

Neutrality of LVT in time.
With Nic Tideman

10 AM

Tues, Dec. 15

Nic will call

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Mon., Dec. 21

Write Nic

Nic Tideman Paper



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April 15, 1983

Professor Mason Gaffney
3040 Tyler Street
Riverside, California 92503

Dear Mason:

Having disposed of various intervening projects, I got back to the enclosed and decided to try to finish it on my own. I was unable to locate the notes of our earlier conversations until just yesterday, but I found myself remembering a number of your points as I revised the paper.

The paper does not yet properly credit your earlier work--I always find it easier to work it out then look it up. I know for example that the paragraph following equation (4) on page 4 is something that I learned 15 years ago from something you wrote--possibly TISTOC.

I would appreciate suggestions as what work of yours ought to be cited, and any other comments as well.

Sincerely,

Nic

T. Nicolaus Tideman
Associate Professor

TNT/jw

Enclosure

A Tax on Land Value is Neutral

Brian Bentsick (1979) has argued that there are circumstances under which a tax on land value is not neutral. David Mills (1981) has extended Bentsick's analysis. This note argues that the tax that Bentsick and Mills analyze cannot reasonably be described as a tax on land value and that a true tax on land value is neutral, just as we have always known.

Bentsick defines the value of land in such a way that the value depends on how the land is used. Such a definition has no special virtue from the perspective of theory and does not correspond to anything that might plausibly be observed empirically. It is quite unsurprising that a tax on "land value" should be found to be non-neutral if land value varies depending on how land is used.

Any theory necessarily abstracts from reality, but the abstractions that are used by Bentsick to characterize land are not only unrealistic, but unrealistic in a way that unnecessarily obscures the very real possibility of a neutral tax on land value. Bentsick (1979, p. 861) says:

Consider a piece of land which may be used in two alternative projects, 1 and 2. Project 1 offers the land immediate rentals of \$1.00 per year in perpetuity, while project 2 offers higher rentals of \$c in perpetuity, but only after a gestation period of T years. This delay might result not only from technology in project 2, forestry and mining for example, but also from need to allow the market to ripen.

The significant element of unreality in this characterization is the supposition that returns to land typically come in the form of contractual payments, the present value of which is readily computed to yield the value of the land. On the contrary, most land is held by individuals or firms that use the land in conjunction with other inputs to yield an aggregate return that cannot be allocated unambiguously among inputs.

If the opportunity cost of every input to a project other than land is known, then the present value of the return to the land over the span of the project can be computed as the present value of the total return less the present value of the opportunity costs of other inputs. It is unlikely that all such opportunity costs could be known, if for no other reason than that one input is specialized entrepreneurship, the quantity of which, as well as the unit opportunity cost, cannot be measured in any reliable fashion.

Even if the opportunity costs of all other inputs were known, it would still not be possible to allocate the return to land among time periods until the pattern of depreciation of all capital improvements on the land had been determined. Later I will argue that the sensible way to determine the depreciated value of improvements is by subtracting the value of the land from the combination of land and improvement value. In the absence of land value information, there is no way to make an unambiguous determination of the depreciated value of improvements from the pattern of returns to the combination of land and improvements.

When economists describe land value as the present discounted value of a stream of future returns net of all other costs, they are not describing an identity that can be verified by undertaking computations with observable magnitudes. Rather, they are describing the subjective process that the bidders and sellers of sites might plausibly be understood to employ to determine their bids and asking prices. Future returns do not exist as observable magnitudes, only as the subjective expectations of traders.

We will never have the chance to check our theory by observing the net return to a parcel of land over the whole span of time that is covered by a present value calculation. The most that we could do would be to observe the selling price of land at two points in time and the net returns over that span. Our theory tells us that with perfect foresight and perfect markets, the first selling price should equal the present value at that time of the combination of the second selling price and the net return over the span of time between the two sales. But we would not discard the theory if we found that that identity did not hold; we would merely say that some event not anticipated with certainty by the market had occurred.

At the level of things that are or might be observed, the value of land is not the present value of future net returns, but rather the price at which land sells. If a piece of land has not sold recently, what is meant by its value is the price at which a reasonably well-informed person thinks it would now sell in a transaction with no unusual features--no need on the part of the seller to sell quickly and no unique value of the land to the buyer.

If the improvements did not depreciate and if comparable improvements were now being placed on comparable sites, then the value of the land could be computed as the sale price of the combination less the value of the improvements. But such circumstances would be rare.

The most sensible meaning to give to the phrase "the value of land" when one speaking of a site endowed with improvements that cannot economically be removed to another site is that the value of land is the estimate of a reasonably well-informed person of the price at which the land would sell under the hypothetical condition that the land was not endowed with any improvements. Such a judgment would be based on the observed selling prices

of vacant land that was either comparable or different only ways for which adjustments could be made, or it would be based on an effort to imagine the calculation of the present value of net returns that buyers and sellers might make.

Such a concept of the value of improved land differs in a very significant way from the concept that is employed implicitly by Bentick and by Mills. Bentick (1979, p. 863) says:

Contrast this tax with an annual wealth tax, also at rate b , which is based on the current market value of the land. Investors will now require a gross rate of return of $r + b$ in order to continue enjoying a net return of r , and capitalization of the tax will cause the present values to decline... [Emphasis added.]

Implicitly Bentick is treating the present value of the future returns to a site that is committed to a given project as if it were synonymous with the current market value of the site. But that is not a reasonable definition, because those future returns cannot generally be observed now.

If the current market value of a site is understood to mean the price at which the site would sell if it were not encumbered by being committed to any particular project, then the value of the site has no necessary connection to the present value of the future returns of a project to which someone committed the site in the past. The value of the site today may be conceived as the highest present value of streams of returns that begin with bare land today, but such streams might look quite different than streams that were optimal from starting times in the past.

Once the value of land at each point in time has been defined, it is possible to define the depreciated value of any improvement to the land at any point in time as the sale value of the combination of land and improvements less the sale value of the land.

Because the value of land is now defined independently of how the land is used, the tax on a parcel of land will be independent of the use of the land, and therefore the tax will be neutral.

Consider a PIPO investment that yields a return of R at time t . The interest rate is r and the property tax rate is m . The carrying cost of the investment, c , will be given by

$$c = (m+r)V$$

where V is the market value of the investment. It is also true that

$$V = R e^{-(m+r)t}$$

so that

$$c = (m+r) R e^{-(m+r)t}$$

and

$$\frac{\partial c}{\partial m} = R \left[e^{-(m+r)t} (1 - [m+r]^2) \right]$$

So that c is not independent of m .

A Rehabilitation of the Neutrality of a Tax on Land Value

T. Nicolaus Tideman

and

Mason Gaffney

We are all familiar with the way that capital depreciates if it is not maintained properly and occasionally rehabilitated. The same is true of our social capital in the form of knowledge. Knowledge can disappear if it is not transmitted, but more commonly it depreciates when a useful generalization is found not to hold in some circumstances. Rehabilitation of knowledge occurs when a useful generalization is re-expressed in a way that incorporates the previous exceptions.

A useful generalization with a long history in economics is that a tax on land value generates no deadweight loss. Brian Bentick has argued that the neutrality of a tax on land value does not hold when improvements are recognized as durable, so that there is interdependence in the rents that a site yields at different times. David Mills has extended Bentick's analysis by incorporating the endogenous adjustments that result from the tax that Bentick describes. This paper argues that the conclusions reached by Bentick and Mills resulted from the particular way in which they defined land value, and that there is an alternative and more natural definition of land value under which a tax on land value is neutral whether or not the rents of different times are interdependent. The alternative definition has two additional advantages: It provides an insight into how assessors ought to administer a tax on land values to insure its neutrality, and it suggests a mode for taxation of extractable natural resources that will preserve neutrality in that context as well.

Bentick and Mills both assert that one tax that is neutral is a tax on the current rental income of land. This is actually not quite true. The fundamental generalization about neutrality is that a tax is neutral if it is a lump-sum tax, that is, if the amount of the tax can not be altered by anything that might be done by the person to whom the tax is assigned. Earning income from land involves a certain amount of entrepreneurship as well as the services of land. A tax on the rental income of land will remove some of the incentive for a land holder to locate the user of the land who would pay the highest rent for it. It would be more appropriate to say that a tax on the rental value of land is neutral, where rental value (unlike rental income) is understood to be independent of any action that a landholder might take.

Bentick's conclusion that a tax on the sale value of land is non-neutral arises from an example like the following: Suppose that a lot could be used either for garden apartments, yielding a rental value after expenses of \$100,000 per year in perpetuity, or it could be left vacant for ten years and then used as the site of a high-rise apartment building, yielding \$200,000 per year in perpetuity. At a discount rate of four percent, the value of the land under the first option is:

$$\int_0^{\infty} \$100,000 e^{-.04t} dt = \$2,500,000, \quad (1)$$

and under the second option it is

$$\int_{10}^{\infty} \$200,000 e^{-.04t} dt = \$3,351,600. \quad (2)$$

An investor would prefer the second option.

Now suppose that an annual tax of six percent of the value of land is imposed. Assuming that the tax is a local tax in an open economy, investors will still require a four percent return after taxes. The value of a site with a perpetual rent of \$100,000 will fall to \$1,000,000 because at an assessed value of \$1,000,000 the annual taxes will be \$60,000 and the net return will be \$40,000, for a four percent return.

Now consider the effect of the tax on land values on a site for which the best use is vacant land for 10 years, and then a high-rise apartment building yielding a net rent of \$200,000. When the high-rise is built, the value of the site will be \$2,000,000. Ten years before it is built, the value of the site will be $\$2,000,000 e^{-10(.04+.06)} = \$735,759$. The site will appreciate in value at ten percent per year. After the six percent tax, the rate of return during the period of appreciation will be the investors' required rate of four percent.

For these two investment possibilities, the second has a higher present value before the tax is imposed, and the first has a higher present value after the tax is imposed. Bentick and Mills assert that this means that a tax on land value will distort investment decisions in such circumstances.

Implicitly, Bentick and Mills are asserting that the value of land depends on how the land is used, that if a landholder has the choice of either building the garden apartments or waiting and building the high rise, then the value of his land will be \$1,000,000 if he builds the garden apartments, but it will be \$735,759 if he waits and builds the high rise.

There is an element of truth in what Bentick and Mills assert. If the investment decisions are taken as given, then the specified values are the only ones that are consistent with the returns achieved.

The difficulty with the above reasoning is that it makes a tax on land values not a lump-sum tax. It implies that an assessor should see what improvements a landholder makes to his land to determine the value of the land. Besides making a tax on land values not a lump-sum tax, such a conception of the value of land is entirely contrary to the history of the meaning of land value.

[Mason, I leave this section for you.]

The value of a parcel of land at any time is the price at which it would sell if it were bare at that time, the present discounted value of future returns net of taxes as of that time, discounted at the investors' required rate of return, assuming that the investments that will be made in the future are those that will maximize that net present value. When the optimal investment path varies with the assumed starting time the computations become complex, but land value at each point in time remains well defined and independent of the actual use of the land.

Consider first the case in which the rent that a parcel of land can earn at time τ , $R(\tau)$, is independent of how the land is used at any other time. The value of the parcel of land at time t , $V(t)$, will be

$$V(t) = \int_t^{\infty} [R(\tau) - T(\tau)] e^{-r(\tau-t)} d\tau, \quad (3)$$

where $T(\tau)$ is the tax levied on the parcel at time τ , and r is the investors' required rate of return. If $T(\tau)$ is the millage rate, m , applied to $V(\tau)$,

then the value of the parcel is defined implicitly by the equation

$$V(t) = \int_t^{\infty} [R(\tau) - mV(\tau)] e^{-r(\tau-t)} d\tau, \quad (4)$$

in which V appears on both sides. It is relatively easy to transform (4) into an equation in which V appears only on the left side. Differentiating (4),

$$\frac{dV(t)}{dt} = -R(t) + mV(t) + rV(t). \quad (5)$$

The solution to the differential equation (5) is

$$V(t) = \int_t^{\infty} R(\tau) e^{-(r+m)(\tau-t)} d\tau, \quad (6)$$

which can be verified by differentiating (6) and observing that (5) is obtained. Thus, in the context in which rental value at any time can be defined independently of how the land is used at any other time, the effect of a tax on land values at a rate of m is to increase the discount rate that must be used to compute land values given rents, from r , the investors' required rate of return, to $r + m$.

