~~322. Surtax, Temp. (Nov. 1)~~

Longevity Bios, Inc. Tax

Longevity bios of Income Tax.

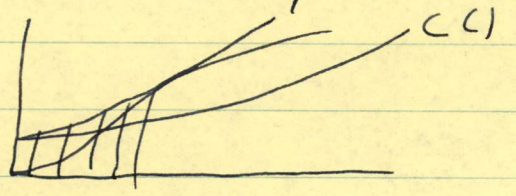
Please destroy
Tue afternoon

Ec. 302

Tues. nite.

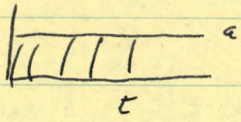
1. Generality of point that longer trees require more space.
Better ~~Longer~~ bldgs. are gently. built w. more space. Limits life, even tho they are built of higher quality mtrls.

2. Cap: output α pd. of inv.



Stagger, & pile the horiz. area vertically.
An important conversion, stressed in Wicksell, algebraically.
Analogies: a) Horiz. & Vert. mult.
b) Cambridge k vs. Velocity.

3. Land: output α pd. of invt.



4. Factor propns. From one cycle, reps. all.

5. Easy solution to oversaving dilemma: no need to reduce invt.; just lengthen pd. of prodn.

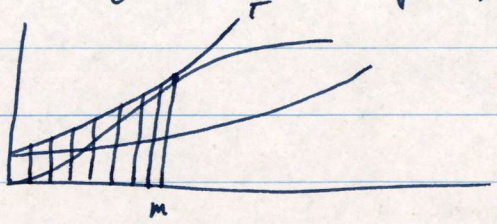
6. Bias Opt. life for
   a. Oil
   b. Bldgs.

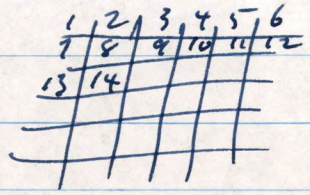
7. Bias to longevity
   a. Taxes
   b. Urban sprawl
   c. Oil

Thurs., Dec. 2, '65

1. If perfect staggering, then you have one bit of capital tied up corresponding to each yr. of life of the tree:

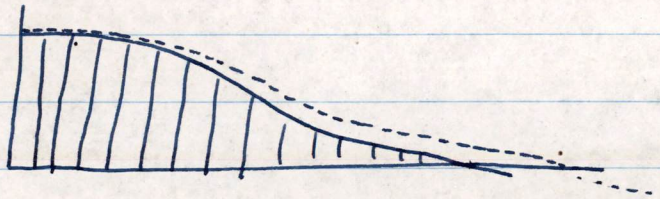
a.)





Total capl. is area under T curve.
Longer cycle, more capl. (& of course more land, too).

b) Depreciz asset.



Same principle, only the older capl. reps. less value.
Here, as you extend life, you must tie up more land, ~~but not much more capt.~~ You could do so w/o any more capl. — & wd., if land became cheaper. But if you build in more longevity, that adds to capl. value right along, rains whole curve a little. (dash line ----)

2. ~~Macro Implics of r.f.~~
Substn. of labor, capl., & land.

a. Wicks's wine.

For simple e.g.s, assume capl. is a mixture of appreig & depreig, & on ave. does neither.

Then required capl. stock to hire labor = yrly. payroll x ave. life.

If add labor to the popn. but no capl. cannot feed the new ones. (In a money ec., can't pay them. In inflationary ec., pay them anyway, take from others (over)) Now then withdraw 1/n th yr. of capl. Thus start off new labor, & also keep the existing cycles going.

Might require new land; or intensify old land.

b. Elast. of substn. depends on proportions.

If labor already gets 90% of pie, a small wage cut sucks in capl. — fast. & v.v., etc.

Thus factor propns. are very changeable, from small cost changes. Given a free ec., free prices.

Thus FICA tax has big impact, unemployment.

Thus wide use of low int. as subsidy to hsg., forestry, munic. & inn. works, etc., diverts capl. to uses in which labor gets small % of product. Takes invt. of new capl. away from labor-intensive uses. Blocks the opn. of the natural equilibrating mechanism.

(over)

Thus artificial abundance of land is bad. a) Requires much capl. of very slow v.f. b) Slows r.f. on all sites; c) scatters popn., slows r.f. of interite capl.

c. The bottleneck of capl. turnover

Getting to full emplmt. Must adapt capl.

by devaluating M.

In a tax ec, can pay them by taxing others.

Each is a forced levy of circ'g. capl. Each is possible, but raises other problems, & is less desirable than flexible capl. stock.

Low $\underline{i}$ also slows output of raw mtrls.

(One sign of capl. shortage is high. of minerals. But doesn't relieve shortage, bcs. devalues. Thus provides rationale for cartel schemes)

stock to complement more labor, à la Wicksell.

If it cannot adapt, you get inflx. of consumer goods w/o being able to employ all labor

Why cd. it not adapt?

a. Wage & int. rigidities.

b. Frozen capl.

3. Macro-ec. imptnce. of r.f.

Any capl. stocks can adapt to the popn. -- down to a bamboo ec.

Thus, the u.i. assoc. w. capl. stock depends on its r.f. — up to max. set by labor supply

Relate this to spending. $I_G \rightarrow Y$. Fast r.f. means hi I_G.

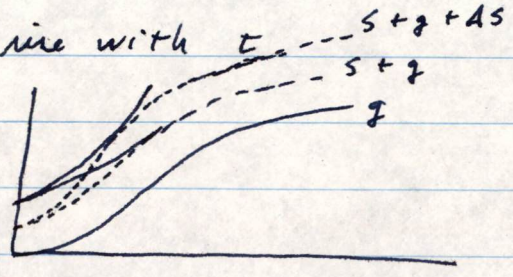
95% of all demand is replacement demand. Fn anticn. of sales.

Ackley says "Consumer is hero" of current thought. But ~~B.S.~~ consumers have very little autonomy. To small extent they spend in anticn. of inc. But not to the extent that

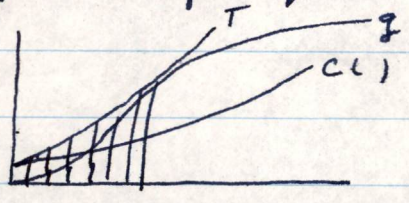
Thus, with

1. Review. Open for questions

2. Let S rise with t



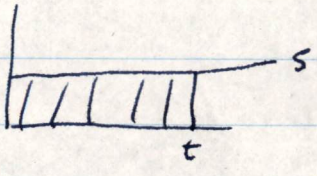
3. Cap./: output & pd. of invt.



$$\bar{T} = C_0(I) + \frac{a}{i}\ddot{s}\overline{|}\,[(1+i)^t - 1]$$

analogies: a, Horiz. & Vert. mult.
b. Cambridge k ratio $= \frac{1}{v}$

4. Land: output do.



5. Forest Service solution.

6. General bias to longevity.
From oil paper

Replacement. Topics -- not in order.

Fn. of capl. as reservoir

Gross invt. → Y.

Less circular system. Y not Y(c); but of replacement.

Hiway stretch out. Historical precedents. German analogy, today.

Waste of unfinished projects.

a. Individual bldgs.

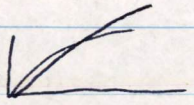
b. Grand programs -- moon race.

Role of land value & rent in replacement. $r = \left[\frac{R_m + S_m}{C_0 + S_0}\right]^{\frac{1}{m}} \cdot ($

Determinants of longevity.

a. Timber

b. Oil

c. Bldgs. 

d. Effect of taxes

e. " " urban sprawl

E. 302, Tues. Ideas from Michèle.

1. Elting's paradox

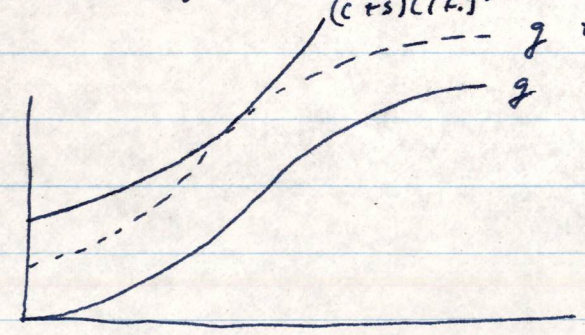
2. Why is it good to max. $\underline{a}$ per yr., [instead of per t yrs., ~~etc~~] or anything else?] a) It maxs. site value. Usually that is intend.

~~One answer~~, b) It does it in one rotation, w/o introducing ∞ time.

c) You can max. anything else, provided use full formula.

3. If max. $\left[g - C_0(1+i)^t\right]$, ~~you use~~ $g' = i\,C_0(1+i)^t$, just earning interest on $C(1+i)^t$, not on upper half of g.

4.

