n - \frac{a}{m}\frac{(1+i)^{n}-1}{i}}{(1+i)^{n}} = \frac{R_n}{(1+i)^n} - \frac{a}{m}\frac{1-(1+i)^{-n}}{i}$$



Let $R_n = 100$, $\frac{a}{m} = 2$, $n = 20$

at 4%, $V_0 = 46 - 2 \times 14 = 18$

at 8%, $V_0 = 21 - 2 \times 10 = 1$

c. So $\Delta i \rightarrow$ less land, less durable cap.

Not much less short cap. of hi c.f.

Labor input probly fixed, so to keep busy, probly turn cap. even faster, by inventy

So gross i.q. goes way up; & net i.q. some

In fact, back up a moment & maintain output w/o imports, just by Δ c.f. onsite. Implies more labor on site. -- how else create all that capl. that is consumed yrly, as higher output. Sub. labor for cap. — w/o much reducing capl.!
Δ i.q.

Might keep static i.q. const., i.e. $\frac{cap}{land + cap}$. But higher c.f. affects output. Distrib. share of cap. input re land needs + Δ much, follows static i.q. Labor share follows dynamic i.q.

2. Annual cost approach

$MR = MC = Vi \pm \Delta V + V\tau$

So $\frac{MR \pm \Delta V}{V} = i + \tau$; $\frac{MR}{V} = i \pm \frac{\Delta V}{V} + \tau$

Note how $\Delta\tau$ is answer to Castro.

2. 2 concepts of factor propns. dynamic & static.

1, above, is dynamic.

Usual instinctive thought is ~~dynamic~~ static: $\frac{\text{Value of cap.}}{\text{Value of land}}$

$$V_0 = \frac{c\,\frac{1-(1+i)^{-n}}{i}}{a/i} = \frac{c}{a}\left[1-(1+i)^{-n}\right]$$

Annual cap. cost

$$c = C_0 \frac{i}{1-(1+i)^{-n}}$$

When n is large, value of $c \propto i$

" " " small, " " " not $\propto i$

c is annual cost of holding asset. Ergo, when i rises, cost of holding durables is more affected.

Effect of i on factor proportions

1. What are factor propns? Ratios of inputs. I.q. = $\frac{\text{non-land inputs}}{\text{output}}$ Barley v. citrus.

Land input = $a_1 = S_e i$

Cap. input = $c = C_o \frac{i}{1-(1+i)^{-n}}$

2 concepts of i.q., at least.
[also a value-added i.q.]

This constitutes return of capl. + return of growth in value of cap. that justifies our having tied it up:

Think of a house as 2 houses, etc.

$$\left[\frac{c}{a} = \frac{C_o}{S_e}\frac{i}{1-(1+i)^{-n}} \qquad \text{Usable?}\right]$$

Effect of higher i.

a) To one individual.

His land cost Δs in prop. to i, bcs $a = S_e i$

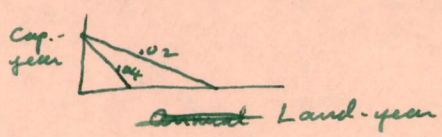
His cap. costs Δ in less prop: $c = C_o \frac{i}{1-(1+i)^{-n}}$

The less durable, nimbler the cap., the less the Δ costs. E.g. if $n = 1$, $c = C_o \frac{(1+i)i}{i} = C_o + C_o i$. Etc. If $n \to 0$, $c \to \infty$

So w. higher i, become more cap. intensive, per unit of land. By fast turnover. Hash house is very cap.-intensive. Skyscraper may not be.

Lo-i will have more standg cap., but less yrly. input. Above all, he gets more land.

Indiff. curves & price lines. ----



b.) To society.

Hi i makes all speed up, Δs use of cap. supply. Makes it more comply. to labor -- frequent conversion -- & less comply to land, bcs standing still requires a site.

Intensity of land use.

2 concepts

$I.q. \equiv \frac{\text{non-land inputs}}{\text{output}}$ Dynamic. Non-land input $= c = C_0 \frac{i}{1-(1+i)^{-n}}$

Static. $\frac{\text{cap.}}{\text{l.v.+cap.}} \cong \frac{\text{int.}}{\text{rent+int.}}$ (a value-added i.q.)

They differ bcs of turnover.

i.r. & intensity.

Δ i.r. tends Δ dynamic intensity, make cap. turn faster.

