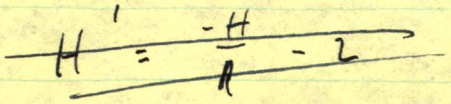
Cylinder Case, 1985

Cylinder case

loose ends.



Const S, max V.

Same 2 conclusion $A = 0$. Same result.

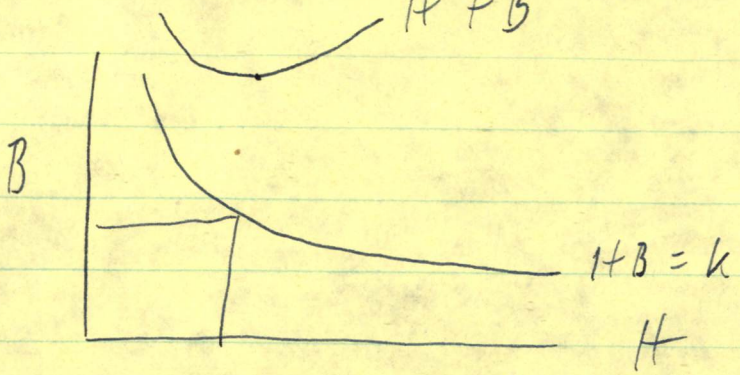
GB 2.4

Lloyd Connelly - D vs. secret service.

Call State Clerk to copy.

Rectangle

I. Const Area



$\epsilon = -1$ means $\Delta(HB) = 0$

Min. $(H+B)$ means $\Delta(H+B) = 0$. This says nothing about ϵ?

$$\epsilon = \frac{b}{h} = \frac{B'/B}{H'/H} = H\frac{B'}{B} \quad \text{--- dead end.}$$

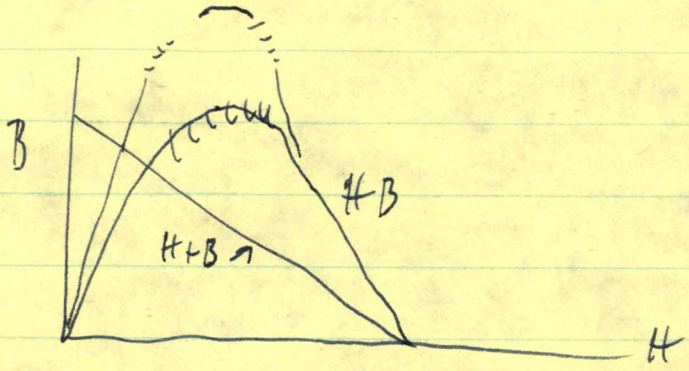
2 conditions must obtain simul:

$$B' = -H' = -1 \qquad (1)$$

$$\frac{B'}{B} = -\frac{H'}{H} = -\frac{1}{H} \qquad (2)$$

Only (2) is couched in ϵ terms.

II. Rectangle.
Const. Perim.



2 simul condns. must obtain:

$$B' = -H' = -1 \qquad (1)$$

$$\frac{B'}{B} = \frac{-H'}{H} = \frac{-1}{H} \qquad (2)$$

Same 2 conditions; same answer.

Cylinder

I. Const. Vol. $= \pi R^2 H$

A. H as indep. vble.

Condition for const. vol.

$$R^2 H = \frac{V}{\pi}$$

$$2 \ln R + \ln H = \ln \frac{V}{\pi}$$

$$2 \frac{R'}{R} + \frac{1}{H} = 0$$

$$2 \frac{R'}{R} = -\frac{1}{H}$$

$$2R' = -\frac{R}{H} \qquad (1)$$

$$\frac{R'}{R} \cdot H = -\frac{1}{2} = \epsilon$$

$$2r = -h$$

$$\frac{r}{h} = -\frac{1}{2} = \epsilon$$

Condition for const. $S. = 2\pi R^2 + 2\pi RH = 2\pi(R^2 + RH)$

$$R^2 + RH = \text{const.}$$

$$2RR' + R + HR' = 0 \qquad (2)$$

Subst. (1), $H = \frac{-R}{2R'}$

$$2RR' + R - \frac{R}{2} = 0$$

$$2R' + 1 = \frac{1}{2}$$

$$2R' = -\frac{1}{2}$$

$$R' = -\frac{1}{4}$$

$$\frac{dR}{dH} = -\frac{1}{4}$$

$$-2 \cdot \frac{1}{4} = -\frac{R}{H}$$

$$H = 2R$$

B. R as indep. vble.

Condition for const. Vol.