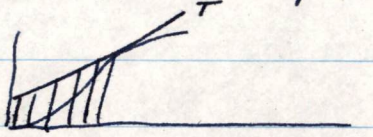


5. As lengthen $\underline{t}$, C_0 falls absolutely (for a fixed land area, or S) because you plant farther apart. But then, so does g fall.

To keep it to just one change, let C & g be constant, but S rises with t, bec. tree requires more space.

Topics, re timber

1. Cap/output ratio varies w. pd. of invt.



Stagger. & capl. at any time is area under T curve.

2. How to max. S

$$S = \frac{(\quad)}{(1+i)^T} + \frac{(\quad)}{(1+i)^{2T}} \cdots + \frac{(\quad)}{(1+i)^{\infty}} = \frac{(\quad)}{1-(1+i)^{-T}} \cdot \frac{r}{1-r}$$

$$= (\quad)\frac{(1+i)^{-T}}{1-(1+i)^{-T}} = \frac{(\quad)}{(1+i)^T - 1}$$

3. Forest service

3-30-65

I. There are Problems w. simple Wicksell solution to fncl. mat. problem.

a. Mgnl. — no return on site

b. Max g — P.V. >> C_0, impossible

c. $\pm \Delta C_0$, no effect on solution. (unless by $\mp \Delta i$, but reasons for that not shown in equ., as plenty remains for i anyway)
This one is esply. bad in W's scheme, bec. he needs to have ΔC_0 lengthen rotations.

d. If $i \to 0$, turnover → max. g_1, < max g/t

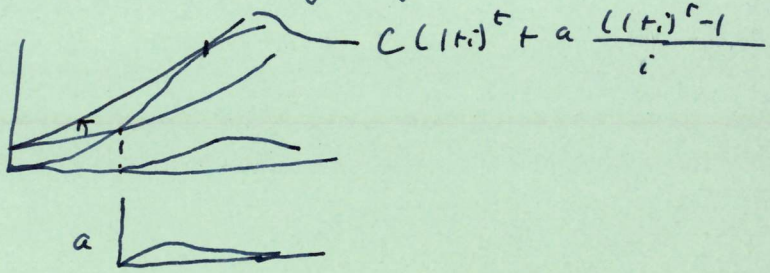
II. Solution

a. Mgnl. ~~Max~~. $g' = g i + a = i[g + S]$

b. Max g. a & S.

✓ Why is that better? Bec. they are net of costs.

How know they are ~~same~~ max. when $g' = g i + a$? Geometry?



$T = (C+S)(1+i)^t - S$

$T' = i(C+S)(1+i)^t$ .. [This can be shown w/o calc.]

~~$a + ai = a(1+i)$~~
~~$a + i \cdot a(1+i) = a(1+i)$~~

~~Now, can show that limit. $g = r$~~
~~$g' = r'$ gives same eq~~

~~as $g' = g i + \frac{[g - C(1+i)^t]\, i}{(1+i)^t - 1}$~~

$$P_0 = a\frac{1-(1+i)^{-t}}{i}$$

$$V_t = a\frac{(1+i)^t - 1}{i}$$

~~But is that a simpler way?~~

~~$a_0 + a + a(1+i)$~~

$1 + (1+i)^{-1} + (1+i)^{-2}$

$(1+i)^t + (1+i)^{t-1} + \dots + (1+i)^1 + 1$

so $\sum = a + a(1+i) + \dots + a(1+i)^t$ .... work backwards

$\sum_{t=0}^{\infty} a_t + a_i$

~~No problem to show that slope of T is $g_i + a$~~

~~$g_i + a = i(g + S)$~~

~~$T =$~~

How know ~~*~~ max. $\underline{a}$ corresponds to $g' = g_i + a$?

Bcs. T curve grows ~~at rate~~ by $Ti + a$, (where T is value realized from Σ of previous Δs.)

$T = (C+S)(1+i)^t - S$

$i[C(1+i)^t + S(1+i)^t]$

→ Growth, aply., is $i[C+S](1+i)^t = i[T+S] = Ti + \frac{a}{i} i = Ti + a$

So $T' = Ti + a$. When $T = g$, $T' = g_i + a$, & $T' = g'$, so $g' = g_i + a$

c. $\pm \Delta C$ now affect solution

i. Mgmt. approach

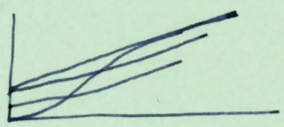
ΔC means lower a, that ~~shortens pd~~ lengthens pd.

Now it is clear we must introduce 3d factor, land, to analyze effect of ~~on~~ wage rates etc. on factor propns.

Higher wages also lowers $\underline{i}$, but what is the mechanism? Wickell (over) Land differs. for macro analysis, bcs. no turnover: requires no inc-creating spending to create; & needs no replacement. Also, slow r.f. calls for large social invt. in land-supporting p.w. to get enough land, & the p.w. themselves turn slowly.

ii. Max. approach

Geometric approach. ΔC means max. $\underline{a}$ comes later.

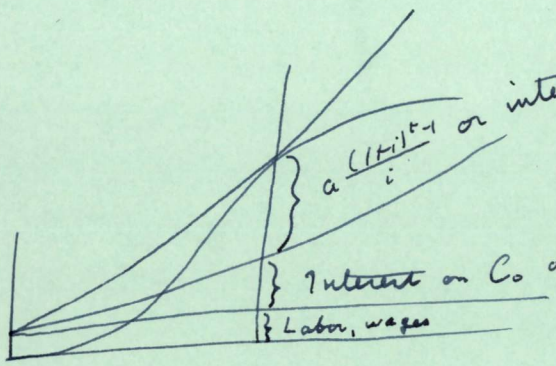


iii. Another approach — The higher C_0, the greater the desire to reduce the freq. of incurring it. Thus reduce annual burden.

d. If $i \to 0$, $a \to \frac{g-c}{t}$, & rule has you max. net inc. per yr, seems rational.

→ Bcs. of this long renewal pd., land may become limitational. Capl. turns over faster, & ∴ a little capl. covers much land. If land comes on the mkt. once in 100 yrs, we need 100 times as much land around as wd. be enough to employ all labor, while capl. turns much faster.

[Also, land can be very unstabilizing. if yrly. supply goes from 1% to 1½%, a 50% rise in supply — or v.v.
a shortage may be met by heavy invt., but that also unstabilizing.



Divide into 2 parts, tu & $\left[a\frac{(1+i)^t-1}{i} - tu\right]$
Use tables to est. values.

has all opns. on mgnl. land, so i must fall directly, & thus ~~lower~~ lengthen rotns.

Actually, process more complex. ↑ could all come out of a, if no land were mgnl.
As Mgnl. land goes out of use, throwing labor & capl. on mkt., & that pushes down i. Also, a falls. Each tends to lengthen rotns, thus producing timber w. less labor

Note well that the longer rotns. produce timber w. more land. as well as more capl. (show graph, formulae.) Replmt. freq. of capl. falls; so does renewal freq. of land. Thus if land is cheap we spread labor around this by renewing sites slowly, so labor applied to fewer of them each yr. (Staggering applies to land, too.)

Note renewal freq. is slower than turn freq. of capl., whether appreciating or depreciating. Bcs. is long pd. w. little capl., either at beginning or end of rotn.

III. Some Morals of this analysis & discussion

A. In free mkt., there are equilibrating mechanisms that tend toward full employment, & limit displacement of one sector by another.

B. Slower turnover of capl. leads to much higher land requirements to furnish given unit of employment, bec. site r.f. much slower.

[Same time, you (probly) put more capl. on ea. site, (even averaged over life), so less land may be brought to use.]

space requirements of a product are a simple multiple of its life. Capl. reqmts are much less. esply. for depreg assets.

C. Invention ≡ capl.

Inventors seek not only to lower labor costs, but all costs.

Many new devices save on capl. & land

(Ask class for illustrations)

(E.g. hot wtr pump vs. ~~radiator~~ gravity radiator system. Saves capl.

(E.g. tractors for horses - saves land)

D. "Technological" unempl. is ∴ not likely, ~~is~~ if prices can adjust.

More likely explanations are in instns. that prevent price adjustments

Suppressed i (note major break w. Keynes.)

Supported wage rates

(Housing, manifests both)

Land - slow renewal policies. Replacement needs a place. Compulsory land consvn. Credit rationing that keeps indivds from feeling sec.-o.c.

Frozen capl. - irreversible errors.

E. ~~Replmt. freq.~~,

The line between micro & macro is dubious.

R.f. is central to macro.

But we came at it as a micro qu. – how sub. capl. for labor

IV. Is replmt. freq. imptnt. in monetary ec?

In real terms, labor paid in final products, demand for labor limited by ripening of goods.

Monetary terms, demand for labor is gross invt., & that is detd. by ripening for replmt.

