

Declining Cash Flow Model

Declining cash flow model
To find opt. replacement time.

$$i = .04$$

$$d = .25$$

11

$$ccf = \text{current cash flow} = 100 - B(1+d)^n$$

~~using~~ $\left[\sum_{i=1}^n \left(\frac{1+d}{1+i} \right)^n \right]$ is a little tricky, when $d > i$. Because

then $\frac{1+d}{1+i} > 1$, and the calculator formulae get distorted,

if you use $i-d$, or $\frac{1+d}{1+i} - 1$, for the I.R. in calc. formulae.

Instead, use this: $\frac{1+d}{1+i} = \frac{1}{1-x}$

$$1-x = \frac{1+i}{1+d}$$

$$x = 1 - \frac{1+i}{1+d} = \frac{d-i}{1+d} = \frac{.25 - .04}{1.25} = .168$$

Terms:

ccf = current cash flow

"100" is a parameter, rep's the positive or gross c.f.

It can be ~~lowered~~ changed.

- In tot's, showing higher rent, or K.

- Stepping down over time

- Declining over time, using another ~~denom~~ for. of n

B is a parameter for current expenses

d " " " " growth of current expenses.

In the present simple form, acct results entirely from rise of $B(1+d)^n$.

But to compare hi + low i.g., raise + lower "100" and $B + d$ together, w.r.t. K_0 .)

K = cap. cost, beginning of yr. 1, (same as t_0)

D_{cct} = Discounted cct

$$a = \text{net} = \text{IPF}_n \left\{ -K + \sum_1^n D_{\text{cct}} \right\}$$

$$= \text{IPF} \left\{ -K + 100 \text{ PVIF} - B \sum_1^n \left(\frac{1+d}{1+i} \right)^n \right\}$$

$$= 100 - \text{IPF} \left\{ K + B \sum_1^n \left(\frac{1+d}{1+i} \right)^n \right\}$$

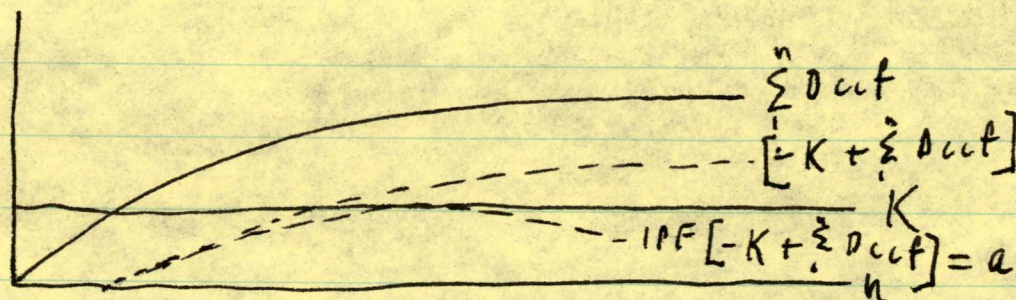
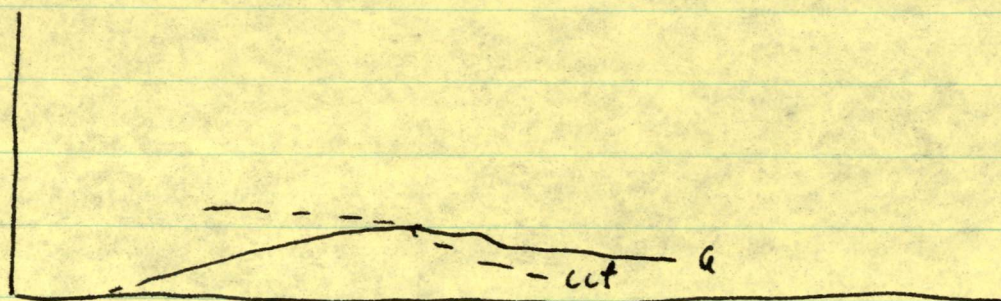


Table 1: $i = .04$ $B = 10$ $d = .25$ $K = 300$

$$cct = 100 - 10 \times 1.25^n$$

$$a = 100 - 1PF_n \left\{ K + 10 \sum_{i=1}^n \left(\frac{1.25}{1.04} \right)^i \right\} \quad \left(\frac{.25-.04}{1.25} = .168 \right)$$

n	$1PF$	$300 + 10 \sum$	$\frac{1PF \times (3)}{(2) \times (3)}$	$a = 100 - (4)$	cct
(1)	(2)	(3)	(4)	(5)	(6)
5	.225	389.8	87.56	12.44	69.5
6	.191	419.9	80.20	19.80	61.9
7	.167	456.1	76.17	23.83	52.3
8	.149	499.7	74.46	25.54	40.4
9	.134	552.1	73.98	26.02 ✓	25.5 ✓
10	.123	615.0	75.65	24.35	6.9
11	.114	690.6	78.73	21.27	-16.4
12	.107	781.5	83.62	16.38	
13	.100	890.7	89.07	10.93	
14	.095				

a max between 8 and 9, where $cct = a$.

cct falls too fast; so use lower d. But first, observe effect of lower K on opt. n (T-2).