$$2\ln R + \ln H = \ln \frac{V}{\pi}$$

$$\frac{2}{R} + \frac{H'}{H} = 0$$

$$h = -2r$$

$$\frac{h}{r} = \epsilon = -2$$

Cond. for const. S

$$R^2 + RH = \text{const.}$$

$$2R + RH' + H = 0 \quad : \quad R(2+H') = -H$$

Subst. (1), ~~$R =$~~ $\frac{H'}{H} = \frac{-2}{R}$; $R = -2\frac{H}{H'}$

$$-2\frac{H}{H'}(2+H) = -H$$

$$-\frac{4H}{H'} - 2\frac{H^2}{H'} = -H$$

$$-\frac{4}{H'} - \frac{2H}{H'} = -1$$

$$4 + 2H = H'$$

$$4 + 2H = -\frac{2H}{R}$$

$$4R + 2HR = -2H$$

$$2R + HR = -H$$

$$R(2+H) = -H$$

$$R = \frac{-H}{2+H}$$

?

Wrong

Const Surface.

A, H as indep. vble.

Cond. for const. S

$$R^2 + RH = k$$

$$2RR' + R + HR' = 0$$

$$2r + 1 + r = 0$$

$$3r = -1$$

$$r = -\frac{1}{3}$$

$$r + r + r + h = 0$$

$$3r + h = 0$$

$$R(R+H) = k$$

$$\ln R + \ln(R+H) = \ln k$$

$$\frac{R'}{R} + \frac{R'+1}{R+H} = 0$$

$$R'R + R'H + R'R + R = 0$$

$$2R' + H\frac{R'}{R} = -1$$

~~Subst.~~

$$G = H\frac{R'}{R} = -1 - 2R'$$

Subst. $G = -\frac{1}{2} = -1 - 2R'$

$$2R' = -\frac{1}{2}$$

Subst. again! $R' = -\frac{1}{4} = -\frac{R}{2H}$; $2R = H$

Vol. cone

R variable. 2 mgnl. conditions simul.

$$S' = 2\pi \cdot 2R + 2\pi R H' + 2\pi H = 0$$

$$2R + RH' + H = 0$$

$$H = -R(2 + H')$$

Method: Eliminate H, find H' (R disappears)

$$R^2 H = \frac{V}{\pi}$$

$$2RH + R^2 H' = 0$$

$$H = \frac{-R^2 H'}{2R} = -\frac{RH'}{2}$$

$$-\frac{RH'}{2} = -R(2 + H')$$

$$\frac{H'}{2} = 2 + H'$$

$$-\frac{H'}{2} = 2$$

$$H' = -4$$

$$2R - 4R = -H$$

$$-2R = -H$$

$$R = \frac{H}{2}$$

Vol. const.

H vble. 2 unspd. condns. simul.

Method: Elim. H

$$S' = 2\pi[2RR'] + 2\pi(R + HR') = 0$$

$$2RR' + R + HR' = 0$$

$$H = \frac{-R(2R'+1)}{R'}$$

$$R^2 H = \frac{V}{\pi}$$

$$H\,2RR' + R^2 = 0$$

$$H = \frac{-R^2}{2RR'} = \frac{-R}{2R'}$$

$$\frac{-R}{2R'} = \frac{-R(2R'+1)}{R'}$$

$$\frac{1}{2} = 2R' + 1$$

$$-\frac{1}{2} = 2R'$$

$$R' = -\frac{1}{4}$$

$$2R\left(-\frac{1}{4}\right) + R\,\cancel{H} - \frac{H}{4} = 0$$

$$R\left(1 - \frac{1}{2}\right) = \frac{H}{4}$$

$$R = \frac{H}{2}$$

$$V = \pi r^2 h$$

$$\frac{dV}{dr} = \pi h \, 2r$$

$$\frac{dV}{dh} = \pi r^2$$

$$S = 2\pi r^2 + 2\pi r h$$

$$\frac{dS}{dr} = 2\pi 2r + 2\pi h = 2\pi(2r + h)$$

$$\frac{dS}{dh} = 2\pi r$$

$$\frac{dV}{dS} = \frac{rh}{2r + h}$$ when h held const.