1. Factor propns are extremely variable.
But we don't realize how variable until we realize role of capl.

2. Prodn. takes time. Role of capl. is to carry labor products over time.
(Also carry products of land: ½ of capl.)

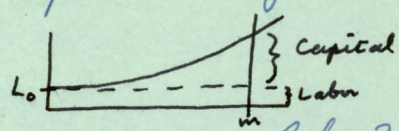
3. E.g. of one-yr. cycle vs. 50-yr. Drop wage 5%, double return to investors in 1-yr. cycle. $i = \left(\frac{P_1}{C_0}\right)^{\frac{1}{1}} - 1$

In 50 yr. cycle, $\frac{P_{50}}{C_0} \cong 10$. Use int. tables, 50th root of $10 \cong 1.0\underline{4}10$ Drop C_0 by 5%, so $\frac{P_{50}}{C_0} = 11$.

But $11^{\frac{1}{50}} \cong 1.0\underline{4}85$ .... difference is hardly noticeable. To Δi from 4% up to 5%, must $\Delta \frac{P_{50}}{C_0}$ from 7.1 up to 11.5. Etc.

(Here is why s.r. $\underline{i}$ is more sensitive in capl. shortage. S.r. can pull capl. away from lang in a shortage. L.r. $\underline{i}$ can't rise much bes long term borrowers can't pay the rates. So rate doesn't rise — but volume of new loans falls.)

4. Thus, a small drop of wages (or Δi) redirects invt. into forms more comply. to labor, to spread the given capl. stock over labor supply seeking work.



[Why are shorter cycles more comply. to labor? Bes. C_0 then makes up higher % of $P_{mat.}$] Note, however, an adjustment problem. Or, at a time, there is less capl. per man.

if periods have gotten too long, ^ or if more labor shows up. There is no immediate Δ demand for labor, ↑ from shortening cycles bcs. liq. capl. wd. have been reinvented anyway. Δ doesn't appear until after 1st cycle. One ~~cycle~~ theory of crises has it that we get stuck here, trying to shorten periods, creates liquidity panic.] [Of course, you can shorten cycles already in being, & in Wicksell's case he had a smooth adjustment occur that way. Problem arises from cycles-in-being that cannot be altered — time depreca. expty.]

5. The harbinger stand

a. Hi turnover means C.i is low % of ~~gross~~ v.a. (& even lower % of gross)

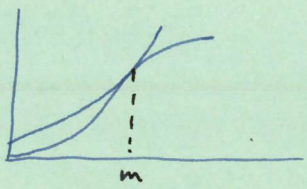
b. Capl. on hand = $\frac{m}{f}$ × wage bill, where $\frac{m}{f}$ ≡ pd. of ~~waiting~~ maturing (over)

c. Assume progressive v.a.

etc.

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6. Wicksell's wine



a. Derive compound int. formula. ~~fig~~

b. Numerical e.g.s.

c. What determines ~~maturity~~ ?

$\frac{f'}{f} = i$. Also, max.s p.v., $\frac{q}{(1+i)^m}$

(He slipped, shd. be $\frac{f'}{q+s} = i$, but will bring that in later)

i. To simplify, abstract from raw mtrls. bought, consider only v.a. (~~a as~~ like assuming full vertical intn. — is not a narrowing assn., but a broadening one — is what you wd. ultly. have to do to see the big picture.)

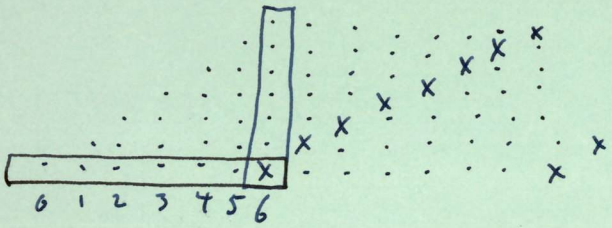
ii. Assume pd. of 7 days — ~~some~~ all v.a. ~~each~~ 1st day, ~~by adding stages of the oper.~~ (later, add complications)

iii. Assume perfect staggering

iv. Begin w. a 7 day ~~invt~~ lead before consumable at X

. X

v. But on ea. succeeding day, begin new cycle



On day 6, are in bus., & capl. = 7 × daily wage, = 7 × gross. ~~assuming~~

(wd. be less if v.a. progressively)

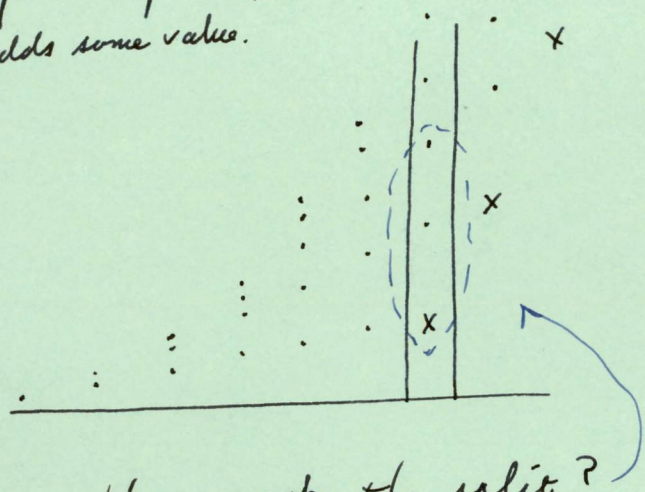
vi. So on any day, capl. = $\frac{m}{7}$ × wage bill. = $\frac{m}{7}$ × gross.

More quickly, but less precisely, capl/wage bill ≈ $\frac{m}{7}$

(In emergency, if can't work out, give them Fncl Mat. in a M) C-0

6. (Durable or fixed capl.)
(Raw mtrls.)

Now suppose vertical disintegration occurs. Break process up into parts. Each adds some value.



How make the split?

Bath tub

Vertical column reps. one firm at one time.

Actual firms have columns of uneven "width", & changing "width" — meaning 3D thickness. Use nos. for dots. Might have lots of shorter cycles
any one piece of durable equipment has the time dimension, or vertical — regard as annulare. Higher dots rep. values further from mat.

Ht. of col. reps $\sum$ of cap. values. (adding 3D, is ht. of all columns.)

No. of dots under a dot reps yrs. to liqn.

Fixed capl., treat same as circ'g, except:

a) Fixed cap. undergoes time-depreen, not use depree., can't be thrown at mtrls.

b) " " runs from top to bttm. of col. Consists of annulae

parts

c) House Bldgs. etc. consist of nn cycles, each. Firm can't stagger them, unless it has 50-60 bldgs. Staggering must be done by ea., via fncl. mkts — big source of instab.

As X is consumed, mentain capl. by a) Replac'g raw mtrls; b)

Adding value at every stage. So output = input, & cap/output ratio is cap. used per hired input

5. d. Bathtub theorem (cover)

6. Vertical disintegration — how to aggregate the turnover of all firms to get cap/output, & to maintain principle that cap/output = 1/turnover.

a. Explain the term — (cd. be shown either vertically or horiz. on one chart).

b. Split into just 2 stages, A & B. [A has 4 yrs, B has 3.]

c. Lowers cap. reqmts. of each; but doesn't lower overall cap/output.

d. Output of B is now v.a.

e. "Turnover" for firm becomes ~~less~~ faster than for entire opn.

Social turnover is slower. Begins w. inc.-creating expenditure; ends w. physical depreen. of capl., (or deplm.) — & not even then, bcs. must get to consumer goods before inst. is liquidated.

Firm's turnover includes an extra transaction.

To get firm's contrib. to social turnover, just use v.a. as output. Count all capl.

f. To get a figure for turnover of firm that ~~corr~~ corresponds to a figure for the whole ec. — & that can be aggregated to get figure for whole ec. — must work out all transfers along the way & count as inputs only factors; & (over)

Insert fm. → p. C-1a

7. Durable goods

a, Take col. as rep'g one firm. A pipe-line, or endless sausage machine. Dots work their way to the outlet at bottom, or X

b. Actual firms have many columns. A 3d D. shown by nos.

c. Usually more short cycles.

d. Bldgs. & plant, regard as annulose.

e. Makes a lump, bcs. all produced the yr. before, if new. (Ecs. of time-scale re longevity, explain why joint prodn. of different yrs. is done)

Turnover = ~~cap~~ output / cap. $= \frac{1}{\text{ave. pd.}}$

If inflow (& outflow) in yr. = stock, then it has bn. enough to change the whole once.

Is not to say it all *did* change. Maybe some changed 20 times, & some not at all. But on ave., all changed once.

count as output only ~~cap~~ v.a. final products ready for immediate consumption. But that is only the 1st step. Must not count all v.a. as output, but only

Thus early stage firms wd. have v.a. but no output

W/o growth, even so, output = $\sum$ v.a., & we cd. msre. ec's cap./output using ~~cap~~ $\sum$ v.a. as output. But if cap. stock grow'g, $\sum$ v.a. > output, & v.v.