In this context, the tax at any time t is

$$T(t) = mV(t) = m \int_t^{\infty} R(\tau) e^{-(r+m)(\tau-t)} d\tau. \quad (7)$$

Such a tax is neutral if it is assessed in such a way that the amount of the tax does not depend on how the land is actually used, but rather on how it might be used. The easiest way for an assessor to accomplish this would be to base the tax not on his estimates of future values of $R(\tau)$ for that site, but rather on observed selling prices of comparable vacant sites.

The problem of computing land value is more complex when there is interdependence in the rents earned at different times. Define $R(\tau, t)$ as the rent that can be obtained from the parcel under consideration at time τ , assuming that it was vacant at the previous time t and that the improvements that would be made on it after time t would be those that would maximize the net income from the parcel after taxes, beginning at time t . Assume that $R(\tau, t)$ is differentiable in t . The value of the site at time t is then

$$V(t) = \int_t^{\infty} [R(\tau, t) - T(\tau)] e^{-r(\tau-t)} d\tau. \quad (8)$$

Assuming again that the tax at time τ is the millage rate, m , applied to $V(\tau)$, value is defined implicitly by the equation

$$V(t) = \int_t^{\infty} [R(\tau, t) - mV(\tau)] e^{-r(\tau-t)} d\tau. \quad (9)$$

Unlike equation (4), equation (9) cannot easily be transformed into an equation in which V appears only on the left side. When equation (9) is differentiated, the result is

$$\frac{dV(t)}{dt} = -R(t, t) + mV(t) + rV(t) + \int_t^{\infty} \frac{\partial R(\tau, t)}{\partial t} e^{-r(\tau-t)} d\tau. \quad (10)$$

Equation (10) makes economic sense. It says that the change in the value of the parcel at time t consists of a diminution arising from the current inflow of rent,* an increase from the current payment of taxes, an increase

*If it is optimal to postpone investment further when the site is vacant at time t , then $R(t, \tau)$ is the amount that can be earned from the site in an unimproved state, e.g., as an unpaved parking lot with a trailer for a collection booth. If it is optimal to commence investment in the site immediately when it is vacant at time t , then $R(t, t)$ is the optimal rate of investment in the site at time t , and is negative.

arising from the fact that future potential rents are drawing closer, and finally a change that consists of the present discounted value of the alteration in future rents that arises from the postponement of the start of the investment path.

While equation (10) makes economic sense, unlike equation (4) it cannot in general be transformed into an equation in which $V(t)$ appears only on the left side. The value of a parcel is defined implicitly by equation (10), but an explicit calculation of value at each point in time from the bivariate function $r(\tau, t)$ is not in general a simple matter. However, further insight into the role of the changing potential of a site can be obtained by considering a special case.

Suppose that $R(\tau, t)$ is exponential in both τ and t . That is, suppose that

$$R(\tau, t) = R_0 e^{g(\tau-t)+it}. \quad (11)$$

In words, whenever the site is developed, returns will grow thereafter at a rate of g . But because of the more extensive development that is optimal if the time of development is postponed, the initial return upon development grows at the faster rate i as the time of development is postponed.

Substituting (11) into (10)

$$\frac{dV(t)}{dt} = -R_0 e^{it} + (m+r)V_t + \int_t^{\infty} (i-g)R_0 e^{it+(g-r)(\tau-t)} d\tau \quad (12)$$

$$= -R_0 e^{it} + (m+r)V_t + (i-g)R_0 e^{it} \int_t^{\infty} e^{(g-r)(\tau-t)} d\tau \quad (13)$$

$$= -R_0 e^{it} + (m+r)V_t + \frac{i-g}{r-g} R_0 e^{it} \quad (14)$$

or

$$\frac{dV(t)}{dt} = -R_0 \frac{r-i}{r-g} e^{it} + (m+r)V_t \quad (15)$$

The solution to the differential equation (15) is

$$V(t) = \int_t^{\infty} R_0 \frac{r-i}{r-g} e^{it-(m+r)(\tau-t)} d\tau, \quad (16)$$

which is confirmed by noting that the derivative of (16) with respect to t is (15). Performing the integration indicated in (16) yields

$$V(t) = R_0 \frac{r-i}{(r-g)(m+r-i)} e^{it} \quad (17)$$

as an explicit formula for the value of land in this special case. (For the formula to make sense, r must be greater than both g and i .) It may be noted that when g and i are equal, so that there is no interdependence, (17) reduces to

$$V(t) = \frac{R_0}{m+r-g} e^{gt} \quad (18)$$

which is exactly what the simpler theory yields. When g and i are both zero (17) reduces to

$$V(t) = \frac{R_0}{m+r} \quad (19)$$

which is also obtained from the theory without interdependence.

The lack of an explicit general formula for the value of land as a function of $R(\tau, t)$ does not pose any problem for the neutral taxation of land. All that assessors' have to do is to follow a rule of assessing all land for

tax purposes according to the current sale value of comparable vacant land. Assessors do not have to derive land values; they only have to observe them. If assessors use such a rule, then those who buy and sell land will implicitly solve equation (10), discounting into their transactions not only the future opportunities of land but also the future higher taxes arising from the yet greater opportunities that the land would have in the future if it were then vacant.

One possible source of confusion that should be discussed is that, because of the nature of the interdependency between land value and taxes on land value, a tax that is in equilibrium a uniform tax on the value of land does not reduce the value of land by a uniform percentage when land rents for different parcels have paths of different shapes. Interdependency in the rents at different times is not needed for this to occur; simple variation in the growth rates of rents is sufficient.

Consider equation (18). If $R_0 = \$100,000$, $m = .06$, $r = .04$ and $g = 0$, then the tax reduces the value of the parcel from \$2.5 million to \$1 million, a 60 percent reduction. But if $g = .02$, then with the same values of m and r the tax reduces the value of the land from \$5 million to \$1.25 million, a 75 percent reduction. A tax on the value of land takes a greater fraction of the value of land arising from the expected future increment in land values than of the component of value rising from current rents.

Now introduce interdependence in the rents at different times. With the previous values of R_0 , m , r , and g suppose that $i = .03$. From (16), the introduction of the tax lowers the value of the site from \$5 million to \$712,857, an 85 percent reduction.

To recapitulate: Land value is the present discounted value of future rents net of taxes. If the flow of rent is uniform, then the value of a site is equal to the annual rent divided by the investors' required rate of return, r , if there are no taxes, and is equal to the annual rent divided by $r + m$ if there is a tax at the rate m on the value of land. If rents are growing at the rate g , then the property will appreciate at the rate g . The appreciation can be conceived as offsetting the tax, so that the value will be the initial rate divided by $r + m - g$. If what is increasing is not the rent associated with a given level of development but rather the potential rent that could be obtained if the site were developed optimally as of times in the future, then if improvements cannot economically be removed after they are put in place, there will be no capital gain for the owner after the site is developed. But the value of the site, if it were bare, will appreciate. The higher tax resulting from that appreciation will be capitalized into the sale value of the site. For such a site if the rate of increase of potential rent is i , then the capitalization factor for the current rent [manipulating (16)] will be not $r + m$ but rather $r + m \frac{r}{r-i}$. When potential rent from optimal construction is increasing at the rate i while at the same time the rent from a given level of construction is increasing at a rate of g , the capitalization factor can be shown to equal $r - g + m \frac{r-g}{r-i}$.

to be revised & extended

The value of the parcel at time t , $V(t)$, will be

$$V(t) = \sum_{\tau=t+1}^{\infty} [R(\tau) - T(\tau)] (1 + r)^{-(\tau-t)} \quad (3)$$

where $T(\tau)$ is the tax levied on the parcel at time τ and r is the investors' required rate of return. If $T(\tau)$ is computed by applying the tax rate, m , to $V(\tau)$, then the value of the parcel is defined by the equation

$$V(t) = \sum_{\tau=t+1}^{\infty} [R(\tau) - mV(\tau)] (1 + r)^{-(\tau-t)} \quad (4)$$

in which future values of V appear on the right side. It is possible to derive from (4) an equation in which V appears only on the left side. The change in V from time $t-1$ to time t is given by

$$\begin{aligned} V(t) - V(t-1) &= V(t) - \sum_{\tau=t}^{\infty} [R(\tau) - mV(t)] (1 + r)^{-(\tau-t+1)} \\ &= -R(t) + mV(t) + V(t) - \frac{V(t)}{1 + r} \\ &= -R(t) + V(t) \frac{m + mr + r}{1 + r} \end{aligned} \quad (5)$$

The solution to this first-order difference equation is

$$V(t) = \sum_{\tau=t+1}^{\infty} R(\tau) \left(\frac{1 + r}{1 - m - mr} \right)^{-(\tau-t)} \quad (6)$$

which can be verified by computing the first difference of (6), namely

$$\begin{aligned}
 V(t) - V(t-1) &= V(t) - \sum_{\tau=t}^{\infty} R(\tau) \left(\frac{1+r}{1-m-mr} \right)^{-(\tau-t+1)} \\
 &= -R(t) + V(t) - V(t) \frac{1-m-mr}{1+r} \\
 &= -R(t) + V(t) \frac{m+mr+r}{1+r} \quad (7)
 \end{aligned}$$

which is the same as (5).

Thus the imposition of a tax at a rate of m changes the discount factor

$$\text{from } 1+r \text{ to } \frac{1+r}{1-m-mr} = 1 + \frac{r+m+mr}{1-m-mr}.$$

A Rehabilitation of the Neutrality of a Tax on Land Value

T. Nicolaus Tideman*

I. Introduction

Like housing, knowledge depreciates if it is not maintained properly and occasionally rehabilitated. Knowledge depreciates when it is not transmitted adequately, and also when a useful generalization is found not to hold in some circumstances. Rehabilitation of knowledge occurs when the examples that had seemed to be exceptions to a useful generalization are explained as falling within a proper understanding of the generalization.

A useful generalization with a long history in economics is that a tax on land value is neutral, that is, it generates no allocative inefficiency, or "deadweight loss." Brian Bentsick (1979) claims to have proven that a tax on land value is not neutral when there is interdependence in the rents that a site yields at different times. Bentsick's result has been cited approvingly by Comolli (1981), Mills (1981), Noguchi (1981), Fischel (1982), Klemperer (1982), and Wildasin (1982). I have argued previously (1982) that Bentsick's conclusion is incorrect. Mills (1982) accepted my argument, while Bentsick (1982) remained unpersuaded.

This paper identifies the error in Bentsick's reasoning more precisely. It then develops in some detail the theory of land value with interdependence in the rents of different times, showing that with time interdependence, a tax on the (equilibrium) value of land can generate a path of land value that falls for a span of time, because of a superior future opportunity. Furthermore, either time interdependence or simple differential growth of land values implies that a uniform tax on land value takes different fractions of

*Virginia Polytechnic Institute and State University. I am indebted to Mason Gaffney and Tony Kuprianov for helpful suggestions.

the pre-tax value of different sites. But a tax on land value is still neutral. This paper also provides insight into the administrative practices that are needed to insure the neutrality of a tax on land value, and the relative merit of taxes on the rental value and sale value of land.

II. Conditions for Neutrality

Bentick (1979, pp. 861-63) asserts that one tax that is neutral is a tax on the rental income of land. This is not quite true. Most fundamentally, a tax is neutral if no action by the person to whom the tax is assigned can alter the amount of the tax. It might be argued that a tax is also neutral if the behavior that maximizes a taxpayer's utility when the tax is present is the same as the behavior that maximizes his utility when the tax is absent. This assumes, however, that maximization involves no cost. To the extent that earning income from land involves entrepreneurship as well as the services of land, a tax on the rental income of land will not be neutral because it will remove some of the incentive for a landholder to locate the user of the land who would pay the highest rent for it. It would be more appropriate to say that a tax on the rental value of land is neutral, where rental value (unlike rental income) is understood to be independent of any action that a landholder might take.