Table 2: $i = .04$

$$d = .25$$

$$K = 200$$

$$B = 10$$

$$ccf = 100 - 10 \times 1.25^n$$

$\underline{n}$ (1)	$\underline{IPF}$ (2)	$\underline{200 + 10 \Sigma}$ (3)	$\underline{IPF(3)}$ (4)	$\underline{a = 100 - (4)}$ (5)	$\underline{ccf}$ (6)
4	.275	264.7			75.6
5	.225	289.8	65.2	34.8	69.5
6	.191	319.9	61.1	38.9	61.9
7	.167	356.2	59.5	40.5 ✓	52.3
8	.149	399.7	59.6	40.4	40.4 ✓
		452.1	60.6	39.4	25.5
		515.0	63.3	36.7	

By lowering K needed from 300 to 200, we lower opt. $\underline{n}$ from 9⁻ down to 7⁺.

And we raise max. rent from 26.⁰² up to 40.⁵⁰, a rise of 56%.

Table 3: $i = .04$

$$d = .15$$

$$B = 15$$

$$K = 200$$

$$ccf = 100 - 15 \times 1.15^{-n}$$

$$\frac{.15 - .04}{1.15} = .0957$$

n (1)	$1PF$ (2)	$200 + 15E$ (3)	$1PF(3)$ (4)	$100 - (4)$ (5)	ccf (6)
7	.167	360.2	60.1	39.8	60.1
8	.149	393.8	58.7	41.3	54.1
9	.134	430.8	57.7	42.3 ✓	47.2
10	.123	471.4	58.0	42.0	39.3
11	.114	517.2	59.0	41.0	30.2
12	.107	567.4			19.7
13	.100	622.8			7.7
14	.095	684.2			-6.1

In T-4, use $d = .10$ and $B = 20$. Then you'll see a flat max, but still a sharp fall of ccf thru the max.

~~In T-4~~ you could add a salvage value. But now, ~~land~~ salvage is already implicit, when you end life ~~to~~ ~~you~~ when there is ccf remaining.

Table 4; $i = .04$

$$d = .10$$

$$B = 20$$

$$K = 200$$

$$ccf = 100 - 20 \times 1.10^n$$

$$\frac{.10 - .04}{1.10} = .0545$$

n	$1PF$	$200 + 20\Sigma$	$1PF(3)$	$100 - (4)$	ccf
(1)	(2)	(3)	(4)	(5)	(6)
3	.360	267.2	96.2		
4	.275	292.2	80.4		
5	.225	318.7	71.7		
6	.191	346.7	66.2		
7	.167	376.3	62.8		
8	.149	417.6	60.7		
9	.134	440.7	59.05	40.95	52.8
10	.123	475.7	58.51	41.49	48.1
11	.114	512.8	58.46	41.54 ✓	42.9
12	.107	552.0	59.06	40.94	37.2
13	.100	593.4	59.34	40.7	31.0
14					24.1

Here we kept rent around 42, but raised it from 9 to 11 by flattening out ccf .

$E \equiv$ Expenses. Here we accepted higher earlier expenses to achieve lower later E . This is a kind of investment. Cfg. T-4 to T-3, in year 11 T-4 has a lower $PV E$, hence a higher PV overall. But T-3 has higher rent, by ending in year 9.

~~888-4181~~

T-4, (cont.) The inv. ^{in added life} that looks good on a PV. basis is bad on a net basis; and cap. is recovered faster.