Also, consumer durables shd. not be treated as output ~~when~~ when pass to cons.s., but as they wear out.

As qual. method, treat ~~cap~~ $\sum$ v.a. as $\equiv$ output.

As a fallacy, cap./v.a. of one firm seems indep. of turnover (except as turnover uses more labor & Δs v.a./capl., but abstract from that). But the firm's turnover consists mainly of non-factor inputs. Wash them out, & see that the amt of capl. that must be carried by Σ firms, relative to labor, depends on time-lapse between v.a. & ligns.

6 ½. In fact, most indies divided into stages that grow highly labor-intensive.

a) E.g. livestock. Cow-calf & cash-grain.
Feeder
Slaughter & packing
Retail
Cooking

b) Oil
Explr.
Prodn.
Xptn.
Refining
Distribn.
Driving

Thus, on the whole, firms in stages of hi gross turnover also have hi net turnover. (over)

c) Unless the product is durable. E.g. lumber. That becomes highly labor-intensive until frozen in house — then becomes again capl. & land intensive

~~Access roads~~
Planting & growing
Cutting (small fellows can buy timber rts)
Hauling
Milling
Shipping
Retailing
Bldg.

Not <u>as</u> hi, but hi — bcs. the reasons they buy & sell quickly are

a) To eize on capl — let someone else carry the bag waiting. This shows up in their also turning fast the capl. needed to carry v.a.

b) The reasons they buy raw mtls. is to complement labor etc. they have in excess. E.g.

Buyers of timber rts

Small steel producers who buy ore from USS

Cattle feeders

Independent oil refiners, e.g. Clark

Hamburger stands

Speed w. which durables are converted to output determines cap/output.

Can have several dots at each cell

If we treat purchased capl. as Δ cap/output, is ok, is offset for seller by counting cap. goods as output.

Everyone has capl. at various yrs. removed from ligr; at any time. Ptr all moves ult. by toward ligr. Capl./output is detd. by how long it takes to get there — weighted.

The capl. propn. in factor propn is int/v.a.

What was problem re hi turnover of cons? It's not turnover? or factr turnover? Or offset? ? ! ?

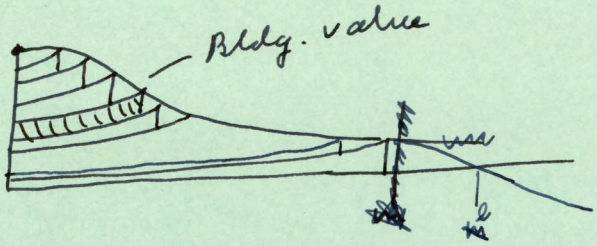
Some inputs, the durable, are not normally recycled smoothly by indiv. firm. Must look to whole ec. for staggering. But don't find it there, either — hence instability

e. (cont.)

But ec. as whole takes care of the staggering. (over)

Thurs. - begin here.

f.



annualize bldg. – valuation as $\sum$ future incs.

1. Each yr. is a stratum
2. The longer the stratum, the more triangular; the shorter, the more rectangular.
3. Ht. of ordinate corresponds to area under curve of p.v.: $\sum$ disc. values.

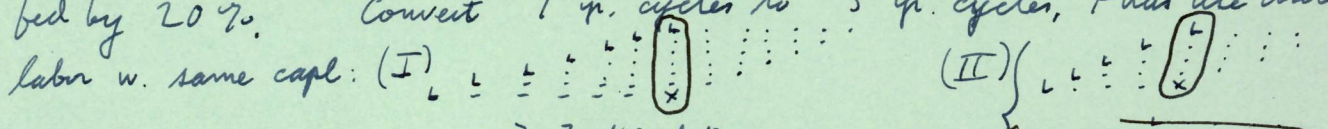
g. Bldgs, unlike W's wine, can't be accel. much. Frozen, once built. Hence, problems of irreverseability. (Hint ahead at macro problem, & monetary reflection thereof.)

~~(Recall W. from class.)~~

8. So capl. intensity depends on capl. longevity.

Substn. of labor for capl. occurs by reducing longevity.

Wicksell's wine shows us how. Cut the life 20%, Δ the labor fed by 20%. Convert 7 yr. cycles to 5 yr. cycles, thus use more labor w. same capl: (I) (II)



~~7 + 7 = 14 units~~

Under II, we have 2 units capl. released, a 2/7 of supply. Can use that to: a) Live for 2 yrs. w/o working; b) Δ output by 2/5.

~~2 × (5 + 5) = 20 units~~

9. So, what dets. longevity of capl?

Price of capl. (int.), relative to labor & land.

Begin w. W.'s wine. Then show inadequacies.

– actually more, bes. the 2 yrs. you cut off require more capl. than the early yrs.

I.e., one bldg. built yesterday gives a smooth flow of output over 50 yrs.

Here, we see that hi cap/output goes w. ~~durability~~ longevity, in another way than the staggering case. The staggering or bath tubs case applies to appreciating assets; adjustments ~~may~~ are needed for depreciating ones.

Staggering still needed, bec. as one bldg. wears out, a new one shld. begin.

Fig. #1

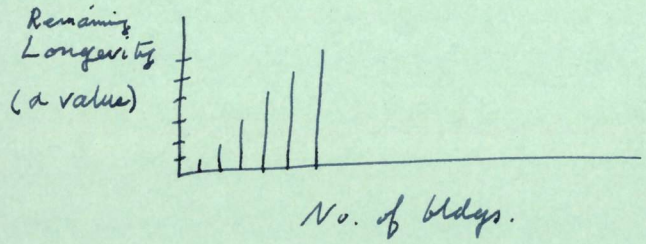
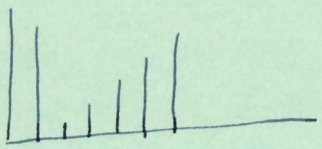


Fig. #2



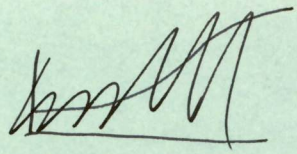
Etc.

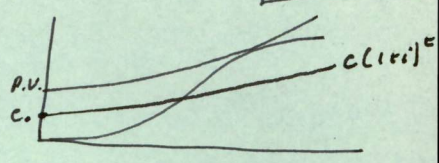
N.B. – New bldg. in position #1 is built using capl. released by liquidation of old ones

If individ. firm has no. of units = ~~n~~ l yrs. (l ≡ life, use instead of maturity), can stagger inside the firm, keep firms capl. const.

Usually, w. depreciating durables, firms don't stagger the very long ones. They borrow – from others whose bldgs. are wearing out. Big ec. problem is, we haven't achieved good staggering

E





Basic rationale of Wick

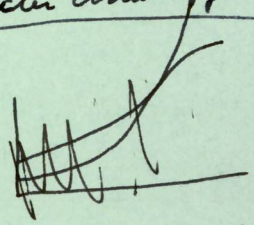
b. Max p.v.

a. $\frac{g'}{g} = i$, or $g' = gi$ (MP of time = MC)

So higher i shortens life

In case of bldgs, higher i shortens life by making longer streets more triangular, so added longevity is not worth the added cost. It accordions together Bldgs: the bottom end of the spectrum. Or, to make a stricter analogy (over)

No return on S



Sudden jump of p.v. from C_0 to $\frac{g}{(1+i)^t}$

No change if costs change — e.g. wages — & that's what we're looking for.

Solution:

a. $\frac{g'}{g+s} = i$

b. Max. p.v. of inf. future stream of $(g-c)$

c. Max $a = i\frac{g - C(1+i)^t}{(1+i)^t - 1}$

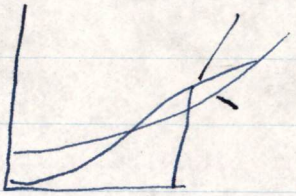
d. Same as max g $S = \frac{a}{i} = \frac{g - C(1+i)^t}{(1+i)^t} + \frac{g - C(1+i)^t}{(1+i)^{2t}}$ etc

to the tree case, the 50th bldg. must be harvested sooner by adding it to earlier bldgs (enhancing quality at expense of longevity — more likely ~~sacrifice~~ enhance quantity, or more likely, quality) tends to go w. longevity)

Where are my notes on taxes?

$$r = \left[\frac{R_m(1-t) + S}{C_0(1-t) + S}\right]^{\frac{1}{m}} - 1$$

Here, tax seems to lower r, even with expensing? Not if value of S falls. How did I show it?



Expensing C_0 is same as deducting $C_0(1+r)^m$ at m

Taxpayer keeps $\left[R_m - C_0(1+r)^m\right](1-t)$

Needs more work – I had a more persuasive way.