Effect on static i.q. less clear. May reduce a static intensity, by making durability unec. Can treat yrs of bldg life indiv. directly, cut off some or all yrs. last yrs., leverage of $\underline{i}$ may be greater. Bring tables.

Gen. ass. $-\Delta$ i.r. $\rightarrow \Delta$ intensity. But:

re dynamic intensity, opposite.

re static, turn up. Buy more land. & Δ durability of cap. im.

In perfect mkt, all drive toward same dynamic intensity.

Next time

Ec. 117. 10-19-62

1. Why max. C_o? Same as max'g i or S_e. Max'g i, doesn't mean we want to max. what we pay for borrow'g, but max. return to us that corresponds to external i. Max'g S_e means internal S_e ... not what we pay, but what we impute.

So max'g C_o means max'g value to us, internal, that corresponds to C_o; that justifies our outlay of C_o.

Wd. be justified if C_o was limited, cd'n't be expanded, & others had mkt. prices.

2. What does p.v. mean, here? Means p.v. of non-land inputs, or net of land inputs — just as site value is net of non-land inputs.

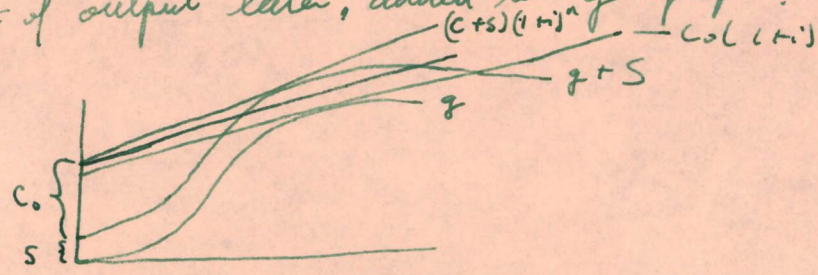
$$V_o = \frac{g - S[(1+i)^n - 1]}{(1+i)^n} \qquad \text{cf. } S = \frac{g - C_o(1+i)^n}{(1+i)^n - 1}$$

3. V_o fn. has Δ & interest as growth in mkt value of stumpage, not for liquidation but for holding to maturity

4. Do cost changes affect solution? Yes, by effect on S_e next time. C_o in current cycle has no effect, bcs becomes a sunk cost immedly. But future C_o shd. have some effect.

5. Can site costs be added to C_o? On graph? Yes, if S is considered part of output later, added to g-graph.

for Takis



In obj. effects on site vs. effects on U, annual cost, have 2 choices.

1. assume Si lowers $\underline{a}$, internal. Then get double effect on site: higher cost, Si; lower aply MR, $\underline{a}$.

2. Assume const. $\underline{a}$, impute no loss to $\underline{a}$. Then M yearly cost (M4C) of carrying U is Ui + a, Si is diluted.

1s diluted even more if a falls, bec i rises. 9s not diluted if use Si, & S const. 3 choices: probly middle one right.

Annual cost of holding timber is $[i\,U - a]$

$$\frac{dU}{dt} - a$$

$$U = C_0(1+i)^n + a\frac{(1+i)^n - 1}{i} = (C+S)(1+i)^n - S$$

$$\frac{dU}{dn} = C_0 p(1+i)^n + \frac{a}{i}p(1+i)^n = (1+i)^n(C_0 p + a) = p(C_0+S)(1+i)^n = Ui + a$$

But interest on U, (Ui), is less than $\frac{dU}{dn}$: $Ui = i[(C+S)(1+i)^n - S]$

$$= \frac{dU}{dn} - a$$

or, $Ui + a = U'$

$MR = U' \cong Ui + a$

$MC = i(S+U) = Ui + a$

But $M_A^N R$ of trees alone $= U' - a$

& MC " " " $= Ui$

Eq. is where $Ui = U' - a$

or

$Ui + a = U'$

We could look at MC of holding timber as $Ui + a$. When i rises, not only is it diluted by $\underline{a}$, but more: $\underline{a}$ falls, so cost of holding timber handily $\downarrow$s w. i.

$S = \frac{x}{(1+i)^n - 1}$, very int.-elastic.

Which is more i-elastic, simple present value, or true p.v. net of rent?

$$u_n = C(1+i)^n + a\frac{(1+i)^n - 1}{i} = \frac{g_m}{(1+i)^{m-n}} - a\frac{(1+i)^{m-n} - 1}{i}$$

$$= \frac{g_m + \frac{a}{i}}{(1+i)^{m-n}} - \frac{a}{i}$$

True p.v. is much more i-elastic, assuming a is const. as Δi.