The example shows the principle, but is very tight, & close. For a clearer eg., raise K to lower d (T-5 and T-6)

Table 5: Low K and high d .

$$i = .04, \quad d = .25, \quad K = 100, \quad B = 10$$

$$ccf = 100 - 10 \times 1.25^n$$

$$\left(\frac{.25 - .04}{1.25} = .168 \right)$$

n (1)	IPF (2)	$100 + 10 \Sigma$ (3)	$IPF \times (3)$ (4)	$a = 100 - (4)$ (5)	ccf (6)
5	.225	189.8	42.71		69.5
6	.191	219.9	42.00	58.00 ✓	61.9
7	.167	256.1	42.77		52.3
8	.149	299.7	44.66		40.4

Table 6: Raise K to lower d

$i = .04, d = .10, K = 300, \beta = 10$

$ccf = 100 - 10 \times 1.1^n$

$\left(\frac{.10 - .04}{1.1} = .0545 \right)$

<u>n</u> (1)	<u>IPF</u> (2)	<u>300 + 10Σ</u> (3)	<u>IPF (3)</u> (4)	<u>a = 100 - (4)</u> (5)	<u>ccf</u> (6)
12	.107	476.0	50.9		
13	.100	496.7	49.7		
14	.095	518.6	49.3		
15	.090	541.8	48.8		
16	.086	566.3	48.7		54.1
17	.082	592.2	48.6	51.4 ✓	49.5
18	.079	619.7	48.9		44.4
19					
20					

This invt. does not pay off, on a rent basis.
Cf. PV basis.

T-6: $NPV = -300 - 10 \Sigma + 100 \frac{1 - 1.04^{-17}}{.04} = -592.2 + 1216.6 = 624$

T-5: $NPV = -100 - 10 \Sigma + 100 \frac{1 - 1.04^{-6}}{.04} = -219.9 + 524.2 = 304.3$

On the simple PV basis, the T-6 invt. seems

better than T-5.

$$\Delta K = 300 - 100 = 200$$

$$\begin{aligned}\Delta Decf &= (1216.6 - 292.2) - (524.2 - 119.9) \\ &= 924.4 - 404.3 \\ &= 520.1\end{aligned}$$

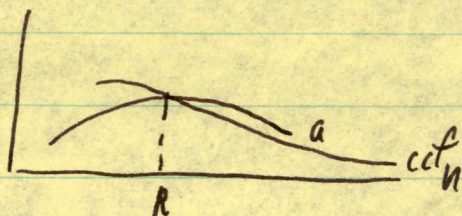
$$\frac{\Delta Decf}{\Delta K} = \frac{520.1}{200} \text{ looks good, superficially. But}$$

it's not the best you can do, $\therefore$ not good.

T-5 also pays out in 6 yrs, vs. 17. We may assume (from prior experience w. formulae) that in equilib., the end of life is also the time of 100% payback. So the effect of rent is to speed payback.

Effect of rent constraint on invs.

1. Cuts off unqual. invt. in longevity, after max $\underline{a}$.



2. Encourages invs. that raise $\underline{a}$ before $n = R$.

Effect on cap. recovery?

Rent, like int., is a prior claim. But with a difference! When expected ccf falls, that reduces PV and repr. a loss of K , reducing future recovery. But it also repr. loss of LV , reducing future rent, w. no loss of K recovery. So how is loss divided between PVK and PVL ? The issue becomes consequential.

$$kr \equiv \text{capl. recovery} = ccf - K_n i - a$$

Table 5A, showing payback. $a = 58.00$

n (1)	cct (2)	K_{n-1} (3)	$.04 K_{n-1}$ (4)	$\frac{cct - a - K_i}{(5) = (2) - 58.00 - (4)}$
1	87.5	100	4.00	25.50
2	84.4	74.50	2.98	23.42
3	80.5	51.08	2.04	20.46
4	75.6	30.62	1.22	16.38
5	69.5	14.24	.57	10.93
6	61.9	3.31	.13	3.77
7	52.3	-.46		

70% of payback is done in 3 yrs.
100% in 6 yrs.

cf. series of ^{cars} ~~trucks~~ of diff. costs + lives.
 For simple model, one ^{net} service flow. Then the lower
 K: IPF is always best

Car A. Shorter life, lower K

$$K_A = 7k$$

$$n_A = 5 \text{ yrs}$$

$$gs_A = 3k/\text{yr}$$

$$e_A = .6k/\text{yr}$$

$$ns_A = gs - e = 2.4k$$

$$a_A = -7 \times IPF^5 + 2.4 = -1.772 + 2.4 = .628$$

Car B

$$K_B = 11k$$

$$n = 8 \text{ yrs}$$

$$ns = 2.4k$$

$$a_B = -11 \times IPF^8 + 2.4 = -11 \times .187 + 2.4 = -2.062 + 2.4 = .338$$

IPF at 10%

$\frac{n}{5}$	$\frac{1.10^n}{1.21}$	$\approx 2 - .18 = 1.82$	$\frac{IPF}{.10}$	$\frac{.10 \times 2}{2 - 1.21}$
			$\frac{1 - \frac{1}{1.21}}{.10}$	
		IPF = .264		$\frac{.2}{.79} = .253$

Add to model:

1. "Fallacy" of Maytag repairman — investing in lower expenses.