Topics

1. $C(1+i)^n = R$

2. How to solve for i

3. Add land -- avoid annoyance.
$(C+S)(1+i)^n = R+S$

4. Lump sum & annuity
   a. Sum of series formula.
   b. P.V. $= \frac{1}{1+i} + \cdots + \frac{1}{(1+i)^{-n}}$

$$\frac{1-r^n}{1-r} = \frac{1-(1+i)^{-n}}{\frac{1+i-1}{(1+i)}} = \frac{1-(1+i)^{-n}}{i}(1+i)$$

5. Graph treatment

6. Tax neutrality.
   a. Point case
   b. Level inv., recovery in terminal yr. case

Tues, Nov. 16, 1965

1. We reduced problem of the upper turning point to one of declining inv. demand:

a. Slower replmt., from bias to longevity

b. Slower net invt.

i. As real values swell, less desire to convert cash into goods — requires more cash to maintain fixed proportions

ii. Less desire to create real capl.

2. Now examine what determines t.f.

a. When to cut a tree.

i. Critics of this kind of capl. thy. have objected that trees are not typical. We will also look at bldgs., & minerals.

ii. Approaches: Max.

Mgnl.

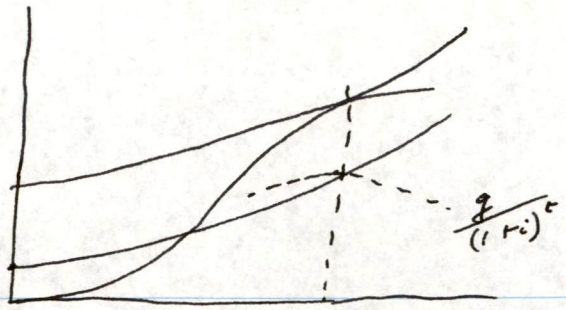
(Tangency, graphic method which combines both)

iii. Rival solutions

a. Fisher

1. $\text{Max } \frac{g}{(1+i)^t}$

Where $g' = gi$



Why does $g' = gi$ when max $\frac{g}{(1+i)^t}$?

P.V. curve is y-intercept of discount curve.

(a) Graphically obvious. Slope of int. growth curve is $\frac{\Delta y}{\Delta x} = \Delta y$. And $\Delta y = V_0(1+i)^t \cdot i$. Or, if $\frac{\Delta y}{y} = i\%$, $\frac{\Delta(1+i)^t}{(1+i)^t} = i\% = i$

(b) Set derivative of $\frac{g}{(1+i)^t} = 0$. Der. $= \frac{(1+i)^t g' - p(1+i)^t g}{(1+i)^{2t}} = 0$ when $g' = gp$

Shortcomings of solution.

(a) Excess of V_0 over C_0 not explained. Can you double p. n immedly?

(b) Growth path of value not shown

(c) Cost of land omitted. Hi or low, no effect on solution.

(d) C_0 no effect on solution

b. Boulding. also show graphically, and marginally.

Max. i_B. (a). C_0, no effect on rotn.

(b). i, " " " "

(c) No pay for sites.

c. Forest Service

Max. $\frac{g}{t}$ or $\frac{g-c}{t}$ Same as Faustman when $i = 0$

d. Faustman.

Max. $S_e = \frac{g - C_0(1+i)^t}{(1+i)^t - 1}$

or $a = i S_e$

d. Faustman, (cont.)

1. As a mgnl. solution, runs into problems, bas. ~~on~~
$g' = gi + a$, but a depends on solution.
Still possible by defining a & substituting.

$$a = i\,\frac{g - C(1+i)^t}{(1+i)^t - 1}$$

$$g' = gi + \frac{gi - C(1+i)^t i}{(1+i)^t - 1} = \frac{gi(1+i)^t - gi + gi - C(1+i)^t i}{(1+i)^t - 1}$$

$$= i(g^t - C)\frac{(1+i)^t}{(1+i)^t - 1}$$

$$\frac{g'}{g - C} = i\,\frac{1}{1 - (1+i)^{-t}} \equiv \text{capl. recovery formula}$$

~~2. As max~~ So cut when your int. yields $> i$.
But the base is net of replacement cost.
Prescribes faster rotn., but not as fast as Bldy.

2. As maxg. soln., max $a = i\,\dfrac{g - C(1+i)^t}{(1+i)^t - 1}$

Methods.

a. Derivative $= 0$

b. Simul slopes & ordinates equal. (See p. 4)

Converting lump sum to an annuity

Finding a, the annual amt of A_t

$$A = a\left[1 + (1+i) + (1+i)^2 + \dots + (1+i)^{t-1}\right]$$

$$S = \frac{1 - r^n}{1 - r} = \frac{1 - (1+i)^n}{-i} = \frac{(1+i)^n - 1}{i}$$

$$A = a\,\frac{1 - (1+i)^n}{-i} = a\,\frac{(1+i)^t - 1}{i}$$

~~$d = A \frac{i}{1-(1+i)^n}$~~

Other forms:

2. $a = A_t \dfrac{i}{(1+i)^t - 1}$ Time-average. Use tables, cf. vs. simple average.

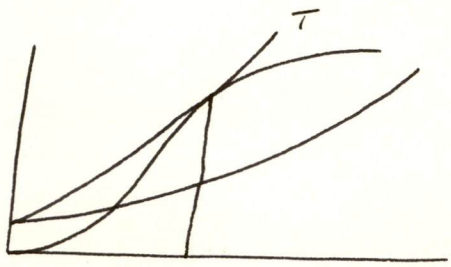
3. $a = A_0 \dfrac{i}{1 - (1+i)^{-t}}$ Capl. recovery formula

4. $A_0 = a\,\dfrac{1 - (1+i)^{-t}}{i}$ Capitalization

So Faustmann maximizes annual net value of site.

As a mgnl. solution, this works out to $g' = i(g+S)$

Show graphically?



Algebraically, slope of $\bar{T}$ curve is $\frac{\Delta}{\Delta t}$ of $\left[iC_0(1+i)^t + a\frac{(1+i)^t-1}{i}\right]$

$= (C_0+S)(1+i)^t - S$.

$\lambda = i(C_0+S)(1+i)^t = i(\bar{T}+S)$, [bcs. $\bar{T} = (C_0+S)(1+i)^t - S$]

$i(\bar{T}+S) = \bar{T}i + Si = \bar{T}i + a$

When $\bar{T} = g$, & $T' \lessgtr g$; then $g' = gi + a$

So, as a mgnl. solution, Faustmann has us earning a return on both stumpage & site.

Solves weaknesses of other approaches.

1. Growth path of invt. value is clear, along $\bar{T}$.
2. Pays all inputs – unlike Bldg., who arrogates.
   (Land rent is most susceptible of arrogation. E.g.s of KCLC; Weyerhaeuser.)
3. Sensitive to Δi

   But less so than Fisher

   They shorten rotations – change factor proportions.

4. Sensitive to ΔC.

ΔC_o lengthens rotations — sub. capl. for labor

sub. land " "

Note role of a. ΔC_o lengthens by making a smaller — not by making i lower. i remains same until unpd land goes out of use

5. Universally adaptable; reversible

Can max. anything - simply adds to the constraints

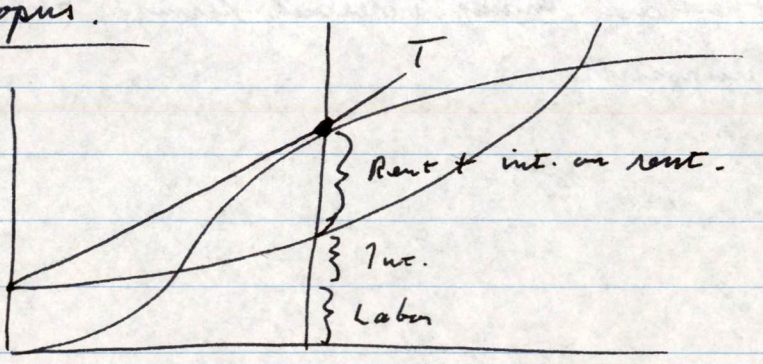
Interest rate ... "residual" is another name.

(over) $a = i \dfrac{g - C_o(1+i)^t}{(1+i)^t - 1}$ Can also solve for i, max. that. Trial & error. Not hand, by unpd. method.

$g' = gi + a = g\,i\left[1 - (1+i)^{-t}\right]$ Easy to tabulate, solve, using different i?

$\dfrac{g' - a}{g} = i$ is better.

Factor propns.



$$a\,\frac{(1+i)^t - 1}{i} - ta = \text{int. on rent}$$

Staggering.

Checkerboard graph. Long rotns. means cap.-heavy, land heavy.

How max. internal i?