So effect of Δi on durability depends partly on cost of acc. rent.

Several steps needed to find temporal margins of MC pricing.

Esply. the beginning. Cut-off not to hard – as soon as SMC > LMC

Inception is tougher? Why not begin as soon as TB > TC, s.r. ... ~~and not~~

Be sure to exclude franchise cost, unless land has to be taken from other uses.

Be sure to consider sq cost alternative

Then turn to p. 94, & thumb thru for discussion.

Ec. 117 11-28-62

1. Gen'l. thy. of MC pricing.
   can't separate distributional from allocational qus.
2. Extreme case
3. But the extreme case characterizes end point of every system
4. SR vs. LR cost curves.
5. LR cost curves presuppose knowledge.

Ec. 117. 11-21-62

2 last topics on i.r.: Effect of antic ΔS on bldg.; & other land uses. .. taxes on alloc.

1. $mec = \left[\frac{R_m + S}{C_o + S}\right]^{\frac{1}{m}} - 1$ or $= \left[\frac{\Sigma[R_n(1+r)^{m-n}] + S}{\Sigma[C_n(1+r)^{m-n}] + S}\right]^{\frac{1}{m}} - 1$

Error

2d form above requires trial & error solution.

Surely, higher S means lower mec, cet. par. I.e., given R_m & C_o. But if can ΔR_m by ΔC_o, can get back.

If higher S is overpricg, means lower mec. If reflects higher potentiality, can be compensated by more intensity.

✓ Now consider spec. bldr. Higher S forces more intensity on him. Maintain proportion of bldg. value to site.

Now consider dynamics. Expect ΔS. $S_m > S_o$. Raises mec. to bldr. Encourages bldg. (My article, to contrary, referred to when no buyg & selling.) Bldr. cd. reap ΔS w/o bldg, but can't afford that, requires both, $S_m - S_o$ and $R_m - C_o$.

✓ Are also those to whom ΔS is enough. So rising mkt. encourage both bldrs & "sitters," hence urban sprawl.

1. Hi-i & higher intensity. Reprise

Big cause is more frequent applns. of C_0; 2dry cause is hi-i redistributes from a to i.

C_0 reps. largely labor & imported funds. Funds we cancel out, because for int. on them, in figuring v.a. or net i.q., so C_0 largely labor. assume $\pi = 0$.

The longer to m, the more cap. intensive. $\frac{C(1+i)^m - C}{} = (1+i)^m - 1$ is $\frac{\text{cap.}}{\text{labor}}$

Note that "discounting" ≡ subtracting capl. input: $V_m - V_0 \overset{C}{\sum}[(1+i)^m - 1] = V_0$

Higher i seems to make more cap. heavy, but shortens life, probably more than compensates.

Promised to do better Weed.! [Note, qualy. cap.-intensity depends on m. Cf. $m = 1$ & $m = 100$. That's horizontally. Vertically, means qualy. hi ratio cap/labor. $\frac{\text{flow}}{\text{fund}}$, or $\frac{\text{labor}}{\text{stock}}$, low. if $m = 100$.]

as to land, tho, hi-i means both lower a & shorter life

Land input, per cycle, $\frac{m}{a}$, falls doubly; labor per decade Δs.

Thus w. same capl. & land, employ more labor, thru higher i.

Promising ave. toward full emplmt.

Note C_0 not capl. input, but labor. Capl. is C_0 over years.

✓ 2. Summarize intensity, algebraically & graphically, fairly qual. expression.