2. Determining opt. u for ea. case, w. falling u_s

3. Rising u_s -- show it tipping balance of advantage to con A

$$i = .10$$

an from 5 to 6

$$\frac{\Delta P}{P} = \frac{4.355 - 3.791}{3.791} = .149$$

Thus, adding 20% to α justifies adding 14.9% to K ,
 L remaining const., this adds less than 14.9% to $(K+L)$
 But you are adding 20% to land-time.

α	IPF	ΔIPF	$\frac{\Delta IPF}{IPF_{n-1}}$
5	.264	-	
6	.230	-.034	-.129

Thus, adding 20% to α justifies adding 12.9% to K .
 But $\text{net} \equiv a = gcf - e - K \cdot IPF$
 rent

$$\text{Rent} \equiv a = ccf - K \cdot IPF$$

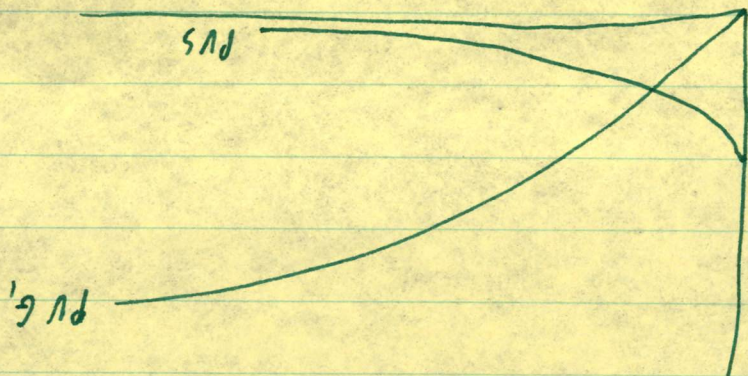
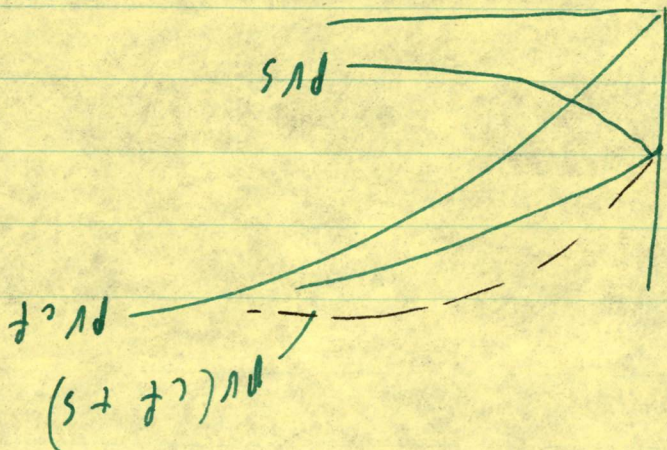
If you add just 12.9% to K , α remaining const.
 But if you reduce $K \cdot IPF$ at all, there is
 leverage, and a higher % Δa

$$= \frac{1}{ccf + I^n(L^n - 1)}$$

$$L^n + K' = ccf \frac{1}{I^n - 1} + L^n I^n = ccf \frac{1}{I^n - 1} + L^n I^n$$

if $\frac{1}{f} = \frac{1}{a}$ but $\frac{1}{a}$ above it, then sum, it is

$$\frac{1}{a} = \frac{1}{a(1 - I^n + I^n)} = \frac{1}{a} + \frac{1}{a(I^n - 1)}$$



$$\Delta u_n = \rho \frac{1}{d} \frac{1 - \frac{1}{d}}{1 - \frac{1}{d^n}}$$

$$u = \frac{1}{d} \frac{1 - \frac{1}{d}}{1 - \frac{1}{d^n}}$$

$$\frac{\Delta u}{\Delta u_n} = \frac{\frac{1}{d} \frac{1 - \frac{1}{d}}{1 - \frac{1}{d^n}}}{\frac{1}{d} \frac{1 - \frac{1}{d}}{1 - \frac{1}{d^n}}} = \frac{[1 - \frac{1}{d^n}]}{[1 - \frac{1}{d^n}]} = \frac{1 - \frac{1}{d^n}}{1 - \frac{1}{d^n}}$$