With respect to taxes on the sale value of land, Bentick (1979, pp. 861-63) says:

Consider a piece of land which may be used in two alternative projects, 1 and 2. Project 1 offers the land immediate rentals of \$1.00 per year in perpetuity, while project 2 offers higher rentals of \$c in perpetuity, but only after a gestation period of T years.... [P]roject 2 will be selected if

$$e^{-rT}c/r > 1/r \quad (1)$$

...

[Now impose] an annual wealth tax,... at rate b, which is based on the current market value of land. Investors will now require a gross rate of return of $r + b$ in order to continue

enjoying a net return of r , and capitalization of the tax will cause the present values to decline so that project 2 will have to pass the more severe test

$$e^{-(r+b)T_c}/(r+b) > 1/(r+b) \quad (3)$$

...

Bentick's error is in the phrase that includes his inequality 3. The expressions he gives for the present values of the two projects are correct if each project is for a site with no alternative use, but if the two projects are competing for the same site, and if taxes with a present value of β are to be levied on the site irrespective of the project that is chosen, then project 2 will be selected if

$$(e^{-rT_c}/r) - \beta > (1/r) - \beta, \quad (1)$$

which is equivalent to the pre-tax condition. Taxes that are not affected by which project is selected cannot affect the relative attractiveness of projects.

This simple explanation does not address the questions of what the value of a site will be, what the taxes on it will be, and how these will change over time when the uses at different times are not independent. A site that is subject to a tax on its value and has a superior competing future use has a path of value through time of a rather surprising shape. But that surprise is best explained after first developing simpler cases.

III. Land Value and Taxes when Rents are Independent

Consider first the case in which the rent that a site can earn at time t , $R(t)$, is independent of how the land is used at any other time. The value of the site at time t , $V(t)$, will be

$$V(t) = \int_t^{\infty} [R(\tau) - B(\tau)] e^{-r(\tau-t)} d\tau, \quad (2)$$

where $B(\tau)$ is the tax levied on the parcel at time τ , and r is the investors' required rate of return. If $B(\tau)$ is the tax rate, b , applied to $V(\tau)$, then the value of the parcel is defined implicitly by the equation

$$V(t) = \int_t^{\infty} [R(\tau) - bV(\tau)] e^{-r(\tau-t)} d\tau, \quad (3)$$

in which V appears on both sides. It is relatively easy to transform (3) into an equation in which V appears only on the left side. If $R(t)$ is continuous, then (3) can be differentiated, yielding

$$\frac{dV(t)}{dt} = -R(t) + bV(t) + rV(t). \quad (4)$$

In words, (4) says that at any point in time the change in the value of a site is composed of three elements. The first is a diminution in value arising from the current inflow of rent, which is therefore no longer a part of the stream of future returns. The second term is an increase in value arising from the current payment of taxes, which are therefore no longer an encumbrance on the site. The third term, $rV(t)$, may be interpreted as the increase in value that arises by virtue of the fact that future net rents are drawing closer.

The general solution to the differential equation (4) is

$$V(t) = \int_t^{t^*} R(\tau) e^{-(r+b)(\tau-t)} d\tau + V^* e^{-(r+b)(t^*-t)}, \quad (5)$$

which may be verified by differentiating (5) and observing that (4) is obtained. When the upper limit of integration, t^* , is finite, the constant V^* is specified by the boundary condition $V(t^*) = V^*$. When t^* is infinity, as in (3), the constant V^* disappears because it is multiplied by $e^{-\infty}$, which is zero, and the solution becomes

$$V(t) = \int_t^{\infty} R(\tau) e^{-(r+b)(\tau-t)} d\tau. \quad (6)$$

The fact that (6) is the solution to (3) says that if the rental value of a site at any time can be defined independently of how the site is used at any other time, then the effect of a tax on land value at a rate of b is to increase the discount rate that must be used to compute land value given rents, from r to $r + b$.

In this context the tax at any time t is

$$B(t) = bV(t) = b \int_t^{\infty} R(\tau) e^{-(r+b)(\tau-t)} d\tau. \quad (7)$$

Such a tax is neutral if it is assessed in such a way that the amount of the tax does not depend on how the land is actually used, but rather on how it might be used. Assessments could in principle be based either on estimates of $R(\tau)$ for all future times or on estimates of the current value of $V(t)$. Estimating $V(t)$ from observed selling prices of comparable vacant sites would seem to be by far the simpler alternative.

Before proceeding to the case of interdependence, consider how the value of land is affected by steady growth in rental values. Suppose that

$$R(t) = R_0 e^{gt}, \quad (8)$$

so that rents are increasing steadily at a rate of g . Then, from (6),

$$V(t) = \int_t^{\infty} R_0 e^{g\tau-(r+b)(\tau-t)} d\tau = \frac{R_0 e^{gt}}{r-g+b}. \quad (9)$$

An intuitive explanation of this result is as follows: If rents are increasing at a rate of g , then the value of the site will be increasing at a rate of g , generating capital gains at a rate of g . If the current income yields a return of $r-g$, the total return from the combination of current income and capital gains will be r . Thus, when there is a perpetual increase in the rent of a

site a rate of g , the current income will be capitalized at a rate of $r-g$ if there is no tax, and at a rate of $r-g+b$ if there is a tax at a rate of b on the value of the site.

IV. Land Value and Rent when Rents are Interdependent

When there is interdependence in the rents earned at different times, a more complex concept of rent is needed to define land value. Define $R(\tau, t)$ as the rent that can be obtained from the site under consideration at time τ , assuming that it was bare at time $t \leq \tau$ and that the improvements that would be made on it after time t would be those that would maximize the present value of the net income from the site, beginning at time t . The value of the site at time t is then

$$V(t) = \int_t^{\infty} [R(\tau, t) - B(\tau)] e^{-r(\tau-t)} d\tau. \quad (10)$$

Assuming again that the tax at time τ is the tax rate, b , applied to $V(\tau)$, value is defined implicitly by the equation

$$V(t) = \int_t^{\infty} [R(\tau, t) - bV(\tau)] e^{-r(\tau-t)} d\tau, \quad (11)$$

in which $V(t)$ appears on both sides. If $R(\tau, t)$ is continuous at (t, t) and the partial derivative of $R(\tau, t)$ with respect to t exists, then (11) may be differentiated, yielding

$$\frac{dV(t)}{dt} = -R(t, t) + bV(t) + rV(t) + \int_t^{\infty} \frac{\partial R(\tau, t)}{\partial t} e^{-r(\tau-t)} d\tau. \quad (12)$$

Comparing (12) with (4) reveals that the introduction of time interdependence in rents has produced two changes in the expression for the derivative of value with respect to time. The first term has changed from $R(t)$ to $R(t, t)$, the rent that the site could earn at time t if were it bare at that time. The second change is the introduction of the fourth term. This term is the present

discounted value of the change in rents at all future times that arises from postponing of the start of the investment path.

Unlike (4), (12) does not have a general solution. The value of a site is defined implicitly by (11), but an explicit calculation of value at each point in time from the bivariate function $R(\tau, t)$ is not in general a simple matter. However, further insight into the role of the changing potential of a site can be obtained by considering a special case.

The Exponential Case of Interdependent Rents

Suppose that $R(\tau, t)$ is exponential in both τ and t . That is, suppose that

$$R(\tau, t) = R_0 e^{g(\tau-t)+it}. \quad (13)$$

In words, whenever the site is developed, returns will grow thereafter at a rate of g . But because of the more extensive development that is optimal if the time of development is postponed, the initial return upon development grows at the rate i as the time of development is postponed. Postponing development could not be attractive unless i were greater than g .

Substituting (13) into (12),

$$\begin{aligned} \frac{dV(t)}{dt} &= -R_0 e^{it} + (r+b)V(t) + \int_t^{\infty} (i-g)R_0 e^{it-(r-g)(\tau-t)} d\tau \\ &= -R_0 e^{it} + (r+b)V(t) + \frac{i-g}{r-g} R_0 e^{it}, \end{aligned} \quad (14)$$

or

$$\frac{dV(t)}{dt} = -R_0 \frac{r-i}{r-g} e^{it} + (r+b)V(t). \quad (15)$$

In view of (6), the solution to (15) is

$$V(t) = \int_t^{\infty} R_0 \frac{r-i}{r-g} e^{i\tau-(r+b)(\tau-t)} d\tau. \quad (16)$$

Performing the integration indicated in (16) yields

$$V(t) = R_0 \frac{r-i}{(r-g)(b+r-i)} e^{it} = \frac{R_0 e^{it}}{r-g + \frac{r-g}{r-i} b} \quad (17)$$

as an explicit formula for the value of land in this special case. (For the formula to make sense, r must be greater than both g and i .) It may be noted that when g and i are equal, so that there is no interdependence, (17) reduces to (9).

Interdependence and Assessed Values

The lack of an explicit general formula for the value of land as a function of $R(\tau, t)$ does not pose any problem for the neutral taxation of land. All that assessors have to do is to follow a rule of assessing all land for tax purposes according to the current sale value of comparable bare land. Assessors do not have to derive land values; they only have to observe them. If assessors use such a rule, then those who buy and sell land will implicitly solve (11), discounting into their transactions not only the future opportunities of land but also the future higher taxes arising from the yet greater opportunities that the land would have in the future if it were then bare.

One way that assessors could increase their information about land values would be by soliciting options to purchase land, if it should become available. Such bona fide offers could complement market transactions in providing information about land values.

The Varying Reductions in Value from a Uniform Tax

When the rents of different sites have paths of different shapes, a tax that is in equilibrium a uniform tax on the value of land does not reduce the value of different sites relative to their pre-tax levels by a uniform

percentage. Interdependency in the rents at different times is not needed for this to occur; simple variation in the growth rates of rents is sufficient.

Consider (9). If $R_0 = \$100,000$, $b = .06$, $r = .04$ and $g = 0$, then the tax reduces the value of the parcel from \$2.5 million to \$1 million, a 60 percent reduction. But if $g = .02$, then with the same values of R_0 , b and r , the tax reduces the value of the land from \$5 million to \$1.25 million, a 75 percent reduction.

Now introduce interdependence in the rents at different times. With the previous values of R_0 , b , r , and g , suppose that $i = .03$. From (17), the introduction of the tax lowers the value of the site from \$5 million to \$714,286, an 85.7 percent reduction. The reason for this greater reduction in value is that the growth of the potential of a site increases the future value of a site, and hence its taxes, without also increasing the return to developing it today.

To recapitulate: Land value is the present discounted value of future rents net of taxes. If the flow of rent is uniform, then the value of a site is equal to the annual rent divided by the investors' required rate of return, r , if there are no taxes, and is equal to the annual rent divided by $r+b$ if there is a tax at the rate b on the value of land. If rents are growing at the rate g , then the property will appreciate at the rate g . The appreciation can be conceived as offsetting the required return, so that the value will be the initial rent divided by $r-g+b$. If what is increasing is not the rent associated with a given level of development but rather the potential rent that could be obtained if the site were developed optimally as of times in the future, then, if improvements cannot be removed economically after they are put in place, there will be no capital gain for the owner after the site is

developed. But the value of the site, if it were bare, will appreciate. The higher tax resulting from that appreciation will be capitalized into the sale value of the site. For such a site, if the rate of increase of potential rent is i , then the capitalization factor for the current rent will be not $r + b$ but rather $r + \frac{r}{r-i} b$. When potential rent from optimal development is increasing at the rate i while at the same time the rent from a given level of development is increasing at a rate of g , the capitalization factor will be $r-g + \frac{r-g}{r-i} b$. In none of these cases does the presence of the tax lead to a path of land value that declines over time.

