



'Time-Averaging' Model, to Determine Life of Capital

By Dr. Mason Gaffney

Article citation:

Gaffney, Mason. "Time-Averaging' Model, to Determine Life of Capital." Econ 162A, October 27, 1982. Study guide.

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"TIME-AVERAGING" MODEL, TO DETERMINE LIFE OF CAPITAL

The premise is that the capital yields a diminishing cash flow over time, due to depreciation and obsolescence. (I failed to make this clear on Tuesday, hence Yukoh and Steve and Chassan very rightly questioned the model.) If the flow never diminished, we would never need to replace (unless the site ripened into a higher use.)

Method: (a) spread the capital cost of building over time, using the installment formula.

(b) spread the cash flow over time in 2 steps. Of course it is already spread over time, but not as a level annuity. We must levellize[†] it to make it commensurable with the levellized capital cost of (a).

Step 1: convert the diminishing cash flow to a PV_0 .

Step 2: convert the PV_0 to the equivalent annuity, using the installment formula.

(c) Now we subtract (a) from (b) to get the annual rent imputable to the site. This is something to max. If we choose life to max this annual rent, we cannot do any better by shortening life to release the site for cycle #2, yielding the same rent again.

Here is a numerical example. A building yields \$2k/year for 5 years, followed by \$1k/year diminishing by 9%/year. $K_0 = \$6k$. When is the site rent a max? (I omit "k" from here on.)

$$P - K = 2 \frac{1 - (1 + i)^{-5}}{i} + \frac{1 - (1.09 + i)^{-(n-5)}}{(.09 + i)(1 + i)^5} - 6 \quad (1)$$

IPF $\equiv$ installment plan factor.

Table 1 shows values when $i = .05$.

Table 1: $PV - K$, IPF and rent, from (1), when $i = .05$.
(Rent = IPF(P - K))

n	5	8	9	10	15	20
P - K	2.66	4.48	4.94	5.35	6.75	7.47
IPM	.23	.15	.14	.13	.10	.08
Rent	.61	.69	.70	.69	.65	.60

The value of (P - K) rises indefinitely. But rent, the "time-average," peaks when $n = 9$.

[†] "Haig-speak" can be useful.

Commentary:

1. A necessary feature of this solution is that cash flow, in the year that maxes rent, equals that max rent. If it were higher it would raise the average; if lower, would lower the average rent, even though it's a "time-average."

Cash flow in this case is the value of the second annuity in (1) after it has been going for 4 years. It begins at 1, and decays 9%/year, so in 4 years the value is 1.09^{-4} or .71. (The slight excess over .70 indicates that the true max actually falls at 9^+ years--we only took values at one-year intervals.)

2. The economic penalty for missing the optimal year of life looks small, if we look just at the average rent. But the marginal penalty is much larger. In year 10, cash flow is 1.09^{-5} , or .65, 9% below max. That means you are getting 9% less from the site in that year than its levelized annual value, which you could get by going to the next use.

Maxing the life-cycle rent is a good tool for planning ahead, estimating optimal life in advance, at time 0. You can be a little vague about it then, 9 years ahead of the event. But in the 9th or other marginal year, the moment of truth itself, the choice is much more critical.

Typically, at that time there is great inertia in the way of a change in land use. Few economic mistakes are more common than that of deferring replacement too long. The penalty is loss of potential rent, but often more than that, loss of entire industries, decay of central cities, etc.

3. A common way of perceiving π_2 is to say that your old building is yielding less than a market return on the site value. Site value is found by:

$$S = \frac{\text{level rent}}{i}$$

Therefore, $S \cdot i = \text{level rent}$, back where we started. S comes from rent; not the other way around.

4. Another approach is possible if S is "exogenous"--i.e. already known. In the above method, we determined max rent and optimal life simultaneously (using trial and error, which a programmable calculator allows). But if you know S --e.g. what another buyer will pay--then optimal life is when current cash flow crosses under $S \cdot i$.

You can easily show that this is the year when you max the PV of (cash flow + $S(1+i)^{-n}$). Try this as an exercise. Let $S = \frac{.70}{.05}$ or 14, and you should get 9 years.

If $S > 14$ you'll get less than 9 years; but you'll also get a life-cycle average rent less than .70[†] (even though the marginal rent is higher). This tells you your whole investment is a mistake. Try again, or sell the site.

If $S < 14$ you'll get more than 9 years. But this tells you that you've undervalued the site. You can do better by going to 9 years. Don't let some dingbat appraiser tell you what you can squeeze from your assets--you may do a lot better!

[†] Remember to follow all of Eqn. (1).