$$\Delta u_n = \frac{1}{d} \frac{1 - \frac{1}{d}}{1 - \frac{1}{d^n}}$$

$$\rho = \frac{1}{d} \frac{1 - \frac{1}{d}}{1 - \frac{1}{d^n}}$$

$$\frac{1 - I^{-(n+1)}}{1 - I^{-n}}$$

$$\frac{\frac{i}{1 - I^{-(n+1)}}}{\frac{i}{1 - I^{-n}}} = \frac{1 - I^{-n}}{1 - I^{-(n+1)}}$$

$$i = .10$$

<u>n</u>	<u>IPF</u>	<u>$\frac{IPF_n}{IPF_{n-1}}$</u>	<u>$1 - \frac{IPF_n}{IPF_{n-1}}$</u>
1	1.10	0	1.00
2	.576	.524	.476
3	.402	.698	.302
4	.315	.785	.215
5	.264	.837	.163
6	.230	.870	.130
7	.205	.893	.107
8	.187	.914	
9	.174	.929	
10	.163	.935	

↑
Drop in IPF, as ratio.

But $K \cdot IPF$ is only the cost side. If Revs. are level, there is leverage - so if only effect of ΔK is on n , the % drop in net will be greater than in $IPF \cdot K$.

\$\Delta K\$ warranted by \$\Delta n\$ above \$n=5\$

$$i = .10$$

$$\text{base life} = 5, \quad \text{IPF} = .264$$

<u>n</u>	<u>IPF</u>	<u>$\frac{n}{5} - 1$</u>	<u>$\frac{\text{IPF}_5}{\text{IPF}_n} - 1$</u>
5	.264		
6	.230	.20	.148
7	.205	.40	.288
8	.187	.60	.412
9	.174	.80	.517
10	.163	1.00	.620

$$K_5 \text{ IPF}_5 = K_6 \text{ IPF}_6$$

$$100 \times .264 = 26.4$$

$$162 \times .163 = 26.4$$

You can show this more directly using PV

$$K_7 = 100$$

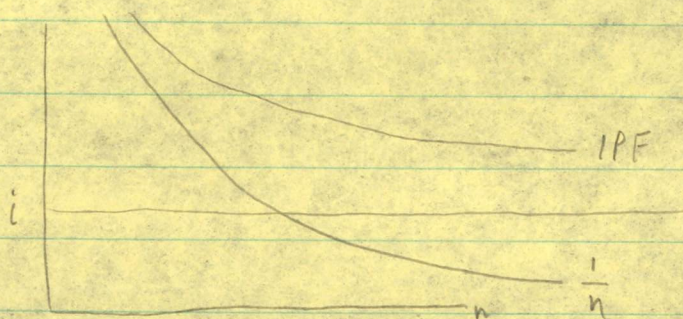
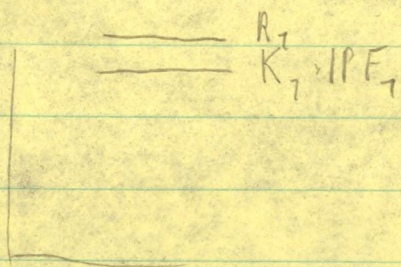
$$IPF_7 = .205$$

$$k_7 = 20,5$$

$$K_5 = 80$$

$$IPF_5 = .264$$

$$k_5 = 21,12$$



$$K = 100$$

$$i = .10$$

<u>n</u>	<u>$\Delta n/n$</u>	<u>IPF</u>	ΔIPF	$\frac{\Delta IPF}{IPF_{n-1}}$	$K \cdot IPF$	gcf	e	ncf	a
5		.264	—	—	26.4	50	20	30	3.6
6	.20	.230	-.034	-.129	23.0	50	20	30	7.0
—									
10	1.00	.163	-.101	-.384	16.3			30	13.7

x	$\frac{1}{x}$	
1.0	1.0	=

1.4	.714	=	$1 - .286 = 1 - \frac{1}{3.5}$
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$\frac{3}{2} =$	1.5	.667	=	$1 - \frac{1}{3} = \frac{2}{3} =$
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$\frac{9}{5} =$	1.8	.556	=	$1 - .444 = 1 - \frac{1}{2\frac{1}{4}} = 1 - \frac{4}{9} = \frac{5}{9}$
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<u>n</u>	<u>PV</u>	<u>ΔPV</u>	<u>$\frac{\Delta P}{P_5}$</u>	<u>IPF</u>	<u>ΔIPF</u>	<u>$\frac{\Delta IPF}{IPF_5}$</u>
5	3.791			.264		
10	6.145	2.354	.621	.163	-.101	-.384

