

Ec. 100B, Prof. Rafferty

P-1

Class notes. Wed. Jan. 28, 1981. ^{Installment}
~~A. #4~~ #5

Payback time, (concluded).

omissions, and clarifications,
1. Errors in notes of Jan. 21.

a. An omission. Put "Installment #4" on heading.

b. An error, p. 3, Eq. (2), should read:

$$K = \sum_{n=0}^{12} (1+i)^n = \text{etc.} \quad (2)$$

This is because the first term of the progression is 1, rather than $(1+i)$. Remember the formula from which we derived the result:

$$S = 1 + r + r^2 + \dots + r^{(n-1)}$$

We have to watch the end points of the progression, to avoid pesky minor errors.

The object of beginning with 1 is to arrive at the simplest possible final form. If we have to or want to begin with $(1+i)$, however, we adjust by multiplying (2) by $(1+i)$. Or, we can add $[(1+i)^{13} - 1]$, which gives the same result.

Payback Time (concluded), p. 2

This is mainly of interest to purists & technicians, and changes nothing of importance.

c. Likewise, Eq. (3) should read:

$$K = \sum_{i=0}^{n-1} (1+i)^n = \text{etc} \quad (3)$$

We are now valuing capital at the beginning (not the end) of each week. For cohort #1, the value is 1, not $(1+i)$; etc.

d. Clarification of Eqn. (4)

When we divide K by V , the $V = (1+i)^n$ (and not $V = (1+i)^{n-1}$). I mentioned this in class, but some may have missed it.

e. On p. 7, Eq. (9), I use " S " (S ales) as the divisor. This is the same as " V " (Volume) used in (4) and (7). For consistency, change S to V in (9).

2. A second approach to showing that Payback Period = Capital $\div$ Volume ($LP = K/V$)

On Jan. 21 we used a "going concern" model to show this. You might ask, what if we don't have a going concern? So let's look at just one cohort of brokers going through its cycle from investment to liquidation.

To avoid imposing a lot of detail on you, we'll just do case 1, as on p. 3 of Installment #4. This is called "P(PO)" (Point-Input, Point-Output). The name refers only to inputs other than capital, which are concentrated at one point of time, when $n=0$. We have seen that there are also regular doses of capital inputs each year, as unpaid interest is plowed back. We'll keep the name "P(PO)" to remind us of how easy it would be to make the mistake of thinking the payback period is $\frac{n}{i}$, where really it is on average less than $\frac{n}{i}$.

a picture will help:

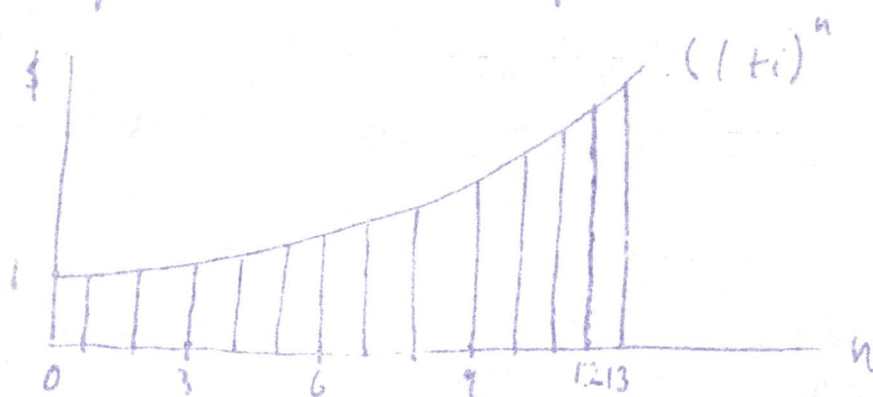


Fig. 1

Eq. (2) is the sum of the columns under the curve. In Eq. (4) we divided this sum by the last column (the right side of the last column,

to be exact, or $(1+i)^{13}$.) We said this ratio of K/V was the average payback period.

The sum of the columns under the curve is the area under the curve. This area can also be calculated as the sum of horizontal bars under the curve, where each bar is one dose of input times its respective payback period, or $k_i n_i$. (the subscript "i" here is not the interest rate; it means the individual unit.)

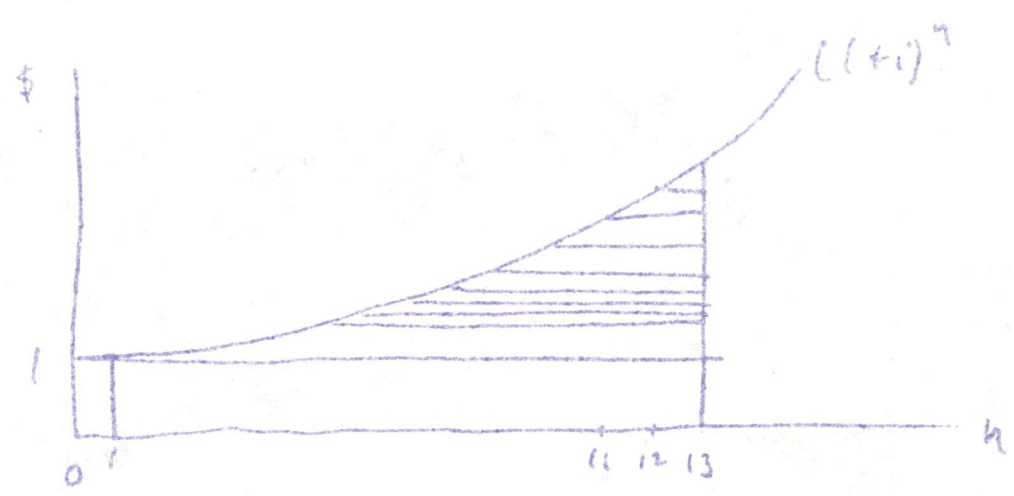


Fig. 2

I have left out the first few bars — they are too close together to draw, except for the very bottom one. [The little triangle in the top right is excluded, because our summation formula values capital only at the beginning of each year. In integral calculus they correct for this minor inaccuracy. Here, we