The Path of Value for Bentick's Example

Now consider an example like Bentick's. Suppose that investments are irreversible, and suppose that a site could be used either for garden apartments, yielding a return after expenses of R_0 per year in perpetuity, or it could be left vacant for T years until a federally-financed subway system is completed in year t_1 , after which the site would be used for a high-rise apartment building, yielding R_1 per year in perpetuity. (It is assumed implicitly in such an example that investments are constructed instantaneously, are infinitely durable, and are financed by 100% mortgages in perpetuity.) In the absence of taxes, the present value of the returns for the first option is

$$\int_0^{\infty} R_0 e^{-r\tau} d\tau = \frac{R_0}{r}, \quad (19)$$

and for the second option it is

$$\int_T^{\infty} R_1 e^{-r\tau} d\tau = \frac{R_1}{r} e^{-rT} \quad (20)$$

The second option is more valuable for $T < T^* = \frac{\ln(R_1/R_0)}{r}$, and the first option is more valuable if one must wait longer than T^* for the subway system to be

completed. The path of the value of the site through time in the absence of taxes, for one set of parameters, is shown by the upper line in Figure 1. Here it is assumed that $R_0 = \$100,000$, $R_1 = \$500,000$, $r = .04$, and $t_1 = 80$. These parameters imply that $T^* = 40.24$. Define $t_1 - T^*$, the time when it first pays to construct the high rise, as t^* . Prior to t^* the value of the site in the absence of taxes is $\$2,500,000$. Between t^* and 80 the site increases in value at 4 percent per year, reaching $\$12,500,000$ in year 80. After year 80 the value of the site remains steady at $\$12,500,000$. The path of value through time in the absence of taxes may be described as the maximum at each point in time of the values associated with the different possible projects.

Now consider the path of the value of the site when there is a tax on land value at a rate of $b = .06$. Equations (11) and (12) are applicable, except that (12) is not defined at $t = t^*$ or $t = 80$, because $R(\tau, t)$ is not differentiable with respect to t at t^* and $R(t, t)$ is discontinuous at both points. For every other value of t , the integral on the right side of (12) is equal to zero. For any time interval that is entirely on one side or the other of t^* , $R(\tau, t)$ reduces to $R(\tau)$ so that (11) reduces to (3). Therefore $V(t)$ is $\$5,000,000$ for $t > 80$, and is $\$5,000,000 e^{-.10(80-t)}$ for $t^* < t < 80$, which implies that $V(t^*) = 89,443 = V^*$.

For $t < t^*$, from (3),

$$V(t) = \int_t^{t^*} [R_0 - bV(\tau)] e^{-r(\tau-t)} d\tau + V(t^*) e^{-r(t^*-t)}. \quad (20)$$

The derivative of (20) satisfies (4), which implies that the solution to (20) is given by (5), so that

$$V(t) = \frac{R_0}{r+b} - \left(\frac{R_0}{r+b} - V^* \right) e^{-(r+b)(t^*-t)}. \quad (21)$$

Equation (21) says that as one proceeds backward in time from t^* , the value of

Figure 1

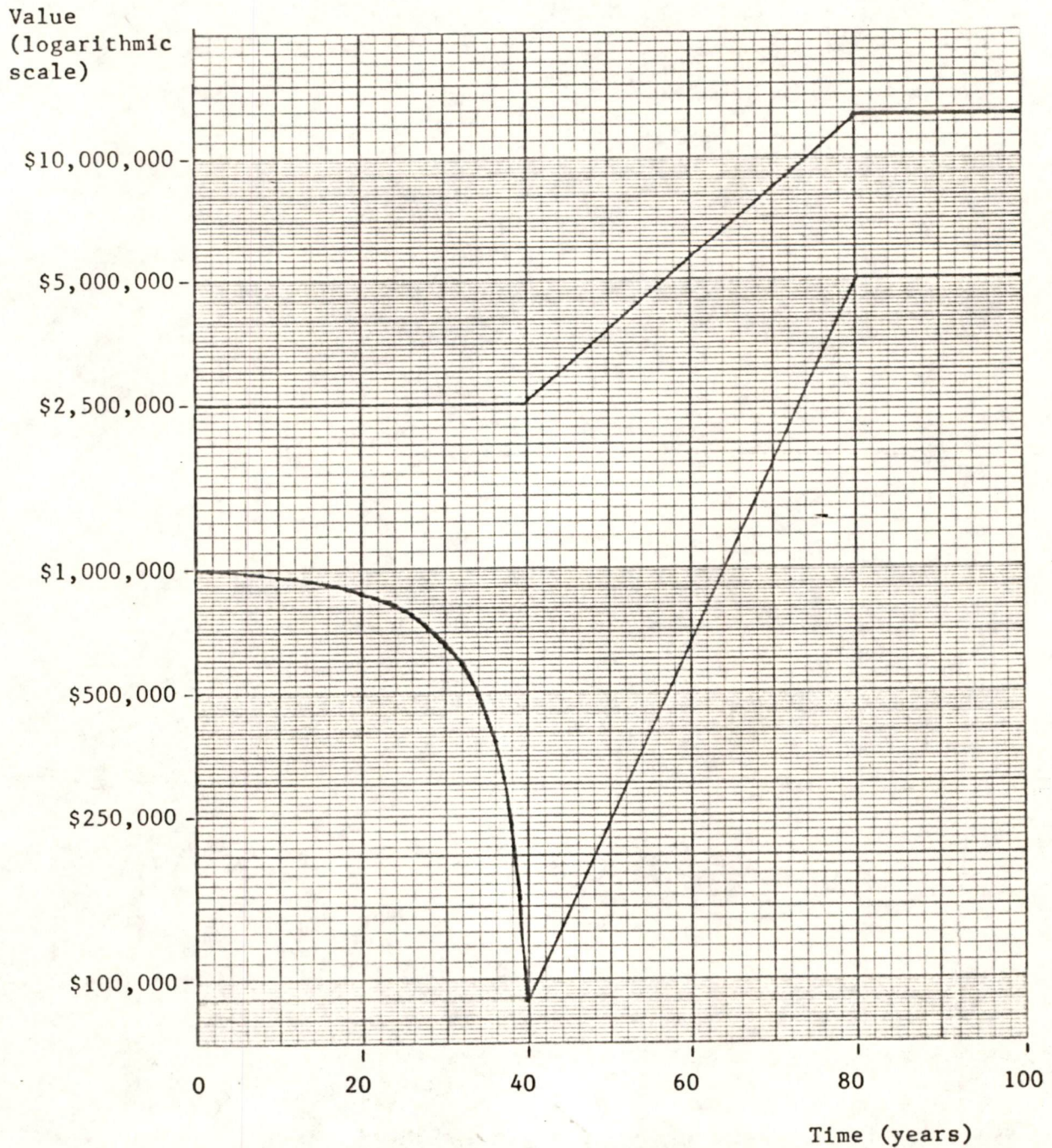


Fig. 1.—The path of land value for a site with a deferred opportunity, with and without a tax on land value, when improvements are irreversible. Assumptions: initial opportunity yields \$100,000 per year; later opportunity (maturing in year 80) yields \$500,000 per year; required return after taxes is 4%; tax rate is 6%.

the site asymptotically approaches, $\frac{R_0}{r+b}$, the value that the site would have were it not for the alternative opportunity, at a rate of $r+b$.

The full path of $V(t)$ in the presence of a six percent tax on land value is shown by the lower line in Figure 1. Prior to t^* the site loses value through time, in anticipation of the higher future taxes that will be assessed on the site. But given the path that value will follow, an investor who builds the garden apartments on the site at any time before t^* will earn the required return of 4 percent. The lower taxes prior to t^* , and for a while thereafter, compensate for the higher taxes later.

Effects of Flexibility

This example involves an extreme fall in value, from \$1,000,000 down to \$89,443. This extreme fall is a consequence not only of the tax on land value in conjunction with the future opportunity, but also of some very special features of the example. One of these features is the permanence of improvements. If the cost of building the garden apartments and then tearing them down at t_1 is $C_0 < \frac{R_1 - R_0}{r}$, then it pays to build them and scrap them when the subway is completed if the time until the subway is completed is greater than the T^* that solves

$$\int_0^{T^*} R_0 e^{-r\tau} d\tau = C_0 e^{-rT^*}, \quad (22)$$

namely $T^* = [\ln(1 + \frac{rC_0}{R_0})]/r$. With renewal possible, the value of the site prior to t^* in the absence of taxes is

$$\begin{aligned} V(t) &= \int_t^{t_1} R_0 e^{-r(\tau-t)} d\tau + \left(\frac{R_1}{r} - C_0\right) e^{-r(t_1-t)} \\ &= \frac{R_0}{r} + \left(\frac{R_1 - R_0}{r} - C_0\right) e^{-r(t_1-t)}. \end{aligned} \quad (23)$$

After t^* the value of the site is the same as in the earlier example. With $C_0 = \$3,000,000$ and the previous values of the other parameters, T^* is 19.71, t^* is 60.29, and $V(t)$ prior to t^* is $\$2,500,000 + \$7,000,000 e^{-.04(t_1-t)}$. The path of this function is shown by line A in Figure 2.

Now add the tax on land value, at a rate of b . As in the previous example, with the tax on land value, $V(t^*)$ is $\frac{R_1}{r} e^{-(r+b)T^*}$. But with the new value of T^* , this is $\$696,487$. The value of the site prior to t^* is therefore given again by (21), but with the new boundary condition the value does not fall nearly so far. The path of value in this case is shown by line B in Figure 2.

Another special feature Bentick's example is that the high-rise cannot earn a positive return prior to t_1 . If the high-rise can earn anything above the interest on its construction cost in the years prior t_1 , then it never pays to leave the site vacant, and the time when the two investments yield the same return comes sooner. If improvements are permanent again, and if the high-rise can earn $R_2 < R_1$ per year prior to t_1 , then the value of the site (in the absence of taxes) after it pays to construct the high-rise is given by

$$V(t) = \frac{R_2}{r} + \frac{R_2 - R_1}{r} e^{-r(t_1-t)} \quad (24)$$

If $R_2 < R_0$, this equals R_0/r , the value of garden apartments, when

$$T^* = [\ln(\frac{R_0 - R_2}{R_1 - R_2})]/r. \quad \text{Line C in Figure 2 shows the path of value in this case.}$$

With a tax on land value at the rate b , the value of a site between t^* and t_1 for this example is given by