$$\frac{dV}{dS} = \frac{r}{2}$$ when r held const.

$$\frac{r}{2} = \frac{rh}{2r + h}$$

$$\frac{1}{2} = \frac{h}{2r + h}$$

$$2r + h = 2h$$

$$2r = h$$

$$r = \frac{h}{2}$$

$$\frac{V}{S} = \frac{\pi R^2 H}{2\pi R(R+H)} = \frac{RH}{2(R+H)}$$

Max. when $r + h = \frac{2(R'+H')}{2R+H}$ etc.

But scale overpowers malprops. when hold R or H const.

$V = \pi R^2 H$

with R = const.

$$\frac{dV}{dH} = \pi R^2$$

with H = const.

$$\frac{dV}{dR} = \pi H 2R = 2\pi HR$$

$$R(R+H) = k$$

$$r + \frac{R' + H'}{R+H} = 0$$

$$rR + rH + R' + H' = 0$$

a. ~~If~~ Let $R' = 1$, solve for H'

$$1 + \frac{H}{R} + 1 + H' = 0$$

$$H' = -\frac{H}{R} - 2$$

$$k = -r - \frac{2}{H}$$

b. Let $H' = 1$, solve for R'

$$2R' + R'\frac{H}{R} = -1$$

$$R' = \frac{-1}{2 + \frac{H}{R}}$$

$$r = \frac{-1}{2R + H}$$

ϵ of subsets is also ratio of ϵs of products.

$$\frac{S}{2\pi} = R^2 + RH$$

W.r.t. R, $\quad \frac{S'}{S} = \frac{2R + RH' + H}{R^2 + RH}$

$$\epsilon_1 = R\frac{S'}{S} = \frac{2R + RH' + H}{R + H} = \frac{R + H + R(1 + H')}{R + H} = 1 + \frac{R(1+H')}{R+H}$$

[By logs, $\ln S = \ln 2\pi + \ln R + \ln(R + H)$

$$\frac{S'}{S} = 0 + \frac{1}{R} + \frac{1 + H'}{R + H} \quad \text{w} = \frac{R + H + R + RH'}{R(R+H)}$$

$$R\frac{S'}{S} = 1 + \frac{R(1+H')}{R+H} \quad]$$

W.r.t. H, $\quad \frac{S'}{S} = \frac{2RR' + R'H + R}{R(R+H)} = \frac{2R' + R + 1}{R + H}$

$$\epsilon_2 = H\frac{S'}{S} = \frac{2R'H + RH^2 + H}{R + H}$$

$$\frac{\epsilon_2}{\epsilon_{H'}} = \frac{2R'H + RH^2 + H}{R + H + R(1 + H')}$$

awkward!

Const. Surface

$$R(R+H) = k = \frac{150}{2\pi} = 7.958$$

$$H = \frac{k}{R} - R = \frac{7.958}{R} - R$$

	R	H	R^2H	r	h
	~~1~~				
	1				
	1.1			.1	
	1.210			.	
	1.331			.	
	1.464	3.972	8.513	.	
✓	1.611	3.329	8.640	.	-.162
	~~1.771~~ 1.774	2.723	8.541	.	-.182
	1.949				
	1.600	3.374	8.637		
✓	1.616	3.309	8.641	.01	-.019
	1.628	3.260	8.641	(.00743	-.015)
	1.632	3.244	8.640	.01	.0196
	1.648			.	
	1.665			.	

$$S = 2\pi R^2 + 2\pi RH$$
$$V = \pi R^2 H$$
$$\pi R^2 = \frac{V}{H} \qquad RH = \frac{V}{R\pi}$$

?. S as fn. of $V = 2\frac{V}{H} + 2\pi\frac{V}{R\pi} = 2V\left(\frac{1}{H} + \frac{1}{R}\right)$

?. a double substn. may be invalid.