where $\pi_n = C_n - L_n$

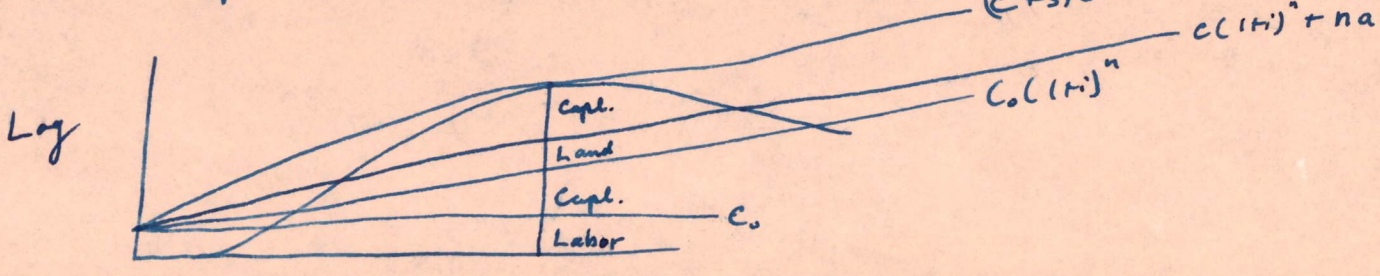
$$\text{i.q.} = \frac{C_0(1+i)^m + S[(1+i)^m - 1 - it] - \pi_0}{g \text{ or } R_m \text{ ~~or } R_n(1+i)^{m-n}} - \pi_0$$

More qualy, $\dfrac{\sum\left[C_n(1+i)^{m-n}\right] + S[(1+i)^m - 1 - it] - \sum \pi_n}{\sum\left[R_n(1+i)^{m-n}\right] - \sum \pi_n}$

Labor $= C_0 - \pi_0$ or $\sum[(C_n - \pi_n)(1+i)^{m-n}]$

Land $= Sit = at$

Capl. $= C_0(1+i)^t - C_0 + S[(1+i)^m - 1 - it]$



Labor loses out as cycles lengthen, & v.v.
as timber cycle lengthens, cap ÷ land Δs
as bldg. " " " " -Δs. -- bec. it is a layer-cake of several appreig strata.

3. Effects of hi i.

a. On allocn. among individs.

i.) Why imptnt.? Capl. mkts imperfect. Spend a few minutes on that.

Surely, hi-i avoids long m.

Land, contracts holdg — unless he can sell fwrd, turn-over stock.

P.v. approach halves it, or more if cappreig. or if deferred income, e.g. $\frac{a}{(1+i)^m - 1}$

& sells, $S_0 = a\frac{1-(1+i)^{-m}}{i} + \frac{S_m}{(1+i)^m}$, can't escape sensitivity to i, but less than halves. Even if he holds it a while

Surely cost, doubles it, or more if appreig. $MC = S_i - \Delta S$.

Even if he will sell land, has to wait to m, surely cost = $S_0 \frac{i}{1-(1+i)^{-m}} - \Delta S$

7 So land is as affected as the mth house by Δi ... & moreso if Δs

---

3. Effects of hi - i

a. On allocn. among indivds.

Imptnt. bec. of imperfect cap. mkts. Brodez. Investor's risk v. lender's risk
Collateral requirements. Costs of banking.

Mtgg. in banking. Differentials Δ w. length of loan. Differentials relatively greater when spectrum of rates moves down. Diffs. greater in recession — & cumulate.

Hi - i avoids long m: land, & durable components or "long strata" of capl.

b. P.v. approach.

i. We've already used this, re ε of p.v. to Δi for diff. m.

ii. Land.

Simple capn., inverse proportionality, $S = \frac{a}{i}$

Unless contemplate resale; $S_0 = a\frac{1-(1+i)^{-m}}{i} + \frac{S_m}{(1+i)^m}$

Reduces i-elasticity of land below that of ${}_0B_m$, where ${}_mB_0$ is the mth bldg., & ${}_0B_m$ is its p.v.

(In fact, moreso, p.v. of B_m is more i-sensitive than simple discount when we also subtract site, so ${}_0B_m = \frac{B_m - S[(1+i)^m - 1]}{(1+i)^m}$ ....)

(But if Δi is qual & lessens S, reduces i-sensy. of ${}_0B_m$)

$\left[a\frac{1-(1+i)^{-m}}{i} + \frac{Sm}{(1+i)^m}\right]$

But, qualy., if m > ca. 20 yrs, ∧ isn't much less i-elast. than $\frac{a}{i}$, except at lo $\underline{i}$, but those w. lo-i aren't going to be doing this, this is an expedient for those w. hi-i trying to hold land. It won't help them much.

opposite situ. more likely
∧ Land w. deferred income, more sensitive to i.