$$P_2 = P_1 (1.621)$$

$$\frac{1}{P_2} = \frac{1}{P_1} \frac{1}{1.621} = \frac{.62}{P_1}$$

$$\frac{P_2}{P_1} = 1.621$$

$$\frac{P_1}{P_2} = \frac{1}{1.621} = .617$$

$\frac{P_2/P_1}{1.621}$	$\frac{P_1/P_2}{.617}$	$= \frac{1.621-1}{1.621} = 1 - \frac{1}{1.621}$
1.05	.952	=
2	$.5 = \frac{1}{2}$	$= \frac{2-1}{2} = 1 - \frac{1}{2}$
3	$1 - \frac{2}{3}$	$\frac{3-2}{3}$
4	$1 - \frac{3}{4}$	
5	$1 - \frac{4}{5}$	
6		

another possible ccf formula.

$$ccf = 100 - 4n$$

$$K = 300$$

$$i = .04$$

$$a = \frac{.04}{1 - 1.04^{-n}} \left\{ -300 + 100 \frac{1 - 1.04^{-n}}{.04} - \frac{4n[1 - 1.04^{-n}]}{.04} \right\}$$

$$= 100 - \frac{.04}{1 - 1.04^{-n}} \left\{ 300 + \right.$$

$$\left. \sum = \frac{-4n}{I^n} \right\}$$

$$- \sum = \frac{1}{I} + \frac{2}{I^2} + \frac{3}{I^3} + \dots + \frac{n}{I^n}$$

$$r = \frac{\frac{n+1}{n}}{I} = \frac{n+1}{nI}$$

$$S = \frac{1}{I} \frac{1 - \left(\frac{n+1}{nI}\right)^n}{1 - \frac{n+1}{nI}} = \frac{1 - (\)^n}{I - \frac{n+1}{n}} = \frac{1 - (\)^n}{1+i - 1 - \frac{1}{n}}$$

$$= \frac{1 - \left(\frac{n+1}{nI}\right)^n}{i - \frac{1}{n}}$$

$$\left| \frac{n+1}{nI} = \frac{1}{I} + \frac{1}{nI} \right.$$

another possible ccf formula_n

$$ccf = 100 - n(1.2)^n$$

($n \leq 10$) then $ccf = 20$

<u>n</u>	<u>1.2^n</u>	<u>$n 1.2^n$</u>	<u>ccf</u>
3	1.73	5.18	94.8
4			91.7
5			81.6
6			82.0
7			74.9
8			65.6
9			53.6
10			38.08
11			20
12			20
13			20

}

Discovery of "x", when $d > i$

$$\begin{array}{r} .16 \\ 12 \overline{) 2.0} \\ \underline{1.2} \\ .80 \\ \underline{72} \end{array}$$

$$1.20$$

$$\frac{1}{1-.20}$$

$$1.20 = \frac{1}{1-x}$$

$$1-x = \frac{1}{1.20} \quad \text{Eq}$$

$$x = 1 - \frac{1}{1.20} = \frac{.20}{1.20} = .16\bar{3}$$

$$100 + B(1-d)^n$$

These errors are tolerable.

$$\frac{1.1}{1.4} = \frac{.79}{1.4} = \frac{.21}{1.4}$$

$$x = \frac{d-i}{1+d} = \frac{.1-.04}{1.1} = .0545$$

$$\frac{1.1}{1.04} = 1.05769 < 1.06$$

n	$\left(\frac{1.1}{1.04}\right)^n$	$\leq$	$\frac{1.06^4}{1.06}$	$\sum$	1.06^n	$\leq$
1		1.06				
2	1.1181	2.19				
3		3.38				
4	1.27	4.65				4.68
	1.35	6.00		5.93		6.04
	1.43	7.43				7.49
	1.52	8.94				
	1.61	10.55				10.68
	1.71	12.26				12.42
10	⁹¹ 1.7522	1.7909			1.7909	14.28
	1.92	15.99		15.64		16.25
	2.04	18.03				18.35
	2.17	20.20				20.59
	2.30	22.49				
	2.44	24.93				
	2.59	27.52		26.63		
20		39.55		37.93		40.78

$D = 1 + i + d$
 Program "Max G"

$$\max G = \frac{b}{i} = \frac{-K + d \frac{1 - D^{-n}}{i + d}}{1 - I^{-n}} \\ 1 - \beta(I - p)^{-n} \\ 1 - \left(\frac{1+p}{1+i}\right)^{sn}$$