Circle

$$A = \pi R^2$$
$$C = 2\pi R$$

$$\frac{dA}{dC} = \frac{dA/dR}{dC/dR} = \frac{2\pi R}{2\pi} = R$$

Cube

$$V = B^3$$
$$S = 6B^2$$

$$\frac{dV}{dS} = \frac{dV/dB}{dS/dB} = \frac{3B^2}{12B} = \frac{B}{4}$$

$$\frac{V}{S} = \frac{B^3}{6B^2} = \frac{B}{6}$$

$$V = \frac{B \cdot S}{6}$$

Screwy!

$$\frac{dV}{dS} = \frac{B}{6}$$

Advantage of understanding E is advantage in quickly grasping + expressing your "subject to"s.

$R^2 H$ = const. as a condition is same as:

1. With H variable,

$$2 \ln R + \ln H = k$$

$$2r + h = 0$$

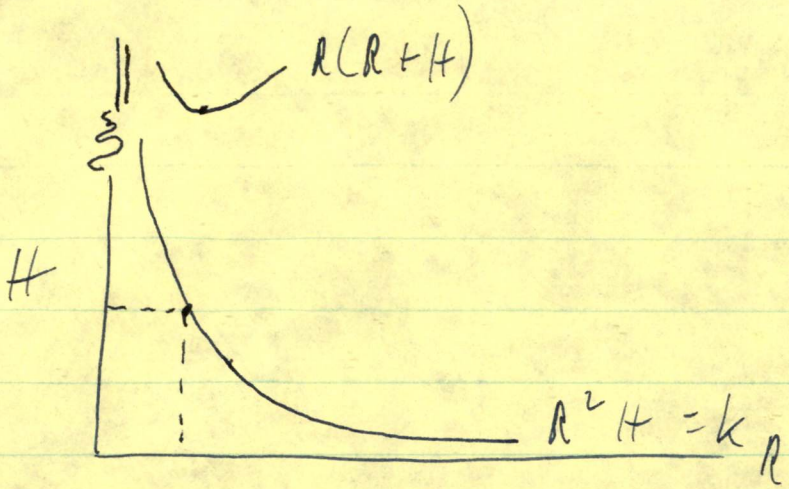
$$r = \frac{-h}{2} \qquad \left(h = \frac{1}{H},\ \text{so } r = \frac{-1}{2H}\right)$$

2. With R variable,

$$2r + h = 0$$

$$h = -2r = -\frac{2}{R}$$

But S = const. is not best expressed in E terms.



$2\pi R(R+H)$

$R^2H = k$

$RRH = k$

$r + r + h = 0$

$2r = -h$

$\epsilon = -1$ or $-\frac{1}{2}$. $2r = -h$ is firmer, less ambiguous.

$R+H$ are locked together by $R^2H = k$ so that it doesn't matter which is indep. vble.

$R(R+H) = k$

$r + \frac{rR + hH}{R+H} = 0$

$rR + rH + rR + hH = 0$

$r = \frac{1}{R}$ when R indep.

$1 + \frac{H}{R} + 1 + hH = 0$

$2 = -H\left(\frac{1}{R} + h\right) = -\frac{H}{R} - H'$

$\frac{H}{R} = -2 - H'$

Ec. 162 F85 Prob. Set #4, 10-25

1. Find ~~min~~ opt. dimensions of a rectangleden fence to enclose a given area with the least fencing.

a. Min $\frac{B+H}{BH}$

$$\frac{bB+hH}{B+H} = b+h$$

$$bB+hH = bB+bH+hB+hH$$

$$bH = -hB$$

$$\frac{H}{B} = -\frac{h}{b} = 1$$

b. Min $B+H$

Subj. to $BH = k$

$$B+H = B+\frac{k}{B}$$

$$1-k/B^2 = 0$$

$$B^2 = k = BH$$

$$B = H$$

b. 2nd approach, using equal. conditions.

$$BH = k$$
$$B(1+b)H(1+h) = BH$$
$$(1+b)(1+h) = 1$$
$$\ln(1+b) + \ln(1+h) = 0$$
$$b + h = 0$$
$$b = -h$$

Min. $B+H$ subject to $b = -h$

Min $B+H$ implies $\Delta B = -\Delta H$

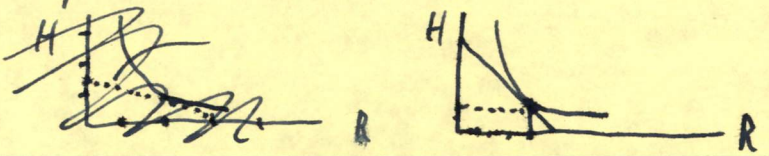
~~$B(1+b$~~

$$bB = -hH$$
$$B = +H$$

Next set we will do same for cylinder.

