



The n% Rule of Thumb

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The n% Rule of Thumb

We saw in the paper on Elasticity of Production that an exponent is an elasticity. The compound interest factor, $(1+i)^n$, is an exponential function of n , where n is number of years. The base of the function is, of course, $1+i$, where i is the rate of growth. The same reasoning as that followed in the paper on Elasticity of Production also tells us that if you raise the base by 1%, you will raise the function by a factor of $[n \times .01]$. Let's try that with numbers and your calculator.

$$1.01^{10} = 1.1046$$

$$1.02^{10} = 1.2190$$

$$1.2190/1.1046 - 1 = .1036$$

You notice, of course, that our answer is not precisely .1000, or 10%. The reason for the slight inaccuracy is that a rise of 1% in the base, small though it may seem, is a finite rise.¹ Recall that we derived this rule from calculus. That means it is precisely true only in the limit where the change in the base approaches zero.

Nevertheless, it makes a useful rule of thumb for roughly gauging the effect of changes in i on F (where $F = (1+i)^n$). Call it the "n% Rule of Thumb." Raise i by one percentage point, and it raises F by a factor of $(n \times .01)$. This holds whether you raise i from .01 to .02, or raise it from .20 to .21. In either case you still raise F by about n%.

Use your calculator and check that out. Notice that the Rule slightly understates F'/F , and the degree of understatement rises with n . The reason becomes clear if we calculate F'/F directly. Take the numerical example above:

$$F'/F = 1.02^n/1.01^n - 1 = [1.02/1.01]^n - 1 \approx 1.01^n - 1$$

¹The reason is not that we raised the base by slightly less than 1%, because that would tend to make our result less than 10%, not more.

But $[1.01^n - 1]$ is slightly greater than $[n \times i]$; and the disparity grows with $\underline{n}$. That is, the fraction

$$\frac{1.01^n - 1}{ni}$$

is an increasing function of $\underline{n}$ - slowly increasing, to be sure, but still increasing. I leave it to you to check that out on your calculator. It's our old friend, the "Com-Simp Ratio" (see "Basic Math," pp. 17-18).

Another inaccuracy in the Rule is, of course, in the approximation that $1.02/1.01 \approx 1.01$. It is actually a little less than 1.01. This error tends to overstate F'/F , and the overstatement rises with $\underline{i}$. Check this, too, out on your calculator. The second error tempers the first. It never offsets it completely, but it's a lot better than if the two errors reenforced each other.

As for the effect of changes in $\underline{n}$, they are self-evident when you think about it. Every time you raise $\underline{n}$ by 1, you raise $\underline{F}$ by $(100 \times i)\%$. For example, $1.15^{11} = 1.15^{10} \times 1.15$, which is 15% more than 1.15^{10} .