STO

K 11

a = 1

i 12

d 13

n 14

LBL 99

upmu $\frac{p}{s} = p$ 15

s 16

n G n

14 5.6

15 10 ~~1.104~~

16 6.1

18 6.4

26 6.8576

27 6.86855 20 - .5

28 6.8681

29 6.8665

30 6.86

35 6.8

40 6.7

I cleared out this program, 24 Apr 86

"REPLACE"

$$\frac{(1+i)^n - 1}{i+d} \quad ||| \quad \frac{1 - (1+i+d)^{-n}}{i+d} - \frac{K}{a}$$

$$\frac{[(1+i)^n - 1] / (1+i+d)^n}{\frac{1 - (1+i+d)^{-n}}{i+d} - \frac{K}{a}} = i$$

$$D \equiv 1+i+d$$

$$\frac{I^n - 1}{i} = \frac{D^n - 1}{i+d} - \frac{K}{a}$$

$$\frac{K}{a} = \frac{D^n - 1}{i+d} - \frac{I^n - 1}{i}$$

$$a = \frac{K}{\frac{D^n - 1}{i+d} - \frac{I^n - 1}{i}}$$

Program

G Max

$$G = \frac{-K + \frac{1 - I^{-x}}{i} + I^{-x} \frac{1 - D^{-(n-x)}}{i+d}}{1 - Q \left(\frac{1+g}{1+d} \right)^{n-x} \times I^{-n}}$$

~~$x = K$~~

$$cct = \frac{1}{(1+d)^{n-x}}$$

N?: Enter ~~100~~ X, 100

To find N: RCL 14 - 1.1

a = Gi = cct at max *

STO

K 11

i 12

d 13

n 14

g 15

s 16

I^{-x} 00

x 10

RCL Y 01

Q 17

* Bug when $d = .20$, $K = 3$, $x = 5$

$$I^{-k} = .61 \quad K = 5 \quad i = .05 \quad d = .08 \quad f = 0 \quad s = 0 \quad x = 10$$

~~10~~ ~~x=10~~

11 $x = 10$

$K = 3$

$x = 5$

$d = .20$

$K = 3$

$x = 5$

$d = .08$

10 7.05

4.08

4.13

11 7.86

12 8.45

4.08 8.59 .28

13 8.88 9.21

10.84

4.13
4.17

14 9.19 9.42 13.23

15 9.56 13.26

6.90

16 9.55 9.65 13.24

6.99

4.05

9.70 13.19

18 9.70 9.72 13.12

7.06

3.92

14 9.72 9.71 13.03

20 9.71 9.69

7.05

22 9.66

25 9.49

30 9.15

1. $K = 3$ $d = 0$ $g = .04$ $x = 1$
 $i = .05$ $n = 16$ $s = 0$
 $G = 55.21$ $Q = 1$
 $iG = 2.76$
 $(1+g)^n = 1.87$

2. $Q = 1.5$
 $g = 0$
 $n = 16$
 $G = 26.50$

3. $s = 0$ $g = 0$ $x = 1$ $d = 0$
 $Q = 1.5$
 $n = 9$
 $G = 47.15$ 124.15

4 Same as #1, $g = .03$
 $G = 29.70$
 $n = 18$

5 Same as #1,
~~1 $f = .02$~~ $g = .02$
 $G = 21.55$
 $n = 23$

This result is
 counter-intuitive!

6. Same as 5; $Q = 1.2$
 $G = 136.7$
 $u = 7$

7. Same, $Q = 1.1$
 $G = 33.02$
 $u = 3$

8. Same, $Q = 1.3$
 $G = 177$
 $u = 10$

So u rises w. Q ; also w. g .

Reason for this counter-intuitive result: G_2 is endogenous — i.e. G_2 rises w. G_1 . By stretching out u_1 , you raise G_1 and thereby raise G_2 (in my formula).

If G_2 is exogenous, this finding does not hold, but is reversed.

What is intuitive is that $Q > 1$ results in shorter u than $g > 0$.