$$V(t) = \frac{R_0}{r+b} + \frac{R_1 - R_2}{r+b} e^{-(r+b)(t_1-t)}. \quad (25)$$

Figure 2

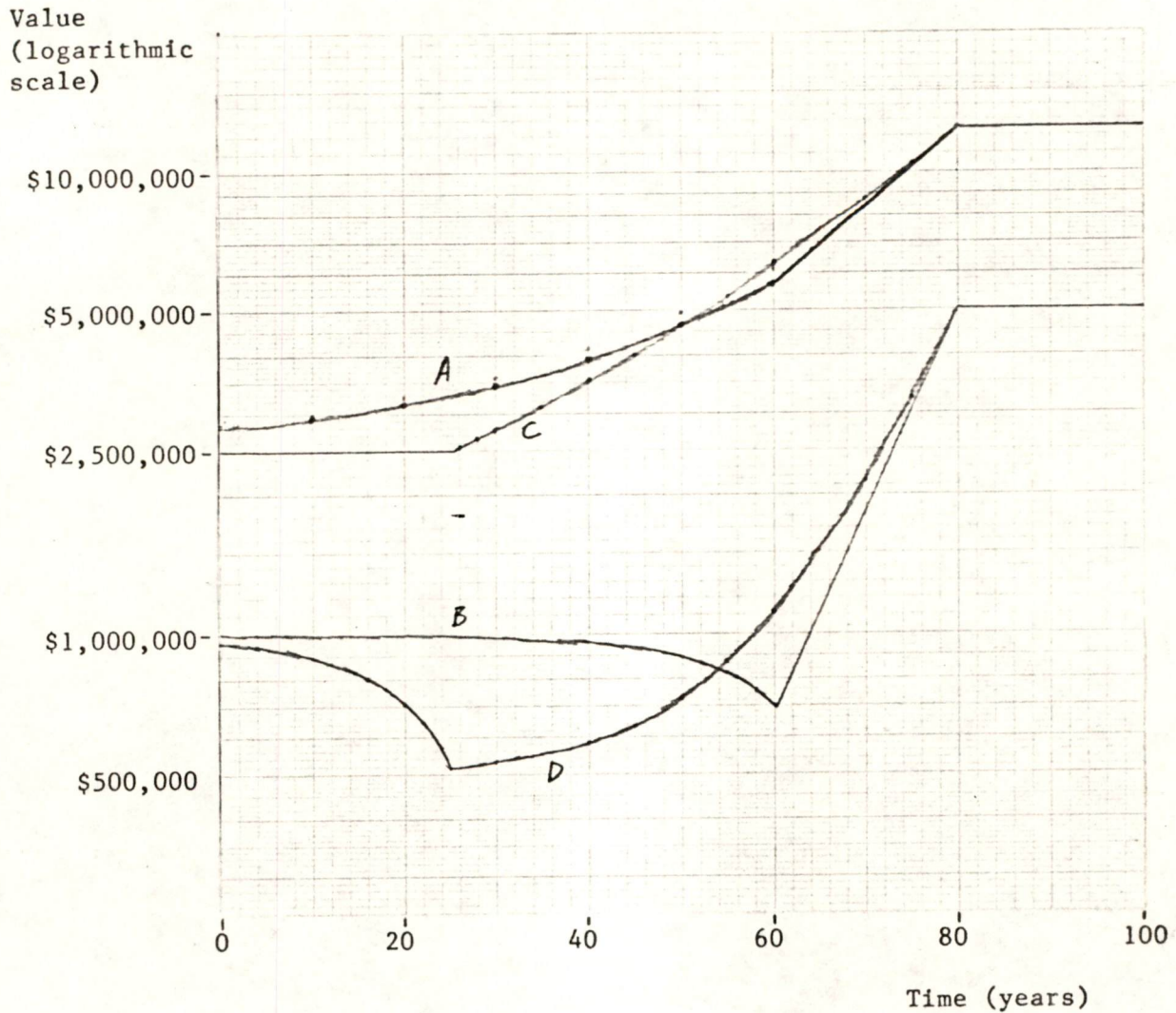


Fig. 2.--The path of land value for a site with a deferred opportunity, with and without a tax on land value, when improvements are reversible (lines A and B), and when improvements are irreversible but the deferred opportunity yields a positive return prior to its maturity (lines C and D). Assumptions: Same as in Fig. 1, plus: cost of reversal is \$3,000,000; premature return on deferred opportunity is \$50,000 per year.

Prior to T^* value is given by (21) again, with the new values of t^* and $V(t^*)$. This path of value is shown as line D in Figure 2. The value of the site would follow the same path if an unimproved parking lot could yield \$50,000 per year.

The general point to be drawn from these examples is that the greater the variety of uses to which a site can be put, the greater the flexibility in changing from one use to another, and the greater the continuity in the returns to those opportunities over time, the less the value of land subject to a tax on land value will fall while high-return opportunities are maturing.

V. The Relative Merit of Taxes on the Sale Value and Rental Value of Land

While a tax on land value is always natural, its characteristic of taking different fractions of the value of sites with different paths of returns might be considered a disadvantage. In contrast, when there is independence in the rents of different times, a tax on the rental value of land takes a uniform fraction on the value of all sites. A tax at the rate of b on the rental value of land lowers all rents to the fraction $(1-b)$ of what they were before the tax and therefore reduces land values to the fraction $(1-b)$ of their pre-tax levels (assuming that the taxes are not spent in ways that raise land rents).

There are two major difficulties with a tax on the rental value of land. First, it is not clear what is meant by the rental value of land when rents at different times are interdependent. Second, even if the rents of different times were independent, it is not clear what potentially observable magnitudes would correspond to the rental value of land. Even if tax base were the rental income of land, as discussed by Bentick (1979), it is not clear what accounting data would be sought to separate the return to land from the returns to improvements and entrepreneurship.

In the case of time independence in rents there is a relationship between rent and value that offers a natural solution to this problem. Moving from

continuous-time models to discrete-time models (which may be justified by the fact that taxes are collected on a discrete rather than a continuous basis), and defining r as the one-period interest rate and $R(t)$ as the rental value of the site for one period beginning at t , it is easy to show that

$$V(t) = R(t) + V(t+1)/(1+r), \quad (26)$$

so that a tax on a base of $V(t) - V(t+1)/(1+r)$ would be equivalent to a tax on $R(t)$. Thus when there is time interdependence in rents, a tax on this base is a natural analog of a tax on rent. At every time the present value of the tax base would be $V(t)$, as with independent rents, so that such a tax would reduce all land values by the same percentage of their pre-tax levels. For an example like Bentick's the tax would be zero between t^* and t_1 .

The administration of such a tax would require knowing the present market values of sites, the market values of sites for delivery one year hence, and the market interest rate. An ordinary tax on land value requires the first of these, and the last can presumably be estimated with adequate precision. The market for future delivery of land is fairly thin, but if assessors were willing to pay for options to deliver land in the future, they might be able to estimate future land values with adequate precision. The fact that land value changes reasonably smoothly with distance means that it would be sufficient to have options on a sample of sites or options that called for delivery of any site satisfying prescribed conditions. Even if the estimates of future land values were somewhat imprecise, a tax on this analog of the rental value of land would probably produce a lower dispersion in the fraction of pre-tax value taken by the tax than a tax on $V(t)$. If dispersion in the fraction of value taken by a tax on land value is a problem, this alternative tax base deserves further consideration.

VI. Concluding Remarks

The fundamental requirement for naturality in a tax is that the amount of the tax be invariant with respect to any action that the taxpayer might take. Both a tax on the rental value of land and a tax on the sale value of land satisfy this condition. A tax need not be uniform to be neutral. If every site were taxed at a different arbitrary rate a tax on the sale value or the rental value of land would still be neutral, as long as the tax rate was truly arbitrary rather than responsive to a taxpayers actions and as long as the tax rate did not exceed 100 percent if the base was the rental value of land.

If uniformity in the fractions of pre-tax value taken by the tax is a desideratum, and if rental values are time independent, then taxing the rental value of land is more attractive than taxing the sale value of land. When
 - rental values are time interdependent, a tax base of $V(t) - V(t+1)/(1+r)$ provides the same advantages.

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Appendix I (Eye)

Sensitivity of Discounted Cash Flow (DCF) to Production

Rates and Interest Rates, Sadlerochit Formation

Table I-1 shows two alternative schedules of the flow of production from the Sadlerochit Formation. These are taken from the Van Poolen report, "Prediction of Reservoir Fluid Recovery, Sadlerochit Formation, Prudhoe Bay Field," Tables XVII and XIX. Both runs result in almost exactly the same ultimate recovery. The difference is that Run 18 starts faster and ends slower than Run 16. Run 18 would require a greater investment in lessee's capital to allow the faster starting rate of production to occur. This is compensated by the higher present value. Table I-2 shows the increase in present value or DCF at different rates of discount. Several points stand out.

DCF is sensitive to production rates. The gain in DCF from choosing Run 18 over Run 16 is a present value increment of \$5.4 billion at a 10 percent rate of discount. This is the maximum gain: it declines in both directions from 10 percent. However, it does not decline very sharply and is surprisingly insensitive to the rate of interest.

DCF is sensitive to the discount rate. For Run 18 it declines from \$47.8 billion down to \$16.1 billion as we go from a 3 percent to a 28 percent discount rate.

Run 16 is more sensitive to the discount rate than is Run 18. This is because Run 16 is slower and therefore more weighted towards the future, where discount factors are more sensitive to the choice of a discount rate.

The third row, which is Run 18 minus Run 16, is not very sensitive to the discount rate. This last point is quite significant. The amount of capital we can justify investing to speed up the flow of production is limited by the gain in DCF that results. Normally, with other kinds of investments, a higher rate of discount means a lesser gain from any investment that increases future production. In this case it means a greater gain, at least until the discount rate reaches 10 percent, after which the gain tapers off again.

Note how this bears on the concern that a higher ad valorem charge (AVC) would cause the last barrel of oil to be extracted too rapidly. When the discount rate rises above 10 percent, the addition of an ad valorem charge will slow down rather than speed up production by limiting the number of wells that may be drilled.

There appear to be two countervailing forces at work here. As we raise the discount rate both series of DCF's become smaller, which tends to bring them closer together, but Run 16 is more sensitive to the discount rate, which tends to pull them farther apart. Between 3 and 10 percent, the second force overpowers the first, and above 10 percent vice versa.

We leave further investigation of this interesting phenomenon to future study. The present point is that the application of an ad valorem charge would not contain much net bias either in the direction of speeding up or slowing down production.

Another important implication is that we may live in a world of uncertainty about future interest rates, but that does not prevent our

scheduling flows with some assurance that our scheduling will be optimal in case future interest rates change. Because future interest rates may change a good deal without changing the optimal application of capital.

Table I-1

Runs 16 and 18 from the Van Poollen Report

<u>Year</u>	<u>Run 16</u> <u>MMB/D</u>	<u>Run 18</u> <u>MMB/D</u>
1	1.4	1.40
2	1.2	1.80
3	1.2	1.80
4	1.2	1.80
5	1.2	1.80
6	1.2	1.80
7	1.2	1.80
8	1.2	1.31
9	1.2	1.10
10	1.2	.91
11	1.2	.89
12	1.2	.87
13	.99	.58
14	.75	.48
15	.56	.35
16	.47	.24
17	.35	.17
18	.27	.16
19	.22	.15
20	.17	.13
21	.13	.12
22	.11	.10
23	.10	.08
24	.08	.06
25	.07	.05
26	.05	--

Table I-2
 Present Values at Different Rates of Discount, (\$ 000, 000, 000)
 Runs 16 and 18, Van Poolen Report

Discount Rate	.03	.05	.08	.10	.15	.20	.28
Run 18	47.8	42.3	35.6	32.7	25.6	21.0	16.1
Run 16	43.5	37.5	30.7	27.3	21.0	16.8	12.6
Run 18-Run 16	4.3	4.8	4.9	5.4	4.6	4.2	3.5
Run 18/Run 16	1.098	1.127	1.160	1.196	1.22	1.25	1.28

Based on \$8/bbl

Thus, each figure in MMB/D is multiplied by 8×365 or \$2920 to get \$MM/year; or, by \$2.92 to get \$MMM/yr. as above.

Nici's MS.

1. Rent needs defining, re long invt. cycles.

2. R-date is not fixed, but depends on rent.

3. We need to show that $cl. = a + T$ is a const., — as it is when future rents are const.

4. How many pp. are we allowed?

3 ~~pp~~ Then, you invest yr. 1 to get higher c.f. beginning yr. 2, etc.

5. p. 1 statement of Mills' claim is unclear.

✓ 6. "Lump-sum" is wrong term. "Indep." of taxpayer action; or "unavoidable".

7. p. 2 tax on rental income is also very hard to define, in context of ripening.

p. 2 8. Conventional neutrality arg. for inc. tax applies best, maybe solely to land rent, where owner's outlays is cost ~~or~~ that is not deductible.

Cost of entrepreneurship cd. will be deducted from rent.

2

This wd. be part of defining rent. Co. be explicit, if
need be — here it done.

But cash flow effects shd. be noted, a wealth
effect. Equally important, as input-effects.

9. Perpetuity e.g. have wants?

It does focus on R-date issue.

But omits most interesting input — sequencing G-2 life.

9A It embeds the assum. of no interim use.

It assumes away the advantage of an earlier G-2
if G-1 is right away.

It assumes away the advantage of reduced obsolescence
when you build less, replace of tenor.

10. Assuming away cash flow effects leaves out
many important facts.

Cap. mkt. for raw land, very broken

— old settlers, no debt, low cc.