Problems beyond
of kinder care.

Graphically, this equal output curve looks so:



If H + R cost the same per foot, the ideal ratio ~~is~~ would be $\frac{R}{H} = 2$, because a 1% rise of R would ~~would then save just as~~ exactly the cost of a 2% ~~rise~~ ΔH.

allow a 2% fall of H, and this would save cost so long as 1% R < 2% H.

But not so simple. Cost is related to surface, not linear feet. The surface of the 2 disks of the cylinder increases with ~~the~~ R^2; and the surface of the tube part rises with ~~the~~ R, as well as H.

So the cost of H is $2\pi R$ ~~ft~~ per foot. The cost of R is $(2\pi R + 2\pi H)$ per foot. ~~So will use less of R, and more of H, would~~
The cost of R is $(R + H) / R$ times the cost of H.

~~So will use less of R, and more of H, would~~

~~$2H = \frac{R+H}{R}$~~

Now a 1% rise of R saves 2% of H, but costs $\left(1 + \frac{H}{R}\right)$ times as much per foot. This pays off if:

~~$\left(1 + \frac{H}{R}\right) R' = -H'$~~ $\qquad H' = -\left(1 + \frac{H}{R}\right) R'$

~~$R' = -H'\left(1 + \frac{H}{R}\right)$~~

or $\quad h H = -\left(1 + \frac{H}{R}\right) r R = -r(R + H)$

But ~~$r = -2h$~~ $\quad h r = -2r$

$\therefore \quad 2H = \left(1 + \frac{H}{R}\right) R = R + H$

$H = R$

$a = b + h$ and $a = 0$, so $b = -h$.

(For finite changes, $0 = b + h + bh$

$b = -h - bh$

$b(1+h) = -h$

$b = \frac{-h}{1+h}$)

4. If B & H cost the same per foot, the ideal shape is a square. b substitutes for h, and 1% of a smaller unit will cost less than 1% of a larger unit.

If H costs more than B -- (e.g. this is the side of a bldg. of construct D) -- then you'll use less H & more B, until what?

To p. 3

~~Say costs of H = 3x cost of B. Then a 3 ft. ΔB is as good as a 1 ft ΔH. So you go to where: $B' = -3H'$ $b = -\frac{1}{3}h$ $bB = -\frac{2}{3}hH$ But $b = -h$ $\therefore B = \frac{2}{3}H$~~

Then you'll pay for a $3\,\Delta B$ to gain a $1\,H - \Delta H$.
This is where:

$$B' = -3H'$$

$$\text{or } \Delta B = -3\,\Delta H$$

But $\Delta = -4$

$$\therefore B = 3H$$

More generally,

$$B = -3H\,\frac{h}{b}$$

$$\frac{B}{H} = -3\,\frac{h}{b}$$

Still more generally,

$$\frac{B}{H} = \frac{-h}{b}\,\frac{P_H}{P_B} \qquad (P_H = \text{Price of } H)$$

Prod. Ecs. Study Qus.

1. The cross-sect. of a pipe grows w. the sq. of the radius.
   a. What is xsect. as a fn. of circ?
   b. What is the ϵ of prodn. of pipe cap. w.r.t. the circumf.
   c. What is the ϵ of $\overset{\text{cap.}}{\text{Cost}}$ $\overset{(K)}{\text{w.r.t.}}$ capac?