$$9. \quad Q = 1.1$$

$$g = .02$$

$$s = 10$$

$$K = 3$$

$$i = .05$$

$$d = 0$$

$$x = 1$$

$$G = \cancel{8.59} \quad 19.91$$

$$n = \cancel{6} \quad 32$$

$$10. \quad \text{Same, } s = 5$$

$$G = 21.47$$

$$n = 23$$

$$11. \quad \text{Same, } s = -5$$

$$G = 325$$

$$n = 7$$

$$12. \quad K = 3$$

$$i = .05$$

$$Q = 1.0$$

$$g = 0$$

$$s = 0$$

$$d = .08$$

$$x = 0$$

$$G = \cancel{2} + \cancel{2.64} = 6.67$$

$$n = 14$$

$$\frac{-3 + \cancel{.95} + \cancel{.95} + 6.30}{1 - .51} = 6.67$$

$$13. \text{ Same, } d = .16$$

$$G = 2.90$$

$$n = 13$$

$$14. \text{ Same, } d = .02$$

$$G = 12.28$$

$$n = 25$$

$$15. \text{ Same, } d = .01$$

$$G = 14.05$$

$$n = 37$$

$$16. \text{ Same, } x = 10$$

$$G = 15.36$$

$$n = 37$$

$$17. \text{ Same, } x = 10, d = .16$$

$$G = 12.76$$

$$n = 13$$

$$18. \quad K = 5$$

$$\bar{r} = .05$$

$$d = .01$$

$$x = 5$$

$$f = 0$$

$$s = 0$$

$$Q = 1$$

$$G = 12.54$$

$$n = 54$$

19. Anne: $K = 7$

$$G = 10.4$$

$$u = 73$$

20. Anne: $K = 9$

$$G = 5.28$$

$$d = .03$$

$$u = 52$$

21. Anne $K = \cancel{10}$

$$G = 6.62$$

$$u = 49$$

22. Anne, $K = 15$

$$G = 7.65$$

$$u = 49$$

- 10 $X = 5$
- 11 $K = 3$
- 12 $i = .05$
- 13 $d = .2$
- 14 $n = \cancel{009.100} \quad n = 10$
- 15 $g = 0$
- 16 $s = 0$
- 00 $1.05^{-5} = .78$
- 01 $RCHY = 8.90$

$$G_{Max} = 8.95$$

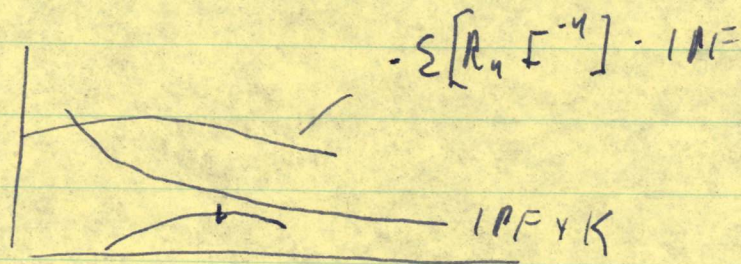
$$n = 9$$

$$ccf = 1.2^{-4} = .48 = .05 \times \underline{8.95}$$

Bldg. ^{is to} last 10 yrs.
 ΔK will add one, on 11th yr. ~~for Δa in any last yr.~~
 Δa_{11} ~~must~~ must come int. on ΔK , and also T_i is 11th yr.

$$\Delta K \leq (a_{11} - T_i) I^{-1}$$

Δa_{10} doesn't need any more land-time. But there are dim. rets. to ΔK on fixed land-time. "As to longevity" come from spreading the ~~land~~ K cost over more yrs.



"As to longevity" is a by-product - hence dimension of the term.

Ec life

$$i = .07$$

$$d = .05$$

$$K/a = \cancel{5}$$

<u>x</u>	<u>n</u>	<u>G</u>	<u>Gi</u>	<u>$1.05^{-(n-x)}$</u>
0	30	3.52	.25	.23
5	28	5.42	.38	.33
10	26	6.73	.47	.46
15	28	7.58	.53	.53
20	32	8.13	.57	.56
40	50	9.02	.63	.61

Ec. life

$$\bar{i} = .07$$

$$X = 10$$

$$d = .05$$

<u>K/a</u>	<u>n</u>	<u>G</u>	<u>Gi</u>	<u>$1.05^{-(n-10)}$</u>
1.2	13	12.0	.84	.86
1.5	14	11.5	.81	.82
2	16	10.8	.75	.75
3	19	9.3	.65	.64
5	26	6.7	.47	.46
7	36	4.4	.31	.28

Ec. life

$$K/a = 5$$

$$i = .09$$

$$d = .05$$

$$x = 10$$

Reg

11

12

13

~~14~~

i	n	G	Gi	$1.05^{-(n-10)}$
.01	19	61.7	.62	.64
.03	21 19	19.1 29.4	.57 .57	.58
.05	23	10.5	.53	.53
.07	26	6.73	.47	.46
.09	29	4.56	.41	.40
.11	34	3.12	.34	.31
.13	40	2.07	.27	.23
.15	49	1.26	.19	.15
.17	64	.60	.10	.07

} Note inaccuracy over $i = .11$

Counterintuitive result! Higher i causes longer lives!

But higher i implies lower K/a , offsetting.