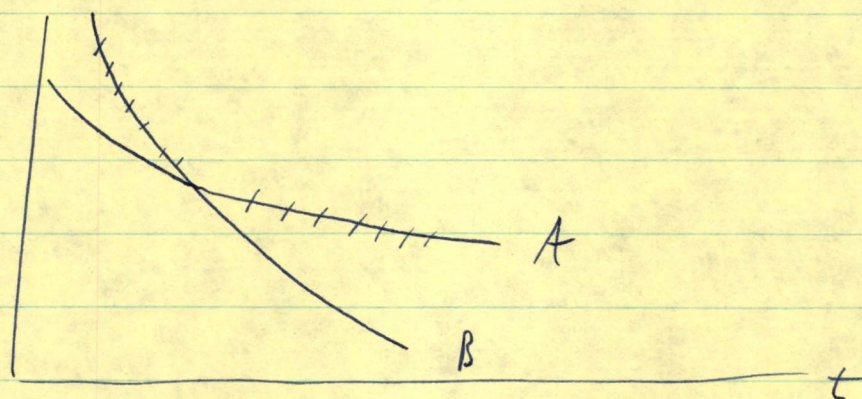
— devers, hi cc, but banks won't lend on

Important effect, L.V.E., is to sort this out.

11. p. 3 Local tax, open ec. ass. seems unrec. here.

12. L.V. based on highest & best use. - a familiar concept to use.

13.



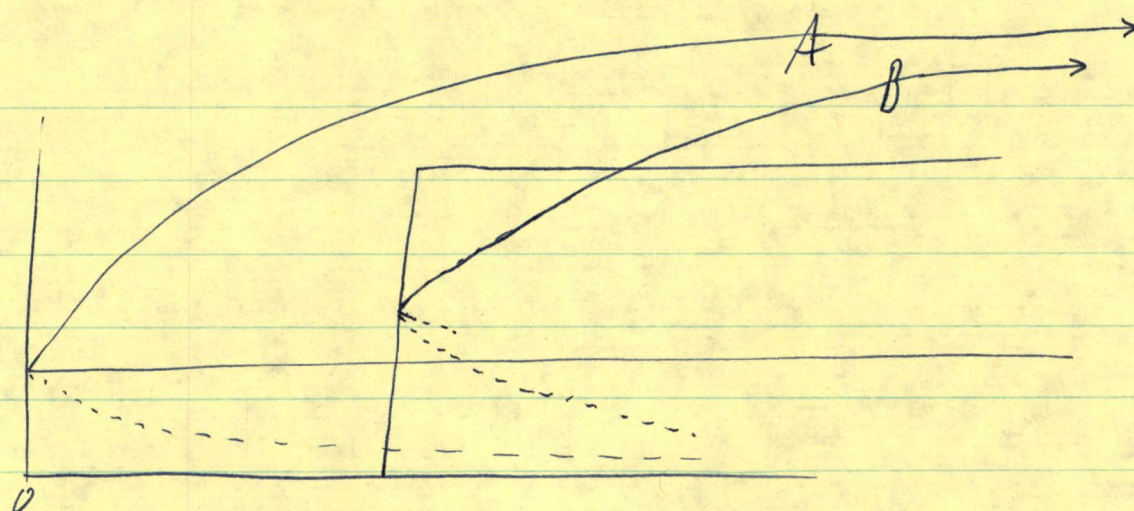
Raising tax shifts best use to A, the ~~less~~ deferred use.

? Case depends basically on deferred rents' being discounted at $(i+t)$. Has Nic really met it? Can one?

Fallacy of May et al. is to assume asmt. frozen at level of ~~deferred~~ improvement.

Bartick ~~is~~ is assuming frozen asmt. for use A.

✓ But as time passes, B becomes higher use, and A is a catastrophe. Give a numerical ex. based on the orig. one. Don't skip around & moralize as Nic. does.



I. No tax, $i = .04$, slow use is higher.

$$L, V_0 = \frac{500,000}{.04} e^{-.04 \times 20} = \$5,616,612.$$

II. 6% tax.

$$V_B \text{ falls to } \frac{500,000}{.10} e^{-.1 \times 20} = \$676,676.$$

$$V_A \text{ falls to } \frac{100,000}{.1} = \$1,000,000$$

13. alt. A: 100 k in pres.

alt. B: 500k in pres. after 20 yrs.

PV_A is allegedly much higher than PV_B by land tax.

But PV_A is based on taxes det. by PV_B , after some time. In yr. 20, $PV_B = \$5m$; taxes @ 6% = 300,000, or 3 times the inc. of alt. A.

Work out numerical ex., directly refute theirs.

14. $1.22^{100} = 432.5 \cdot 10^6$
 $e^{22} = 3584.9 \cdot 10^6$ or 8.3 times as much
So, ~~but~~ because of elegance w. high exponents.

$$\left(1 + \frac{.22}{12}\right)^{1200} \approx 2900 \cdot 10^6 \text{ or } 82\% \text{ of } e^{22}$$

It cd. be useful to assume monthly rent, and annual tax, dramatizing this difference and its importance at $\ln i$.

also, monthly int. prnts. vs. annual tax.
This raises effective i.r.

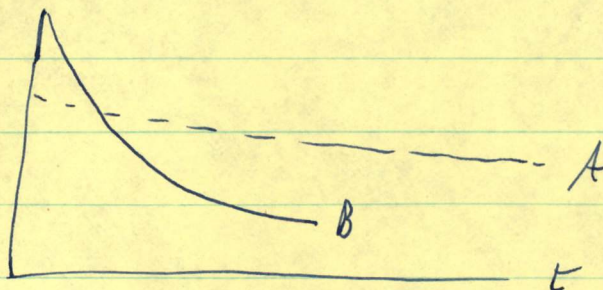
also, loss in ass. value may be built in.

$$p = \ln(1+i)$$

$$e^p = 1+i$$

13. (cont.) Summary.

a. Bentsen's Model:



$$A = \frac{100k}{i+t}$$

$$B = \frac{500k}{i+t} \cancel{(i+t)}^{-20} (1+i+t)^{-20}$$

b. Correct model

$$A_{\text{correct}} = \frac{100k - T}{i+t} \quad (\text{heuristically})$$

$$T = tB \quad (\text{not } tA \text{ as assumed})$$

$$\text{Rather, } T = T(t, B)$$

c. But tB is not const. like tA , bc B rises over 20 yrs. Work out this integral, have good solution (to this numerical exp.)

15. Consider that tax capn. reflects tax of after-tax rent,

Maybe Bentick has stumbled on a weakness resulting therefrom.

Tax shd. be the highest rent the land is capable of producing over time. But this leaves open the qn. of how to distr. tax over time.

a. Annual cycles, rising a .

Rent rises ea. yr. Tax shd., too.

An alt. is a level annual tax of $= PV$:

$$V_0 = \frac{a}{i-g}$$

$T = t V_0$ in perp. $T_0 > a_0$, but this gradually changes.

This alt. is less beneficial than having tax rise over time, so fisc "participates rather than anticipates."

b. Periodic discrete cycles, w. initial down time,
Fisc cd. participate.

If take 100% rent from yr. 20,
there is no prior tax base.
To be finished.

16. p. 7. 2d para. is abrupt.
17. There may be a case, but Bent. hasn't made it.
Can we? Have Shoup et al?
18. What has Nic in mind for my part?
19. pp. 4, 5.
Value does not depend on "the" investor r , but
 r of the hi bidder.
20. p. 5 Equ. (5), (6) is JPE stuff, not NTJ.
Screens out 90% NTJ readers. Can step 4 to 5 be
explained otherwise?
21. p. 5 How about setting $\epsilon = 0$, obviate confusion
of ϵ + τ ?
22. "Mullage rate" is confusing. "90 tax rate" is intended.

Call Wed

A Rehabilitation of the Neutrality of a Tax on Land Value

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and

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We are all familiar with the way that capital depreciates if it is not maintained properly and occasionally rehabilitated. The same is true of our social capital in the form of knowledge. Knowledge can disappear if it is not transmitted, but more commonly it depreciates when a useful generalization is found not to hold in some circumstances. Rehabilitation of knowledge occurs when the examples that had seemed to be exceptions to a useful generalization are explained as falling within a proper understanding of the generalization.

A useful generalization with a long history in economics is that a tax on land value generates no deadweight loss. Brian Bentick has argued that the neutrality of a tax on land value does not hold when improvements are

recognized as durable, so that there is interdependence in the rents that a site yields at different times. David Mills has extended Bentick's analysis by incorporating endogenous[?] adjustments that would occur in the framework that Bentick describes. This paper argues that the conclusions reached by Bentick and Mills resulted from the peculiar way in which they defined land value, and that under a more natural and customary definition of land value a tax on land value is neutral whether or not the rents of different times are interdependent. The alternative definition has two additional advantages: It provides an insight into how assessors ought to administer a tax on land value to insure its neutrality, and it suggests a mode for taxation of extractable natural resources that will preserve neutrality in that context as well.

* Here or somewhere, we need a clearer statement of their theses.

Bentick and Mills both assert that one tax that is neutral is a tax on the rental income of land. This is actually not quite true. The fundamental generalization about neutrality is that a tax is neutral if it is a lump-sum tax, that is, if the amount of the tax can not be altered by anything that might be done by the person to whom the tax is assigned. Secondly, a tax may be argued to be neutral if the behavior that maximizes a taxpayer's utility when the tax is present is the same as the behavior that maximizes his utility when the tax is absent. This assumes, however, that the act of maximization involves no cost. To the extent that earning income from land involves entrepreneurship as well as the services of land, a tax on the rental income of land will not be neutral because it will remove some of the incentive for a land holder to locate the user of the land who would pay the highest rent for it. It would be more appropriate to say that a tax on the rental value of land is neutral, where rental value (unlike rental income) is understood to be independent of any action that a landholder might take.

Bentick's conclusion that a tax on land value is non-neutral arises from an example like the following: Suppose that a lot could be used either for garden apartments, yielding a rental value after expenses of \$100,000 per year in perpetuity, or it could be left vacant for twenty years and then, after a federally-subsidized subway system is completed, used as the site of a high-rise apartment building, yielding \$500,000 per year in perpetuity.

too
petty

(It is assumed implicitly in such an example that improvements are constructed instantaneously, are infinitely durable, and are financed by mortgages in perpetuity.) At a discount rate of four percent, the value of the land under the first option is:

$$\int_0^{\infty} \$100,000 e^{-.04t} dt = \frac{\$100,000}{.04} = \$2,500,000, \quad (1)$$

and under the second option it is

$$\int_{20}^{\infty} \$500,000 e^{-.04t} dt = \frac{\$500,000}{.04} e^{-.04(20)} = \$5,616,666. \quad (2)$$

An investor would prefer the second option.

Now suppose that an annual tax of six percent of the value of land is imposed. Assuming that the tax is a local tax in an open economy, investors will still require a four percent return after taxes. The value of a site with a perpetual rent of \$100,000 will fall to \$1,000,000, because at an assessed value of \$1,000,000 the annual taxes will be \$60,000 and the net return will be \$40,000, for a four percent return.

Now consider the effect of the tax on land values on a site for which the best use is vacant land for 20 years, and then a high-rise apartment building yielding a net rent of \$500,000. When the high-rise is built, the value of the site will be \$5,000,000. Twenty years before it is built, the value of the site will be $\$5,000,000 e^{-20(.04+.06)} = \$676,676$. The site will appreciate in value at ten percent per year. After the six percent tax, the rate of return during the period of appreciation will be the investors' required rate of four percent.

For these two investment possibilities, the second has a higher present value before the tax is imposed, and the first has a higher present value after the tax is imposed. Bentick and Mills assert that this means that a tax on land value will distort investment decisions in such circumstances.

✓ Implicitly, Bentick and Mills are asserting that the value of land depends on how the land is used, that if a landholder has the choice of either building the garden apartments or waiting and building the high rise, then the value of his land will be \$1,000,000 if he builds the garden apartments, but it will be \$676,676 if he waits and builds the high rise.