Ans. a. $A = \pi r^2$

$C = 2\pi r$

$r = \frac{C}{2\pi}$

$A = \pi \frac{C^2}{4\pi^2} = \frac{C^2}{4\pi}$

b. $\epsilon = 2$

c. $\text{Cost}(K) = k\,C$

$C = 2\pi r$

$A = \pi r^2$

$r = \frac{1}{\pi^{\frac{1}{2}}} A^{\frac{1}{2}}$

$C = 2\pi \pi^{-\frac{1}{2}} A^{\frac{1}{2}} = 2\pi^{\frac{1}{2}} A^{\frac{1}{2}}$

$\epsilon = \frac{1}{2}$

$$\frac{dV}{dS} = \frac{dV/dr}{dS/dr} = \frac{2\pi rh}{4\pi r + 2\pi h} = \frac{rh}{2r+h}$$

$$\frac{dV}{dS} = \frac{dV/dh}{dS/dh} = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$$

or, if $h = kr$

$$V = \pi r^2 kr = \pi k r^3$$

$$S = 2\pi r^2 + 2\pi k r^2 = 2\pi r^2(1+k)$$

$$\frac{V}{S} = \frac{\pi k r^3}{2\pi r^2(1+k)} = \frac{kr}{2(1+k)}$$

C. Remove constraints on scale, see changes in ϵ

~~I.~~ ~~Square~~ Rectangle

~~A. Fixed~~ ~~Dim. vars.~~

$$A = BH. \quad (1)$$

$$P = 2(B+H) \quad (2)$$

A. ~~Dim.~~ Variable propns.

~~a.~~ Hold B const

$$\frac{dA}{dH} = B$$

$$\frac{A'}{A} = \frac{B}{BH} = \frac{1}{H}$$

$$\epsilon = \frac{A'/A}{1/H} = 1$$

$\frac{A}{P}$ keeps rising, bec. B const

$$\frac{A}{P} = \frac{BH}{2(B+H)}$$

B. Variable scale, fixed propns.

~~b.~~ Enlarge, with $B = kH$, $A = kH^2$

$$\frac{dA}{dH} = 2kH \; ; \; \frac{H}{A} = \frac{1}{kH}$$

$$\epsilon = \frac{A'/A}{1/H} = \frac{H \cdot 2}{H} = 2$$

C. Substn. of inputs, ~~Variable propns.~~, fixed output or fixed input

~~c.~~ (2.) Min. P subj. to fixed ~~const~~ A.

Subst. (1) in (2)

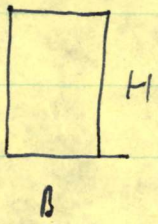
$$P = 2\left(\frac{A}{H} + H\right)$$

$$\frac{H}{P} = \frac{H}{2\left(\frac{A}{H} + H\right)} = \frac{1}{2\left(\frac{A}{H^2} + 1\right)}$$

$$\frac{dP}{dH} = 2\left[-AH^{-2} + 1\right]$$

$$E = \frac{H}{P} P' = \frac{2\left[-AH^{-2}+1\right]}{2\left(AH^{-2}+1\right)} = \frac{1 - AH^{-2}}{1 + AH^{-2}} = \frac{H - \frac{A}{H}}{H + \frac{A}{H}}$$

$$= \frac{H - B}{H + B}$$



(1.) Fixed input (P), variable output (A)

From (2), $B = \frac{P}{2} - H$

$$A = \left(\frac{P}{2} - H\right) H = \frac{HP}{2} - H^2$$

$$\frac{dA}{dH} = \frac{P}{2} - 2H$$

$$\frac{A'}{A} = \frac{\frac{P}{2} - 2H}{\frac{HP}{2} - H^2} =$$

$$E = \frac{P/2 - 2H}{P/2 - H} = \frac{H + B - 2H}{B}$$

$$= \frac{H + B - 2H}{B} = \frac{B - H}{B} = 1 - \frac{H}{B}$$

Here, you ΔH by taking from B, to change A. A is const., $\therefore$ max, when $H = B$. When $H > B$, E is neg, so you shd. reduce H. Thus this tells you more than deriv. = 0, which cd. max. or min.

(2.) From p. 1

(1) Use deriv = 0, for check & comparison.

$$\frac{dA}{dH} = \frac{P}{2} - 2H = 0$$

$$B + H = 2H$$

$$B = H$$

~~[~~$A'' = -2$~~]~~

(2).

$$P' = 2\left[-AH^{-2} + 1\right] = 0$$

$$1 = \frac{A}{H^2}$$

$$H^2 = A = BH$$

$$B = H.$$

More random ideas on

Cy Linden.