p. 5

accept the ~~approx~~ approximation based on columns with squared-off tops -- it is plenty accurate for most purposes.]

Now observe that the area under the curve is the sum of the horizontal bars, or $\sum_{i=0}^{12} k_i n_i$, where i is the date of each dose of

capital, and n_i is its individual payback time. We already know the area under the curve, so we have:

$$\sum_{i=0}^{12} (k_i n_i) = \frac{(1+i)^{13} - 1}{i} \quad (10)$$

We can also see that $\sum_0^{12} k_i$ is the height of the last column, which we know is $(1+i)^{13}$.

$$\sum_0^{12} k_i = (1+i)^{13} \quad (11)$$

(10) is the sum of payback periods weighted by the amount of capital with that period.
(11) is the sum of all capital. Dividing

(10) by (11)^(which is the sum of the weights) therefore gives the average payback period of all the capital:

$$\frac{(10)}{(11)} = \frac{\sum_0^{12} (k_i u_i)}{\sum_0^{12} k_i} = \frac{(1+i)^{13} - 1}{i(1+i)^{13}} = \frac{1 - (1+i)^{-13}}{i}$$

(12)

But (12) is the same as (4), Q.E.D.
The average payback period equals the capital/output ratio, or K/V .

3. Shorter periods are more labor-intensive.

In Eq. (1), p. 3, Installment #4; and Fig. 1, p. 3, Installment #5, we see that output is $(1+i)^{13}$, or generally $(1+i)^n$.
In the PPO case, labor is applied once, when $n=0$, and is equal to \$1 in value. Labor's share of output is therefore:

$$\frac{\text{Wage Bill}}{\text{Output}} = \frac{1}{(1+i)^n} = (1+i)^{-n} \quad (13)$$

Capital's share is:

$$\frac{(1+i)^n - 1}{(1+i)^n} = 1 - (1+i)^{-n} \quad (14)$$

In a going concern model, the result is the same. Capital's share is:

$$\frac{Ki}{V} = \frac{(1+i)^n - 1}{i} \times \frac{i}{(1+i)^n} = 1 - (1+i)^{-n} \quad (15)$$

Note that capital's share increases with n ; labor's share falls ~~with~~ as n rises. This is not because the rate of wages or interest change: they don't. It is because capital adds a higher share of the value of output as n rises.

We can now perceive the power of

short payback periods to employ idle labor in two ways:

a) Shorter paybacks means more payoffs are advanced by the same capital;

b) Shorter paybacks means production is more labor-intensive.

4. Effect of higher interest rates on choice of payback periods.

The interest rate is the economic constraint or discipline that makes us economize on capital when it is scarce. The following diagrams illustrate how this works. We will discuss them in class.

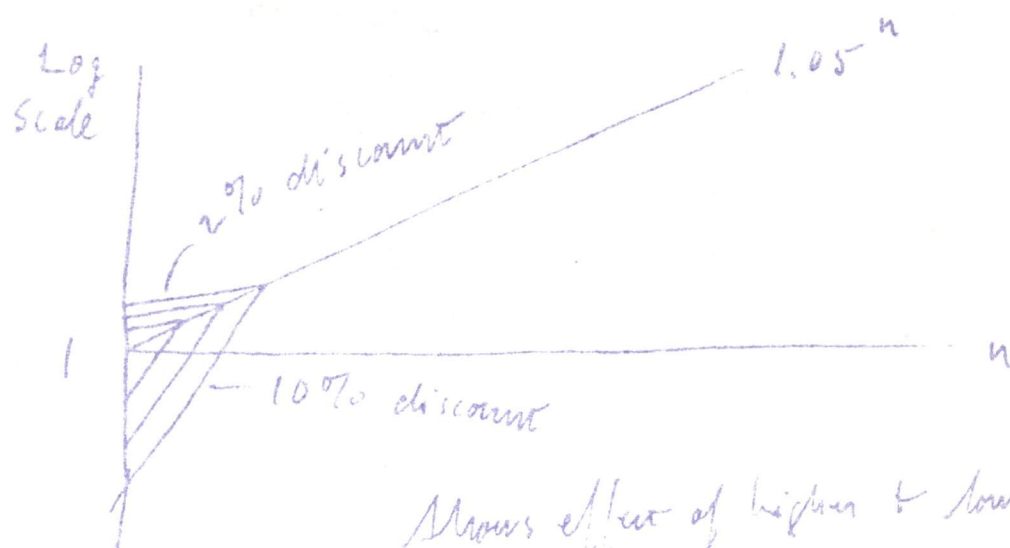


Fig. 3

Shows effect of higher & lower i .



Fig. 4

Shows effect of higher wage rates