Write basic relationship, solve for i.

$$V_m = C_0(1+i)^T + S[(1+i)^T - 1] = (C_0 + S)(1+i)^T - S$$

$$i = \left[\frac{V_m + S}{C_0 + S}\right]^{\frac{1}{T}} - 1$$

a. Better than Bldg. – no $i = \infty$ if $C_0 = 0$.

b. Apply to inventories, or poultry on floor space, etc.

c. Where there are many intermed. terms, becomes a little harder, but not impossible

Effect of $ wages, etc., on propns.

$ Co. longer notns.

Universal formula.

$$a = \frac{i \sum \left[R_n (1+i)^{t-u} \right]}{(1+i)^t - 1}$$

Comments on Wicksell, Value, Capl. & Rent, pp. 97-119

Laborious!

1. Opens w. an error. Defining capital, chides BB for saying M yielded interest bec fruitful objects can be bought for money. -- M is sterile, its interest-bearing power is borrowed.

✓ 2. Capl. is stored wealth.

Contrasts to land, bec. land yields a flow, undemandable in time, and stores yield optional sequence. ~~Later~~ He modifies this by defining Rent-goods, houses & esply. land improvements that become part of the land.

He errs by saying ores are like stores.

3. Smith & B-B distinguished social capital from pvt. means of acquisition. W. misses main point -- obligation vs. prodn. -- and hits them on cons. capl. vs. business capl.

But includes consumer capital with capital? No, denies it. p. 102. For half-wrong reason. Bec. subsistence held by non-consumers serves to buy productive services.

✓ 4. "Means of subsistence" is capital. Serves to hire labor, to replace it.

~~Subsistence~~ Subsistence of landowners is also capital. If prodn. takes years, they get their rents as an advance from capitalists.

Subsistence of capitalists is also capl.
(Many, e.g. Walras, miss nature of capital, treat it simply as durable goods, but not raw materials, half-finished products & subsistence. See p. 167, too. To understand capl. must regard it as subsistence, an advance to labor & land which do not maintain themselves during production.)

✓ 5. p. 105 Most useful classification of capl. is by durability.

Rent-goods — highly durable

Capital-goods (or capl. in the narrower sense) — consumable or quickly exhausted goods, prodn. goods or cons. goods, (unless in hands of final consumer).

6. B-B loves to pile up theoretical difficulties, to remove them later on. (So does W.!)

7. Interest is an agio. Exch. between present & future

8. Time-consuming methods of prodn are more productive.

9. Skip pp. 111-15

Role of capital in prodn: introduces time between beginning and end of prodn.

10. Free capital (a good term) consists of means of subsistence, or consumption goods, advanced to workers and landowners by capitalists.

It is not always on hand, but "realizable successively."

11. The sum of capital indicates the length of the period of production. Take a cross-section of prodn. at any time.

(Illustrn. Take one timber cycle. To even out inflow-outflow, stagger. Then capl. on hand at any time is area under one bell T-curve. [Interestingly, while the g has no simple formula, the T does, can integrate neatly])

(He does not here present his lengthening & deepening of capital idea, in graph. Find and use that, too.)

12. More capl. means lower % of workers in final stage of prodn. There "produce" more each (he means their gross productivity)

13. Given same labor & land, if they use more capl., pd. of prodn. must be longer and they produce more. p. 116

14. New technical advances need not be capital-using; but "in most cases" they are. (Probably an error.) E.g.s of innovations that save capl, use labor & land: pumps in hot water heating; basic oxygen technique replacing blast furnace; lighter mtrls. in homes; plastic for wood & metal; cars & tractors for horses; trailers; paper disposable napkins for cloth; constant shortening of rotations of timber, broilers, cattle, etc.; inventory control; fork lift.

Even computers are labor-intensive. Rapid obsolescence, very short life; hi op'g cost.

Freezing of capl. is from urban sprawl, public sector.

15. Why is capl. productive. He barely touches on this.

[a. Helps divn. of labor thru time -- coopn. need not be simul., but store up labor of one for later work by others.

b. Helps exchange

c. Meets shortages

d. Ecs. of scale in time -- e.g., bldg. covers more yrs., that is "scale", ~~just~~ like covering more acres. Get more output from bldg.

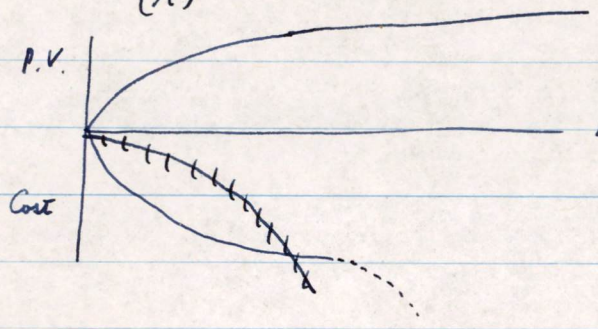
e. Etc.

16. Labor's gross product Δs w. capl., or pd. of prodn.

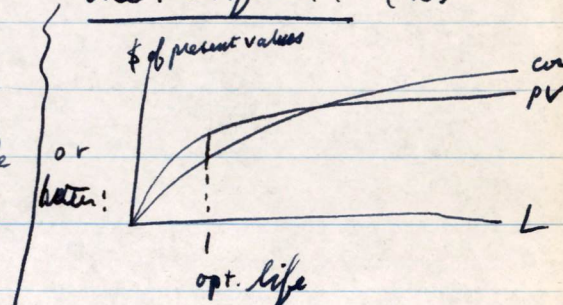
17. pp. 118-19 The sum of all, the capital which determines wages (& rent payments) is not the total of all capital values, but the free capital, (or liquescent). Cf. wages fund

(This is very obscurely written, but I think that's the thrust.)

18. (A)

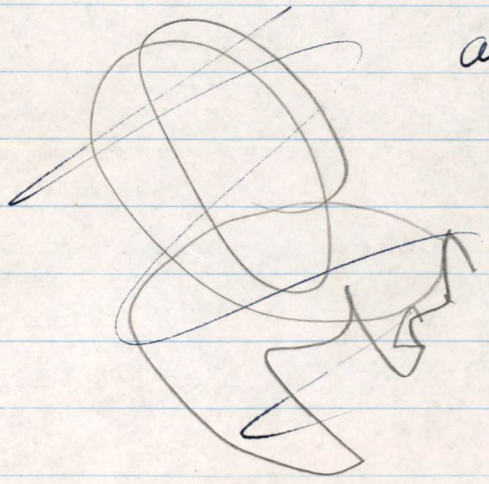


Use this form: (B)



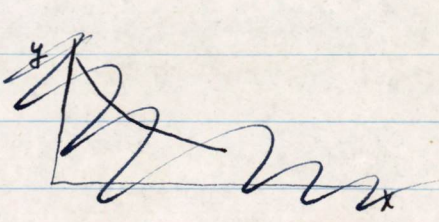
He doesn't quite say the above in a finite, but that is relevant idea.

I dont like etc. is: add longevity until MC = Δ P.V.



Algebraically: $L = C^{\frac{1}{1+x}}$ or $C = L^{\frac{1}{1+x}}$

$$P = E\,\frac{1-(1+i)^{-L}}{i}$$



End of assigned pp.

Fig. 18B might be modified to see its relation to other expressions I have used for the same thing, putting cost & value on annuity basis

PV basis is better to emphasize discounting of future revs. The annuity formula does that, but it is obscured in the shuffle. Annuity approach wd. make that clearer if I discounted before annualizing, rather than compounded before annualizing:

$$a = \sum_{0}^{m} \left[R_t (1+i)^{-t} \right] \frac{i}{1-(1+i)^{-t}}$$

Annuity approach is better bec. it emphasizes replacement qtn.

The "opt. life" in Fig 18B, p.4, is not really opt., for that reason!

to show students all 3 approaches, [18A, 18B, and annuity] to push the idea of expressing same idea in different forms; but annualizing involves another idea, after ~~on~~ discounting, and beyond simple averaging

pp. 120 ff., math model.

The "pd. of production" is so concerned.

Implicit model is a long productive process during which you produce something w. steady flow of labor, $\underline{l}$, over $\underline{t}$ years, and on completion it is consumable. (In fact, anything taking so long to produce would be durable. Approximate, tho, by saying it has a PV when complete, and that is indep. of interim rate of int., which W. calls $\underline{z}$ (z ins))

$\underline{s}$ is value of final product, from cost side.

$\underline{l}$ is yrly. wage

lt is total subsistence advanced

simple interest is $1 + tz$

" " on the mean input is $1 + \frac{tz}{2}$

$$s = tl\left(1 + \frac{tz}{2}\right)$$

[Correct formula wd. be $s = l\,\frac{(1+z)^t - 1}{z}$].

This is ~~an example~~ a formulation hard to exemplify. One can take the aggregate of final goods of any time & conceive of them as resulting from a long, steady process.