Cylinder.

Problem: To min. surface, S, for a given Volume, V.

Method: Express S as $f(V)$.
Then min. S with V held constant.

(1) $V = \pi r^2 h$

(2) $S = 2\pi r h + 2\pi r^2$

(3) $= 2\frac{V}{r} + 2\pi r^2 \qquad \left(2\pi r h = 2\frac{V}{r}\right)$

(4) $\frac{dS}{dr} = -r^{-2} \cdot 2V + 2 \cdot 2\pi r = 0$

$$4\pi r = \frac{2V}{r^2} = \frac{2\pi r^2 h}{r^2} = 2\pi h$$

(5) $2r = h$

Note that Step (3) eliminated h. Would it have worked to elim. one r? Yes!

$$S = 2\pi r h + 2\frac{V}{h} \qquad 2\pi r^2 h = 2V$$

hold r and V const.

$$\frac{dS}{dh} = 2\pi r - 2 \cdot 2V h^{-2} = 0$$

$$\pi r = \frac{2V}{h^2} = \frac{2\pi r^2 h}{h^2} = \frac{2\pi r^2}{h}$$

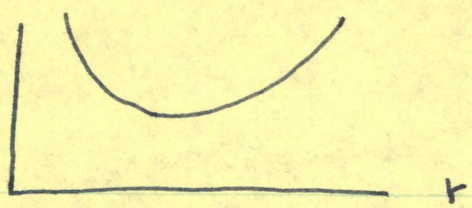
$$\pi r h = 2\pi r^2$$
$$h = 2r$$

This is a pure trade off between ht. and radius. Can it be expressed that way? Maybe using ϵ?

ϵ of S wrt r is: $\frac{2\pi h + 4\pi r}{2\pi r h + 2\pi r^2} \times r = \frac{2\pi h + 4\pi r}{2\pi h + 2\pi r}$

ϵ of S wrt h is:

Substituting ht. and radius.



Cylinder.

$V = \pi r^2 h$

$\left(\therefore\ h = \frac{V}{\pi r^2}\right)$

Method:

1. Hold V const; minimize S

2. Use r as indep. vble.

$S = 2\pi r h + 2\pi r^2$

Avoid temptation of factoring out $2\pi r$!

$2\pi r h = 2\frac{V}{r}$

$\therefore\ S = \frac{2V}{r} + 2\pi r^2$

$\frac{dS}{dr} = -r^{-2} \cdot 2V + 2 \cdot 2\pi r$

$S' = 0 \quad \text{when} \quad \frac{2V}{r^2} = 4\pi r$

$\pi h = 2\pi r$

$h = 2r$

Omit pp. 5 - 7

So a 1% ~~fall~~ rise of R is a ~~fall~~ rise of S of

$\left(1 + \frac{R}{R+H}\right)$ %. Thus, a 1% ΔR costs more than a 1% rise of H.

~~Then you'll pay for a $\left(1 + \frac{R}{R+H}\right)$ ft.~~

a 2% rise of H is a 2% rise of S. But a 1% rise of R is a $\left(1 + \frac{R}{R+H}\right)$ % rise of S.

Treat this as a cost factor. The cost of R is $\left(1 + \frac{R}{R+H}\right)$ x cost of H. Then you'll pay for a

$\left(1 + \frac{R}{R+H}\right)$ ft. ΔH to gain a 1 ft - ΔR. This is

where:

$$H' = -\left(1 + \frac{R}{R+H}\right) R'$$

or $h H' = -\left(1 + \frac{R}{R+H}\right) r R$

But $h = -2r$

$$\therefore \quad H' = -\left(1 + \frac{R}{R+H}\right) \frac{1}{2} R$$

In terms of our general formula.

$$\frac{P_R}{P_H} = 1 + \frac{R}{R+H}$$

$$\frac{R}{H} = \frac{-h}{r}\,\frac{P_H}{P_R} = +2\,\frac{1}{1+\frac{R}{R+H}} = \frac{+2(R+H)}{2R+H}$$

$$2R^2 + RH = 2RH + 2H^2$$

$$2R^2 = RH + 2H^2 = H(R+2H)$$

Cylinder.