Such reasoning would indeed make a tax on land value not a lump-sum tax. It would imply that an assessor should see what improvements a landholder makes to his land to determine the value of the land. Besides making a tax on land values not a lump-sum tax, such a conception of the value of land is entirely contrary to the history of the meaning of land value.

[Mason, I leave this section for you.]

What's it called? History of land value. I guess.

The value of a parcel of land at any time is the price at which it would sell if it were bare at that time. The selling price is equal to the present discounted value of future returns net of taxes as of that time, discounted at the investors' required rate of return, assuming that the investments that will be made in the future are those that will maximize that net present value. When the optimal investment path varies with the assumed starting time, the computations become complex, but land value at each point in time remains well defined and independent of the actual use of the land.

Consider first the case in which the rent that a site can earn at time t , $R(t)$, is independent of how the land is used at any other time. The value of the parcel of land at time t , $V(t)$, will be

$$V(t) = \int_t^{\infty} [R(\tau) - T(\tau)] e^{-r(\tau-t)} d\tau, \quad (3)$$

where $T(\tau)$ is the tax levied on the parcel at time τ , and r is the investors' required rate of return. If $T(\tau)$ is the millage rate, m , applied to $V(\tau)$, then the value of the parcel is defined implicitly by the equation

$$V(t) = \int_t^{\infty} [R(\tau) - mV(\tau)] e^{-r(\tau-t)} d\tau, \quad (4)$$

in which V appears on both sides. It is relatively easy to transform (4) into an equation in which V appears only on the left side. Differentiating (4),

$$\frac{dV(t)}{dt} = -R(t) + mV(t) + rV(t). \quad (5)$$

The solution to the differential equation (5) is

$$V(t) = \int_t^{\infty} R(\tau) e^{-(r+m)(\tau-t)} d\tau, \quad (6)$$

which can be verified by differentiating (6) and observing that (5) is obtained. Thus, in the context in which rental value at any time can be defined independently of how the land is used at any other time, the effect of a tax on land values at a rate of m is to increase the discount rate that must be used to compute land value given rents, from r to $r + m$.

In this context the tax at any time t is

$$T(t) = mV(t) = m \int_t^{\infty} R(\tau) e^{-(r+m)(\tau-t)} d\tau. \quad (7)$$

Such a tax is neutral if it is assessed in such a way that the amount of the tax does not depend on how the land is actually used, but rather on how it might be used. The easiest way for an assessor to accomplish this would be to base the tax not on his estimates of future values of $R(\tau)$ for that site, but rather on observed selling prices of comparable vacant sites.

Before proceeding to the case of interdependence, it may be worth considering how the value of land is affected by steady growth in rental values. Suppose that

$$R(t) = R_0 e^{gt} \quad (8)$$

so that rents are increasing steadily at a rate of g . Then, from (6),

$$V_t = \int_t^{\infty} R_0 e^{gt-(r+m)(\tau-t)} d\tau = \frac{R_0 e^{gt}}{r-g+m}. \quad (9)$$

An intuitive explanation of this result is as follows: If rents are increasing at a rate of g , then the value of the site will be increasing at a rate of g , so that the owner of the site will be receiving capital gains on the site at a rate of g . If the current income yields a return of $r-g$, the owner will still be earning a total return of r from the combination of current income and capital gain. Thus, when there is a perpetual increase in the rent of a site at a rate of g , the current income will be capitalized at a rate of $r-g$ if there is no tax, and at a rate of $r-g+m$ if there is a tax at a rate of m on the value of the site.

When there is interdependence in the rents earned at different times, land value is related to rents in the following way. Define $R(\tau, t)$ as the rent that can be obtained from the site under consideration at time τ , assuming that it was vacant at the previous time t and that the improvements that would be made on it after time t would be those that would maximize the net income from the site after taxes, beginning at time t . The value of the site at time t is then

$$V(t) = \int_t^{\infty} [R(\tau, t) - T(\tau)] e^{-r(\tau-t)} d\tau. \quad (10)$$

Assuming again that the tax at time τ is the millage rate, m , applied to $V(\tau)$, value is defined implicitly by the equation

$$V(t) = \int_t^{\infty} [R(\tau, t) - mV(\tau)] e^{-r(\tau-t)} d\tau. \quad (11)$$

in which $V(t)$ appears on both sides. When equation (11) is differentiated (assuming that $R(\tau, t)$ is differentiable with respect to t), the result is

$$\frac{dV(t)}{dt} = -R(t, t) + mV(t) + rV(t) + \int_t^{\infty} \frac{\partial R(\tau, t)}{\partial t} e^{-r(\tau-t)} d\tau. \quad (12)$$

In words, equation (12) says that at any point in time, the change in the value of a site is composed of four elements. The first is a diminution arising from the current inflow of rent, which is therefore no longer a part of the stream of future returns.* The second is an increase arising from the

*If it is optimal to postpone investment further when the site is vacant at time t , then $R(t, t)$ is the amount that can be earned from the site in an unimproved state, e.g., as an unpaved parking lot with a trailer for a collection booth. If it is optimal to commence investment in the site immediately when it is vacant at time t , then $R(t, t)$ is the optimal rate of investment in the site at time t , and is negative.

current payment of taxes, which are therefore no longer an encumbrance of the site. The third element, the term $rV(t)$, may be interpreted as the increase in value that arises by virtue of the fact that future rents are drawing closer. When there is no interdependence in the rents of different times, these three elements constitute the whole change in the value of a site. When interdependence is present there is a fourth term: the present discounted value of the change in rents at all future times that arises from postponement of the start of the investment path.

Unlike equation (4), equation (12) cannot in general be transformed into an equation in which $V(t)$ appears only on the left side. The value of a parcel is defined implicitly by equation (12), but an explicit calculation of value at each point in time from the bivariate function $R(\tau, t)$ is not in general a simple matter. However, further insight into the role of the changing potential of a site can be obtained by considering a special case.

Suppose that $R(\tau, t)$ is exponential in both τ and t . That is, suppose that

$$R(\tau, t) = R_0 e^{g(\tau-t)+it}. \quad (13)$$

In words, whenever the site is developed, returns will grow thereafter at a rate of g . But because of the more extensive development that is optimal if the time of development is postponed, the initial return upon development grows at the faster rate i as the time of development is postponed.

Substituting (13) into (12)

$$\begin{aligned}
 \frac{dV(t)}{dt} &= -R_0 e^{it} + (m+r)V_t + \int_t^{\infty} (i-g)R_0 e^{it-(r-g)(\tau-t)} d\tau \\
 &= -R_0 e^{it} + (m+r)V_t + (i-g)R_0 e^{it} \int_t^{\infty} e^{-(r-g)(\tau-t)} d\tau \\
 &= -R_0 e^{it} + (m+r)V_t + \frac{i-g}{r-g} R_0 e^{it}
 \end{aligned} \tag{14}$$

or

$$\frac{dV(t)}{dt} = -R_0 \frac{r-i}{r-g} e^{it} + (m+r)V_t \tag{15}$$

The solution to the differential equation (15) is

$$V(t) = \int_t^{\infty} R_0 \frac{r-i}{r-g} e^{it-(m+r)(\tau-t)} d\tau, \tag{16}$$

which is confirmed by noting that the derivative of (16) with respect to t is (15). Performing the integration indicated in (16) yields

$$V(t) = R_0 \frac{r-i}{(r-g)(m+r-i)} e^{it} = \frac{R_0 e^{it}}{r-g + \frac{r-g}{r-i} m} \tag{17}$$

as an explicit formula for the value of land in this special case. (For the formula to make sense, r must be greater than both g and i .) It may be noted that when g and i are equal, so that there is no interdependence, (17) reduces to (9).

The lack of an explicit general formula for the value of land as a function of $R(\tau, t)$ does not pose any problem for the neutral taxation of land. All that assessors have to do is to follow a rule of assessing all land for tax purposes according to the current sale value of comparable vacant land. Assessors do not have to derive land values; they only have to observe them. If assessors use such a rule, then those who buy and sell land will implicitly solve equation (12), discounting into their transactions not only the future opportunities of land but also the future higher taxes arising from the yet greater opportunities that the land would have in the future if it were then vacant.

The necessity of observing the sale prices of comparable sites in order to administer a neutral tax on land value might seem to cast doubt on the possibility of having such a tax. How often does vacant land change hands in a built up area? In a study of land values in Milwaukee, one of us (Gaffney) found that 25 percent of property sales in the downtown area were of vacant sites or of sites with buildings about to be demolished.

One way that assessors could increase their information about land values would be to solicit options to purchase land, if it should become available. Such bona fide offers could complement market transactions in providing information about land values.

One possible source of confusion that should be discussed is that when the rents of different sites have paths of different shapes, a tax that is in equilibrium a uniform tax on the value of land does not reduce the value of different sites relative to pre-tax levels by a uniform percentage. Interdependency in the rents at different times is not needed for this to occur; simple variation in the growth rates of rents is sufficient.

Consider equation (9). If $R_0 = \$100,000$, $m = .06$, $r = .04$ and $g = 0$, then the tax reduces the value of the parcel from \$2.5 million to \$1 million, a 60 percent reduction. But if $g = .02$, then with the same values of m and r the tax reduces the value of the land from \$5 million to \$1.25 million, a 75 percent reduction. A tax on the value of land takes a greater fraction of the component of the value of land arising from expected future increases in rents than of the component of value arising from current rents.

Now introduce interdependence in the rents at different times. With the previous values of R_0 , m , r , and g suppose that $i = .03$. From (17), the introduction of the tax lowers the value of the site from \$5 million to \$712,857, an 85.7 percent reduction. The reason for this greater reduction in value is that the growth of the potential of a site increases the future value of a site, and hence its taxes, without also increasing the return to developing it today.

To recapitulate: Land value is the present discounted value of future rents net of taxes. If the flow of rent is uniform, then the value of a site is equal to the annual rent divided by the investors' required rate of return, r , if there are no taxes, and is equal to the annual rent divided by $r+m$ if there is a tax at the rate m on the value of land. If rents are growing at the rate g , then the property will appreciate at the rate g . The appreciation can be conceived as offsetting the required return, so that the value will be the initial rent divided by $r-g+m$. If what is increasing is not the rent

associated with a given level of development but rather the potential rent that could be obtained if the site were developed optimally as of times in the future, then, if improvements cannot economically be removed after they are put in place, there will be no capital gain for the owner after the site is developed. But the value of the site, if it were bare, will appreciate. The higher tax resulting from that appreciation will be capitalized into the sale value of the site. For such a site, if the rate of increase of potential rent is i , then the capitalization factor for the current rent will be not $r + m$ but rather $r + \frac{r}{r-i} m$. When potential rent from optimal development is increasing at the rate i while at the same time the rent from a given level of construction is increasing at a rate of g , the capitalization factor will be $r - g + \frac{r-g}{r-i} m$.