Now he seeks ~~to~~ $\underline{t}$ to max. $\underline{l}$: $\frac{dl}{dt} = 0$

$$l = \frac{s}{t}\,\frac{2}{2 + tz};$$

$\frac{dl}{dt} = 2s[$ -- etc.] Messy, and dubious. This section is both fuzzy

and doubtful.

Following this, just an occasional good page stands out:
127, 129-30, 146-53 (re rent); 167

Ec. 302. Tues. Oct. 18, '66

A. Clippings

1. "Building moves slowly"

a. Constrn. is big. $51b, '66.

Cf. net personal saving = ca. $30b

Cf. "bus. fixed inv." = ca. 75b (overlap)

Cf. "Bus. exp. on new plant & equipment" = ~~ca~~ $60b.

(of which $\frac{1}{4}$ is constrn.)

(& only $28 is mfr. & mining; the rest is utilities & commercial)

b. Note misleading habit of B & M etc. to write of machines in factories as the typical invt., & to think that way.

In fact, only c. $28b $(1 - \frac{1}{4}) = \$21$ is mfr. other than constr.

c. Constrn. is discretionary & autonomous.

d. Constrn. is hit by tight money, bec. durable. Use int. tables.

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e. Demand is not the bottleneck.

f. $13b in streets, utilities, sewer & water.

+ 21b resid.

$34b is largest single item.

+ 4 edl. etc. makes indy. & commerce look small.

A, cont

2. Corp. cash – 20 yr. decline.

Liq. dropping, V rising. "harder working M".

Liq. " rel. to sales, & even interest to liabs.

Liq. is > M

M dropping fast, from 60% to 30% of liq. assets.

3. Banks liq. low.

Cash inflow from loan repayment depends on ave. maturity of loans.

Mortgages 4d, repayments slowed down, hence tight money.

Ratio of loans to deposits rising – fast. Banks sold govt. bonds to make loans. Low liq.

4. Graph, "Climbing Cost of Money".

The problem they got into –

For years, could lower i by ΔM.

Now, antic. inflm. + slow repayments on long maturities caused Δi. As inflm. began to get out of hand, tightened up hard.

5. Top execs urge tax hike now.

They think it will stop spending – but it won't.

Chaps 7, 8, 9.

Capital theory.

1. How to find r.

Simple cases.

a. $C_0 (1+r)^t = T_t$

b. Perpet. $r = \frac{a}{C_0}$

c. If $C_0 = \frac{a}{r}\left[1 - (1+r)^{-t}\right]$

looks impossible, but easy w. tables.

$$\frac{C_0}{a} = \frac{1 - (1+r)^{-t}}{r}$$

If $\frac{C_0}{a} = 10$, & $t = 21$, $r = 8\%$

2. Overhead & land costs.

Often overlooked, understated, exaggerates return

C. Brown.

p. 526

a. Notes that expensing "neutralizes" "effect on inv. incentives", but fails to note that it fails to collect taxes. (later he does.)

b. Under ec. life deprecn, incentives to invest are more hurt ~~for~~ the longer-lived the asset.

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p. 527 For rate of discount, uses rate entr. must pay for funds. (i.e. qual., that corresponds to r after taxes.) But assumes the tax does not reduce supply price of capl., borrowed or internal. (Thus in effect assumes full shifting.

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p. 529 C. Brown says he makes it clear why tax reduces inv. incentives: bec. PV of Σd_x < C.t.

But that is to say the reduced incentive stems from there being a tax!

Bec. if PV of Σd_x = C.t (expensing) there is no tax.

C. Brown gets bias to brevity from ~~need to~~ advantage of ~~accel~~ earlier deprn. ✓
He overlooks disadv. of earlier tax payments.

S. McD. gets bias to brevity from easier shifting of taxes on shorter investments. But overlooks that shifting itself betrays withdrawal of resources from shorter investments. also that there is less need to shift the longer ones, bec. r has been less reduced by the tax.

p. 529 "Shorter-lived assets would move up the scale (of profitability) relative to the longer-lived." Because "the longer the tax rebate is postponed ..., the smaller is its present worth." So he's 180° wrong, and has confused the issue badly.

Leg. Replacement.

M. A. Copeland, Our Free Enterprise Economy
pp. 181-209

Cited by Adams & Dirlam, AER p. 165, 3-66

Inc. tax & longevity

$$i = \left[\frac{R_x}{C_0}\right]^{\frac{1}{x}} - 1$$

$$\Phi_x = \sum_0^x \left[R_n (1+r)^{x-n}\right]$$

after tax,

$$\Phi_x = \sum \left[\left[R_n (1-T) + \left(\frac{C_0}{x}\right)_n T\right](1+r)^{x-n}\right]$$

$$r = \left[\frac{\Phi_x}{C_0}\right]^{\frac{1}{x}} - 1$$

$$C_0 = \Phi (1+r)^{-x} = \sum_0^x \left[\left[R_n (1-T) + \left(\frac{C}{x}\right)_n T\right](1+r)^{-n}\right]$$

Let $R_1 = R_2 = R_n = \dots = R_x$

$$C_0 = \left[R(1-T) + \frac{CT}{x}\right] \frac{1-(1+r)^{-x}}{r}$$

Before tax, $\left[\frac{\Phi_x}{C_0}\right]^{\frac{1}{x}} = \left[\frac{\Phi_y}{C_0}\right]^{\frac{1}{y}}$

$$y \ln\left[\frac{\Phi_x}{C}\right] = x \ln\left[\frac{\Phi_y}{C}\right]$$

$$\frac{\ln \frac{\Phi_x}{C}}{\ln \frac{\Phi_y}{C}} = \frac{x}{y}$$

Let $k \equiv$ yr. of write-off

$$(1+r)^n = (1+i)^n(1-t) + t_k(1+r)^{n-k}$$

If $k = 0$

$$(1+r)^n(1-t) = (1+i)^n(1-t)$$

$$r = i$$

In general, n is life, $k \equiv$ yr. of write-off:

$$(1+r)^n\left(1 - t(1+r)^{-k}\right) = (1+i)^n(1-t)$$

$$(1+r)^n = (1+i)^n \frac{(1-t)}{1 - t(1+r)^{-k}}$$

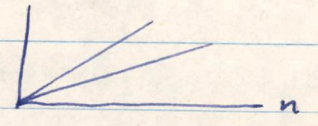
$r \propto \frac{1}{k}$ all the way, given n.

Now let $k = \frac{n}{2}$, how does r vary w. n?

$$1+r = (1+i)(1-t)^{\frac{1}{n}}\left[1 - t(1+r)^{-\frac{n}{2}}\right]^{-\frac{1}{n}}$$

or

$$\frac{(1+r)^n}{(1+i)^n} = \frac{1-t}{1 - t(1+r)^{-\frac{n}{2}}} \qquad \propto \frac{1}{n}$$

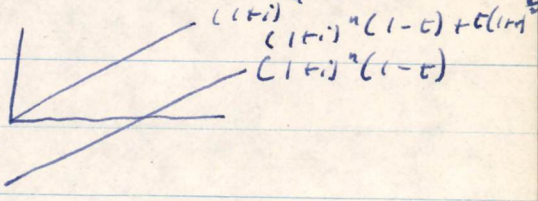


1	1
2	4
4	16
8	64
16	
32	

or

$$(1+r)^n - t(1+r)^{\frac{n}{2}} = (1+i)^n(1-t)$$

$$(1+r)^n = (1+i)^n - t\left[(1+i)^n - (1+r)^{\frac{n}{2}}\right]$$



$(1+r)^n = ((1+i)^n(1-t) + t = (1+i)^n - t[(1+i)^n - 1]$

Tax Base

n	$(1+i)^n - 1$	$\left[(1+i)^n - (1+r)^{\frac{n}{2}}\right]$	$(1+i)^n(1-t) + t(1+r)^{\frac{1}{2}}$
0	0	0	1
1	i	$i - \frac{1}{2}r$	$\frac{1}{2}r$
2	$2i + i^2$	$2i + i^2 - r$	
3	$3i + 3i^2 + i^3$	$3i + 3i^2 + i^3 - \frac{3}{2}r$ etc.	
4	$4i$ --	$4i$ .... $-2r$ ...	
8	$8i$ --	$8i$ ... $-4r$ ---	
∞	∞	$\infty - \infty$	

So it needn't approach lower asymptote

Compare one long inv. w. several short ones of same total length, e.g. 10 yrs. There seems a very slight comp. int. advantage in writing off 5 2 yr. invts. in one yr. each; but they are almost the same.

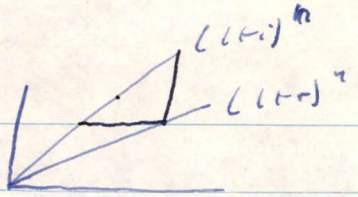
If r is to remain const. w. n, then $\frac{\log(1+r)^n}{\log(1+i)^n}$ must remain const. So solve for ratio of $\log$'s & see if varies w. n.