6. Sensitivity analysis of compounding and discounting

~~a. Using semi-log scale

~~~ i) Comparing  $1.072n$  and  $1.036n$ 

~ The vertical difference of  $1.072n$  and  $1.036n$  waxes wider as  $n$  increases. On the semi-log scale, this means that the percentage difference rises.

~ If the ordinate on Figure 3 were a common scale the lower line would be always just half the upper one in numerical value, because geometrically it is always just half the height. But on the semi-log scale, where the ordinate scale is logarithmic, the lower line changes from being much more than half when  $n$  is small to much less than half when  $n$  is large. It is exactly half when  $n$  is about 20, and the ratio is  $4/2$ . Table 1, "Value of  $1.036n$  as a fraction of  $1.072n$ ", gives the numbers.

Table 1: Value of  $1.036n$  as a fraction of  $1.072n$ 

| $n$ | $1.072n$ | $1.036n$ | $1.036n/1.072n$ |
|-----|----------|----------|-----------------|
| 10  | 2.       | 1.42     | .71             |
| 20  | 4.       | 2.00     | .50             |
| 30  | 8.       | 2.89     | .36             |
| 40  | 16.      | 4.00     | .25             |
| 50  | 32.      | 5.86     | .18             |
| 60  | 64.      | 8.00     | .12             |
| 70  | 128.     | 11.90    | .09             |

## ~~~ ii) Line of constant fractional value

~ A line whose value is any constant fraction of the top line is parallel to it. A line plotting  $1/2$  of the top line in Figure 3 begins from  $1/2$  when  $n=1$ , and intersects the  $1.036n$  line at  $n=20$ . Draw this on Figure 3 yourself. Don't fear to "cross the equator" and go below the abscissa -- it's only a number.

~ (Optional. Draw a set of rays from the origin to this 50% line, at various values of  $n$ . The slopes of these rays are the rates of return that would be earned on an investment whose payoff was always  $1/2$  of  $1.072n$ .)

## ~~~ iii) Discounting and present value

~ On the semi-log scale, the reciprocal of a function is the mirror image of the function below the abscissa. Thus discount curves are the mirror image of the corresponding compounding curves. And all the rules above, from (5,a) forwards, apply to discount curves as well, mutatis mutandis. ("Mutatis mutandis" is a handy Latin phrase which means "when you have made the appropriate and obvious adaptations, so don't come at me with any dumb quibbles." Those old Romans sure packed a lot into a few words.)

~ As to sensitivity we are now ready for a big payoff. It follows that the PV of any FV is the more sensitive to  $i$  the farther in the future it is. More tersely, the sensitivity of  $P$  to  $i$  increases with  $n$ .

~ Draw the mirror images of the curves on Figure 3, and work out a table corresponding to Table 1. If your credit is shaky and you have to pay a premium above the prime to borrow, what does your table tell you about what kind of assets you cannot afford to buy?

~ Look out for one of the "mutatis" which you have to make when you go below the abscissa and look at discounting. A given vertical distance is always the same percentage change we have said, but that means percentage of the value at the bottom of the vertical reach, not the top. Going from 1 down to  $1/2$  is 100% of  $1/2$ , but only 50% of one. There cannot be a 100% or more drop from any value on the log scale, because it never gets down to zero.

~ Don't despair, there are two solutions. One, whenever possible express any % change as an increase from the lower value. Two, if you have to go from the top value downwards, use the rule that when a denominator rises by  $x\%$  the quotient falls by  $x/(1+x)\%$ .

~~ b. Sensitivity, using calculus

~ We have seen above (5,h) that a change in  $\ln y$  equals the fractional change in  $y$  itself. In calculus that is called the "log derivative".

$$\frac{d \ln y}{dx} = \frac{dy}{dx} \cdot \frac{1}{y} \quad (3)$$

~ It is reassuring that the calculus rule comports with what we infer from the semi-log scale.

~ Our "y" is  $I_n$ . The shortest way to the log derivative is first to find the log.

$$\ln I_n = n \ln I \quad (4)$$

$$\text{But } \ln I = i, \text{ so } \ln I_n = ni \quad (5)$$

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(Optional note. For those who care, and remember how the Napierian constant  $e$  is derived, here is why  $\ln I = i$ .)

$$(1+i)^n = e^{ni}$$

$$1+i = e^i$$

$$i = \ln(1+i) = \ln I$$


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$$d \ln I_n = d(ni) = n \, di \quad (6)$$

~ (6) says that when  $i$  rises by a percentage point (e.g. from .07 to .08) then  $I_n$  rises by about  $n\%$ . Table 2, "Sensitivity of  $I_n$  to  $i$ ", shows this for  $n = 20$ .

Table 2: Sensitivity of  $I_n$  to  $i$ , when  $n = 20$

| $i$ | $(1+i)^{20}$ | % rise |
|-----|--------------|--------|
| .00 | 1.000        | ----   |
| .01 | 1.220        | 22.0   |
| .02 | 1.486        | 21.8   |
| .03 | 1.806        | 21.5   |
| .04 | 2.191        | 21.3   |
| .05 | 2.653        | 21.1   |
| .06 | 3.207        | 20.9   |
| .07 | 3.870        | 20.7   |
| .08 | 4.661        | 20.4   |
| .09 | 5.604        | 20.2   |
| .10 | 6.727        | 20.0   |
| .11 | 8.062        | 19.9   |
| .12 | 9.646        | 19.7   |
| .13 | 11.523       | 19.5   |

~ Column 2 looks like a chaotic jumble of numbers, but Column 3 finds order in chaos. The rise is always about 20%. (You may ignore the minor drift from 22% down to 19.5% -- it has a technical cause which need not concern us here.) It gives an accurate, memorable rule of thumb you can always retain and use. The rule is in the paragraph above the table. To repeat, a rise of one point in  $i$  gives a rise of  $n\%$  in  $In$ . Memorize that.

~ We need to label that rule so we can retrieve it from our memory banks at will. Let's christen it "The  $n\%$  rule of growth".

~ From the viewpoint of the aggressive and/or acquisitive investor, let's apply that rule to compounding. "The  $n\%$  rule of growth" says that you can increase your future wealth by  $n\%$  for every rise of one point in your rate of return on investment. Raising the rate from 20% to 21% is just as effective as raising it from 0% to 1%: there are no diminishing returns to this effect. And it is so simple to remember. Our travail has led to a simple, useful end.

~ Applied to discounting and valuation, The  $n\%$  rule of growth says that every drop in the discount rate of one % point raises the PV of 5-year futures by 5%; of 30-year futures by 30%; of 100-year futures by 100%; and so on. The drift away from exactitude which we observe in Table 2 means that for abnormally high values of  $n$  and  $i$  there is less precision than Einstein would want, but still enough for most business purposes.

~ Do be cautious however not to apply the rule to drops in PV caused by rises in  $i$ . Thus when  $n=100$  a rise of  $i$  by one point does not cause a drop in PV of 100%. Rather it doubles the compounding factor which is the divisor, and doubling a divisor cuts a quotient only by 50%. Watch that. Sorry that life never goes on being as simple as the simplest rules we can devise, but that's the way it is and you had best learn to handle it that way.

#### ~~ c. Sensitivity to $i$

~Equation (6) is symmetrical between  $n$  and  $i$ . (6) says the sensitivity of  $In$  to  $i$  is  $n$ . The symmetry of it tells us therefore that the sensitivity of  $In$  to  $n$  is  $i$ . Actually we already knew that, that is what compound interest is all about. It just says that when  $n$  grows by one (i.e. another year passes) our money grows by  $100i\%$ .