$$V = \pi R^2 H$$

$$S = 2\pi R^2 + H 2\pi R = 2\pi R(R+H)$$

$$\frac{V}{S} = \frac{\pi R^2 H}{2\pi R(R+H)} = \frac{RH}{2(R+H)}$$

$\frac{V}{S}$ max when: $\frac{r+h}{\cancel{2}} = \frac{\cancel{R}\cancel{(1+H/R)}\,2(R'+H')}{2(R+H)} = \frac{rR+hH}{R+H}$

$$\cancel{rR} + rH + hR + \cancel{hH} = \cancel{rR} + \cancel{hH}$$

$$rH = -hR$$

$$\frac{r}{h} = -\frac{R}{H}$$

$$S = V\,\frac{2(R+H)}{RH}$$

V held const., so $\frac{h}{r} = -2$

S min. when:

$$\frac{2(R'+H')}{2(R+H)} = r + h$$

$$\frac{r}{h} = -\frac{R}{H} = -\frac{1}{2}$$

$$\therefore \quad 2R = H$$

$$S = 2\pi R(R+H) = 2\pi R^2 + 2\pi RH$$

$$\frac{dS}{dH} = 2\pi R$$

$$\frac{dS}{dR} = 4\pi R + 2\pi H = 2\pi(2R+H)$$

So a 1% rise of H causes s/s of $\frac{2\pi R}{2\pi R(R+H)}$

$$= \frac{1}{R+H}$$

a 1% rise of R causes s/s of $\frac{2\pi(2R+H)}{2\pi R(R+H)}$

$$= \frac{2R+H}{R(R+H)}$$

$$= \frac{R+R+H}{R(R+H)}$$

$$= \frac{1}{R} + \frac{1}{R+H}$$

$\frac{V}{S}$ of Cylinder

R	H	$\frac{RH}{2(R+H)}$
1	1	$\frac{1}{4} = .25$
1	2	$\frac{1}{3} = .33$
2	1	$\frac{1}{4} = .25$
2	2	$\frac{1}{2} = .50$

$S = 2\pi R^2 + 2\pi RH$

$2\pi R^2(1+r)^2 + 2\pi R(1+r)H(1+h) = 2\pi R^2 + 2\pi RH$

$2\pi R^2[(1+r)^2 - 1] + 2\pi RH[(1+r)(1+h) - 1] = 0$

$R[(1+r)^2 - 1] + H[(1+r)(1+h) - 1] = 0$

$R[2r + r^2] + H[hr + rh] = 0$

$R[2 + r] + H[\frac{h}{r}r + h] = 0$

$(1+r)(1+h) =$
$1 + h + r + rh$

$\frac{R}{H} = \frac{-(1 + h\frac{r}{r} + \frac{h}{r})}{2 + r}$

Subst. from p. 1 $\quad r = \frac{-h}{2}$

$\frac{R}{H} = \frac{-(1 + h\frac{r}{r} + \frac{h}{r})}{2 - \frac{h}{2}}$ or $= \frac{-(1 - 2 + \frac{r}{r} - 2)}{2 + r} = \frac{-(-1 \cdot 2r)}{2 + r} = \frac{1 + 2r}{2 + r}$

$\frac{\frac{h}{r}}{1 + h} = \frac{1}{\frac{2}{h} + 2}$

dead end?

$$2\pi R^2(1+r)^2 + 2\pi RH(1+r)(1+h) = 2\pi R(R+H)$$

$$R(1+r)^2 + H(1+r)(1+h) = R+H$$

Subst. $2r = -h$; $h = -2r$

$$R(1+r)^2 + H(1+r)(1-2r) = R+H$$

$$(1+r)(1-2r) = 1-2r+r-2r^2 = 1-r-2r^2$$

$$R(2r+r^2) + H(-r-2r^2) = 0$$

$$Rr(2+r) = Hr(1+2r)$$

$$\frac{R}{H} = \frac{1+2r}{2+r}$$

$$\frac{2+r}{H} = \frac{1+2r}{R} = \frac{1}{R} + \frac{2r}{R}$$