$\frac{n\log(1+r)}{n\log(1+i)} = -n\log(1+i) = \log(1-t) - \log\left[1 - t(1+r)^{-\frac{n}{2}}\right]$

Sorry, can't do it.

Promising Approach Same day.

Try to express $(1+r)^n$ as fn of $(1+i)^n$;
if r is to remain const.



$$\text{Log}(1+r)^n = \cancel{\frac{q}{b}} \log(1+i)^n = \log(1+i)^{\frac{q}{b}n}$$

$$(1+r)^n \doteq \cancel{k}\,(1+i)^{\frac{q}{b}n} \qquad \frac{q}{b} < 1$$

Condition of constant r is that $\dfrac{(1+r)^n}{(1+i)^n} = \dfrac{(1+i)^{\frac{q}{b}n}}{(1+i)^n}$

Does any $=n$ meet the condition?

i.e.

$$(1+r)^n \stackrel{?}{\doteq} (1+i)^{\frac{q}{b}n}, \quad \left(\cancel{\frac{q}{b}} < 1\right)$$

$$1+r \stackrel{?}{\doteq} (1+i)^{\frac{q}{b}}$$

But $(1+r)^n \doteq (1+i)^n(1-t) + t(1+r)^{\frac{n}{2}}$

$$(1+i)^{\frac{q}{b}n} \stackrel{?}{\doteq} (1+i)^n(1-t) + t(1+i)^{\frac{q}{b}\frac{n}{2}}$$

~~$(1+i)^{\frac{q}{b}} \stackrel{?}{\doteq} (1-t) + t(1+i)^{\frac{q}{2}}$~~

$$1 \stackrel{?}{\doteq} (1+i)^{n-\frac{q}{b}n}(1-t) + t(1+i)^{\frac{q}{b}n\left(\frac{1}{2}-1\right)}$$

$$\frac{a^{xy}}{a^y} = a^{xy}\cdot a^{-y}$$

$$= a^{xy-y} = a^{y(x-1)}$$

$$1 - t(1+i)^{-\frac{q}{b}n} \stackrel{?}{\doteq} (1+i)^{n(1-\frac{q}{b})}(1-t)$$

$$\frac{t(1+i)^{\frac{q}{b}\frac{n}{2}} - 1}{t(1+i)^{\frac{q}{b}\frac{n}{2}}} \stackrel{?}{\doteq} (1+i)^{n(1-\frac{q}{b})}(1-t)$$

$$t(1+i)^{\frac{q}{b}\frac{n}{2}} - 1 \stackrel{?}{\doteq} t(1-t)(1+i)^{\frac{q}{b}n\left(1-\frac{q}{2}\right)}$$

$$\left[W_i - T\right]^{\frac{j}{i}} = W_j - T$$

$$W_j = W_i^{\frac{j}{i}}$$

Insoluble for T?

$T + y$:

$$\left[W_i + t\right]^{\frac{j}{i}} = W_j + t = W_i^{\frac{j}{i}} + t$$

$$t^{\frac{j}{i}} = t$$

There is no solution except $t = 1$

$$\text{Log } X^2 = 2 \log X$$

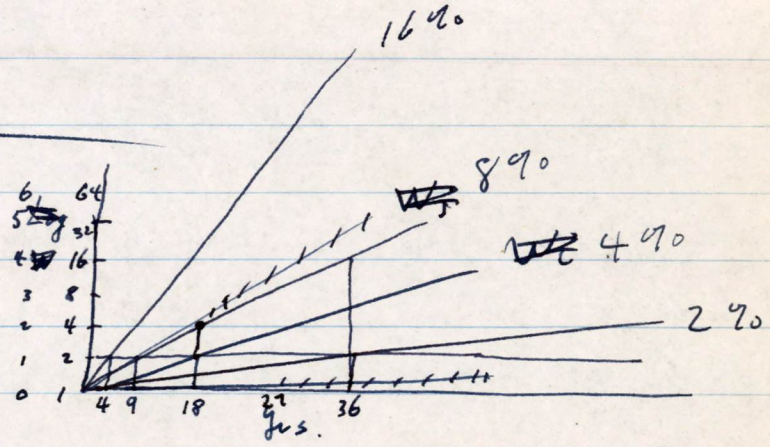
$$\text{Log}_e X^2 = 2y$$

$$e^y = X$$

$$e^{2y} = X^2$$

$$W_j = W_i^{\frac{j}{i}}$$

$$\log W_j = \frac{j}{i} \log W_i$$



a tax that -Δs $\log W_j$ & $\log W_i$ the same is neutral.

a tax that -Δs W_i & W_j by same amt obviously is not neutral — it lowers W_j by a smaller %, biases to future.

a tax that lowers W_j & W_i by same % is still not neutral, bcs. it lowers $\log W_i$ by a larger % than $\log W_j$

a neutral tax maintain ratio of the logs of W. Anything less encourages

longevity.

~~$[\text{Log } W_i][1-t] = \sum_j [\text{Log } W_i](1-t)$~~

$$\frac{\text{Log } W_9}{\text{Log } W_{18}} = \frac{1}{2}$$

$$\frac{\text{Log } R_{18}}{\text{Log } R_9} = \frac{18}{9} = 2$$

$$\frac{\text{Log}[R_{18}(1-t)]}{\text{Log}[R_9(1-t)]} = 2$$

$$\text{Log}(ab) = \log a + \log b$$

$$\frac{\text{Log } R_{18} + \text{Log}(1-t)}{\text{Log } R_9 + \text{Log}(1-t)} = 2$$

$$\frac{\text{Log}\,[R_s(1-t)]}{\text{Log}\,[R_r(1-t)]} = \frac{s}{r} \quad (=2)$$

$$\frac{\text{Log}\,R_s + \text{Log}\,(1-t)}{\text{Log}\,R_r + \text{Log}\,(1-t)} = \frac{s}{r}$$

$$\left(\frac{\text{Log}\,R_s}{\text{Log}\,R_r} = \frac{s}{r}\right)$$

$$= \text{Log}\,R_s$$

$$\text{Log}\,R_s + \text{Log}\,(1-t) = \frac{s}{r}\,\text{Log}\,R_r + \frac{s}{r}\,\text{Log}\,(1-t)$$

$$\text{Log}\,(1-t) = \frac{s}{r}\,\text{Log}\,(1-t)$$

$$s = r$$

$$\text{Log}\left[R_s\left(1-(1-t)^{\frac{1}{s}}\right)\right]$$

$$\text{Log}\,R_s\left[1-\left[(1+t)^s-1\right]\right]$$

$$\text{Log}\,R_s^{\frac{1}{\text{Log}}}$$

Send to: Bank:

1. another card of change of officers
2. paper of authorization

$$\frac{\text{Log } R_s}{\text{Log } R_r} = \frac{s}{r}$$

$$\frac{\text{Log } R_s^{(1-t)}}{\text{Log } R_r^{(1-t)}} = \frac{(1-t)\ \text{Log } R_s}{(1-t)\ \text{Log } R_r} = \frac{\text{Log } R_s}{\text{Log } R_r}$$

$$-t\frac{(1+r)^{\ell}-(1+i)^{\ell}}{(r-i)(1+i)^{\ell}(1+r)^{\ell}} = t(1-t)\frac{1-(1+r)^{-\ell}}{r} - \frac{1}{a}$$

$$\boxed{\frac{1-(1+i)^{-\ell}}{i} = (1-t)\frac{1-(1+r)^{-\ell}}{r} + t\frac{(1+r)^{\ell}-(1+i)^{\ell}}{(r-i)(1+i)^{\ell}(1+r)^{\ell}}}$$

When $\ell = 1$, $\frac{1-(1+i)^{-1}}{i} = \frac{1-(1+r)^{-1}}{i} + t\left(\frac{(1+i)^{-1}-(1+r)^{-1}}{r-i}\right)$

$$-1 = t-1 + t\frac{r-i}{(r-i)(1+i)(1+r)} = t\frac{1}{(1+i)(1+r)}$$

$$-t = \frac{t}{(1+i)(1+r)}$$

$$\boxed{r = i(1-t)} \quad -i$$

$$r - ri - i$$

$$1 = 1 + i + r + ri$$

$$1-(1+r)^{-1} = 1-(1+r)^{-1}$$

$$\frac{1}{1+i} = (1-t)\frac{1}{1+r} + t\frac{r-i}{(r-i)(1+i)(1+r)} \qquad + (1+r)^{-1} + (1+i)^{-1}$$

$$1+r = (1-t)(1+i) + t = 1+i-t-ti+t$$

$$r = i - ti = i(1-t)$$