If ~~pds~~. periodic, $S = \frac{Sm}{(1+i)^m - 1}$ This is more common than steady income

If increasing arithmetically, $S \doteq \frac{a}{i} + \left[\frac{\Delta a}{i^2}\right]$

If " geometrically, $S = \frac{a}{i-r}$

iii. Effects of Δi on ∧ p.v. of capl. ~~improvement~~ ~~values~~

Take depreig capl. as a whole, less i-elast. Cite sheet passed out.
Divide into ~~Take~~ individ. ∧ strata, i-elast. rises w. life-span, or length of strata.

$${}_oB_m = \frac{B_m - S[(1+i)^m - 1]}{(1+i)^m}$$

(or, using Warren Hall-inspired technique, ${}_oB_m = \frac{B_m - a_m}{(1+i)^m}$)

Strata longer than 15-20 yrs, or say m > 15 or 20, more i-elast. than land.

But effect of Δi tempered by −Δa corresponding.)

iv. Net result

Hi-i ~~eats~~ uses less land per $ of capl., on whole. Even tho long strata are more affected, entire package is less affected. The more ben capl. can be altered, shortened

Using ~~much~~ more cap/land; and turning cap. faster; means much more labor/land

Paradox: hi-i Δs cap. intensity. But cf. firms by size, any indy, consistent pattern, larger ones have higher assets ÷ output.

~~Land~~

Re emplmt, small share of our land & capl. are employing large share of people.

Omitted this section, Fall, 1962.

c. Yrly cost approach (& yrly revenue)

i. Land

Yrly cost = Si

[Unless contemplate resale, recover invt. as well as return on it, resembles capl. — ~~but~~ ~~as~~ $s = S \frac{i}{1-(1+i)^{-m}}$. But then must also adjust rev. side. treat S_m as rev, annual rev. = $a + \frac{S_m i}{(1+i)^m - 1}$, fall of revs, partly compensates for reduced rise of costs. w. Δi]

ii. Capl.

Yrly cost = $C_o \frac{i}{1-(1+i)^{-m}}$. (This isn't the capl. input, but cap. plus labor etc. in Co. Correct earlier misstatement.)

Less sensitive to i. ~~But~~ why How reconcile w. p.v. approach, where long streets are more sensitive to i.r. than l.v.? Bcs. Δi lowers yrly. or annualized value, a.v., of R_m when m large, thus making less attractive "c" is less sensitive Bcs. c includes deprecn. as well as int. ~~c~~ ? in an individ.

year, cost = $Vi + |\Delta V|$, and c reps. an ave. of that over m.

iii. Complications ... bring in effects on rev. side

1. apprecg. land

Yrly cost = $Si - \Delta S$. Leverage. Eqn. where $\frac{MR}{S} = i - \frac{\Delta S}{S}$

MR may = 0. Note how t wd. swamp out i ... is what Cuba etc. need.

2. Deferred income land

Yrly cost = Si

Value of revenue to individ. = $a = \frac{Si}{(1+i)^m - 1}$ goes down ....

double effect

3. Ease land cost by contemplating resale

Treat S_m as part of product, rather than something you keep forever

Then cost, $s = S_0 \frac{i}{1-(1+i)^{-m}}$

That is as i-elast. as yrly cost of B_m, & more elast. than yrly cost of rest of bldgs., B_0 to B_{m-1}

But now must add S_m as rev. As rev., $s_r = S_m \frac{i}{(1+i)^m - 1} = S_0 (1+i)^{-m}$

Higher i is yrly cost, reduces yrly. rev.

But is also a yearly rev., $\underline{a}$, so $\sum$ revs. is $a + s_r$, and $\sum$ cost is $= s$.

$s = s_r + a$. Result is less i-sens. than ${}_0B_m$; but more i-sens. than early B_n; & than $\sum B_n$.

iv. Net result

as before,

a) More cap ÷ land, bcs Si is more i-elast. than $C \frac{i}{1-(1+i)^{-m}}$

b) Higher c.f. of cap., bcs. shorter $\underline{m}$ reduces sensitivity to i of both c and a.v. of R_m

c) Only exception to (a), cut back on long strata of cap. more than on land. Possible bcs. capl. is divisible in time, land not.

Can select the kind of capl. least affected by i

v. Cf. vs. "Unwieldy....." conclusion. Present anal. more advanced - that was capitalized, & failed to emphasize divisibility of capl.

c. Effect of gmrl. hi $\underline{i}$ on intensity of land use

i. Reduce differentials

ii. " land ~~use~~ constraint on waste of time. accel. hist. of appreig. but slow demolition. Cause migration.

iii. Etc.