Return now to the example of the site with a current use of garden apartments and a deferred use of high-rise apartments yielding five times the net income before property taxes. How will the value of such a site vary over times? The answer is shown in Figure 1. To show the whole pattern, it is assumed that the alternative use will not be economical for 80 years. Consider first the path of site value in the absence of taxes. The explanation is easiest when the progression is backwards through time. After the subway is completed, the value of the site is $\frac{\$500,000}{.04}$, or \$12,500,000. While that opportunity is maturing, the site will grow in value at 4 percent per year. But it cannot be worth less than \$2,500,000, because that is how much the site could yield in

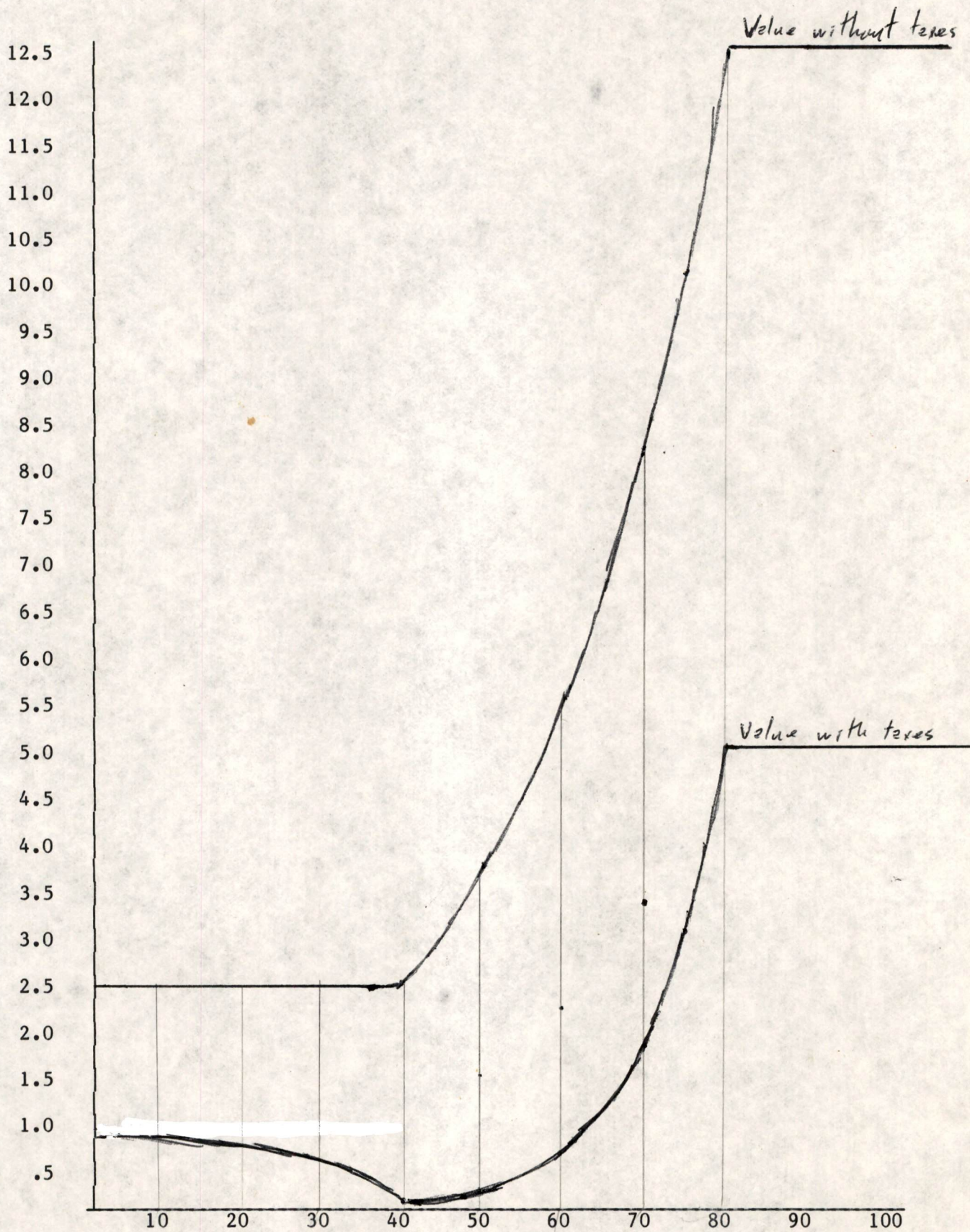


FIGURE 1

garden apartments. The two uses yield the same value when

$$\$24,500,000 e^{-.04(80-t^*)} = \$2,500,000, \quad (18)$$

that is, when

$$e^{-.04(80-t^*)} = \frac{1}{5} \quad (19)$$

or $t^* = 39.764$. Before that time it will be most economical to build the garden apartments and ignore the high-rise possibility; after that time, if the site is bare, it will be most economical to leave it bare until the time for the high-rise comes.

Now consider the path of the value of the site in the presence of a 6 percent tax on land value. For $t > 80$, the value of the site will be $\frac{\$500,000}{.04 + .06}$ or \$5,000,000. As that opportunity matures, the site will increase in value at a rate of 10 percent per year, so that at the time when the two opportunities are equally attractive the value of the site will be $\$5,000,000 e^{-(.04+.06)(40.236)} = \$89,443$. The present value of the taxes on the site at that point will be

$$\begin{aligned} T^* &= \int_0^{40.236} .06(\$89,443 e^{.1\tau}) e^{-.04\tau} d\tau + \int_{40.236}^{\infty} .06(\$5,000,000) e^{-.04\tau} d\tau \\ &= \$5,367 \int_0^{40.236} e^{.06\tau} d\tau + \$300,000 \int_{40.236}^{\infty} e^{-.04\tau} d\tau \\ &= \$89,443(e^{2.414} - 1) + \frac{\$300,000}{.04} (e^{-1.609} - 0) \\ &= \$910,557 + \$1,500,000 = \$2,410,557. \end{aligned} \quad (20)$$

When t^* is subtracted from \$2,500,000, the present value of the return to garden apartments before taxes, the result is \$89,443, the previously computed value of the site at that time, which confirms that the two opportunities are equally profitable after taxes at that time, as well as before taxes.

Prior to t^* the best use of the site is garden apartments. The value of the site at any time prior to t^* is the present value of rents minus taxes, or

$$V(t) = \frac{\$100,000}{r} - \int_t^{t^*} mV(\tau)e^{-r(\tau-t)}d\tau - T^*e^{-r(t^*-t)} \quad (21)$$

where T^* is the present value of future taxes as of t^* . Differentiating (21)

$$\frac{dV(t)}{dt} = -mV(t) - r \int_t^{t^*} mV(\tau)e^{-r(\tau-t)}d\tau - rT^*e^{-r(t^*-t)} \quad (22)$$

From (21),

$$-r \int_t^{t^*} mV(\tau)e^{-r(\tau-t)}d\tau - rT^*e^{-r(t^*-t)} = rV(t) - \$100,000 \quad (23)$$

Substituting from (23) into (22),

$$\frac{dV(t)}{dt} = (m+r)V(t) - \$100,000 \quad (24)$$

With the boundary condition that $V^* = V(t^*) = \$89,443$, the solution to the differential equation (24) is

$$\begin{aligned} V(t) &= \frac{\$100,000}{r+m} - \left(\frac{\$100,000}{r+m} - V^* \right) e^{-(r+m)(t^*-t)} \\ &= \frac{\$100,000}{r+m} - T^* e^{-(r+m)(t^*-t)} \end{aligned} \quad (25)$$

which is confirmed by noting that the derivative of (25) is (24). Thus as one proceeds backward in time from t^* , the value of the site asymptotically approaches $\frac{\$100,000}{r+m}$ or \$1,000,000, the value that the site would have were it not for the alternative opportunity.

Prior to t^* , the site loses value through time, in anticipation of the higher future taxes that will be assessed on the site. But given the path that value will follow, an investor who builds the garden apartments on the site at any time before t^* will earn the required return of 4 percent. The lower taxes prior to t^* , and for a short while thereafter, compensate for the higher taxes thereafter.

This example involves an extreme fall in value, from \$1,000,000 down to \$89,443. This extreme fall is a consequence not only of the tax on land value in conjunction with the future opportunity, but also of some very special features of the example. One of these features is the permanence of improvements. If the present value at t^* of building the garden apartments at that time and then tearing them down in year 80 is less than \$2,000,000, then it would pay to build them with the expectation of tearing them down, and it would pay to build them at times less than t^* , so that the value of the site would not fall as far before it would begin to rise.

Another special feature of the example is that the high-rise cannot earn any positive return before year 80. If the high-rise can earn anything above the interest on its construction cost prior to year 80, then it never pays to leave the site vacant, and the time when the two investments yield the same

return comes sooner. For instance, if the high rise can earn \$50,000 per year, half of what the garden apartments yield, prior to year 80, then the first time that it pays to construct the high-rise is $\frac{\log 9}{.04} = 54.93$ years before its full maturity. The value of the site during those 54.93 years follows the equation

$$V(t) = \$500,000 + \$4,500,000 e^{-(.04+.06)(80-t)}, \quad (26)$$

and the lowest level that the value reaches is \$518,518. The value of the site would follow the same path if an unimproved parking lot could yield \$50,000 per year.

The general point to be drawn from these examples is that the greater the variety of uses to which a site can be put, the greater the flexibility in changing from one use to another, and the greater the continuity in the returns to those opportunities over time, the less the value of land subject to a tax on land value will fall while high-return opportunities are maturing.

to be revised + extended

Beutick, JPE, 29, Aug.

Nic T.

Re App. I

1. The 2 paths are not general enough, to find generalities. One path always dominates the other: $18 > 16$.

Nic.

1. His eq. is 60% t.r.

2. Rent.

Buntick thinks rent is highest & best use.

Nic wants to avoid trying to resolve this.
But throw ball to Mills.

3. Send ref. to N. Ture

Nic. 8/24

1. Continuous, use p

✓ 2. 4 paper to be submitted immediately: send suggestions.

3. Opt. inv. date advanced by tax?
Use as finite - note he has neglected other lit.?

4. $cc \stackrel{?}{=} a$ when $g > 0$?

$cc = V(m+r)$ He Will think more.

5. Tax inc. from land.

4. Define rent as $(a + \Delta V)$?

$$V_1 = V_0(1+r) - I_0$$

$$I_0 = -V_1 + V_0 + rV_0 = V_0 - V_1 + Vr$$

Ecs. Dept. Va. Tech. 24061
Blacksburg

N: Tideman

Papers alleging LVT not neutral

1. JPE Buttik 8/~~7~~79 : 859
2. Miller, David NTG 3/81
3. Roger Smith
4. Mowry, Don
5. Worcester, Dean. AER '46.
6. Ely
7. Krutilla.

Nic Tidman, 7/13/81

$$V = \frac{1}{.15} + \frac{e}{.15} (e^{-.15n})$$

$$V_T = \int_t^{\infty} r(\tau) - \tau(\tau) \times e^{-r(\tau-t)} d\tau$$

$$r(\tau) = r_0 \cdot e^{\gamma \tau}$$

$$G(\tau) = m \cdot V(\tau)$$

$$"V of t" = V(\tau) = \int_t^{\infty} r_0 e^{\gamma \tau} - [m V(\tau)] e^{-r(\tau-t)} d\tau$$

$$\frac{dV(\tau)}{d\tau} = -r_0 e^{\gamma \tau} + m V(\tau) + r V(\tau)$$

~~$r V(\tau)$~~ where $\gamma = r$

diff. eq whose solution

$$V(\tau) = r_0 e^{\gamma \tau} \times \frac{1}{m + r}$$

Tideman article

THE NON-NEUTRALITY OF LAND VALUE TAXATION**

DAVID E. MILLS*

A neutral tax is one that does not distort resource allocation. Its economic effect is purely distributional. It is well established that the real property tax is non-neutral. This is because the supply of structures and improvements is not fixed and the imposition of a tax on them distorts land-use decisions. Even so, that part of the property tax which falls on unimproved land is widely thought to be neutral.¹ Since the supply of land is fixed, the tax is said to be unavoidable. Landowners, therefore, are not induced to change their land-use plans when the tax is imposed, or its rate changed.

This view of the tax on land is badly mistaken. It is true that a (less than 100 percent) tax on land *income* is neutral, but this does not extend necessarily to a tax on capitalized land *value*, or changes therein. The reason is that the discounted sum of payments with the latter tax is not invariant to the intertemporal characteristics of the income stream produced by land. Among options with equal present value, it is greater for income streams skewed to the distant future than for those skewed to the near future.

Brian L. Bentick demonstrated as much in a recent paper.² He constructed a counterexample to the neutrality proposition by showing that an annual tax on land whose base is current market value favors land uses with early-payoff income streams.³ Making explicit assumptions about the supply of competing development projects available to a landowner, he offered a measure of the resource cost of the tax, and found it to be of significant magnitude for plausible values of his model's parameters. He also showed that the resource cost can be reduced significantly if the tax base is changed from current market value to the hypothetical value where the current use is presumed by the appraiser to be maintained perpetually. This hypothetical value is sub-

ject to change only when the "current" use changes.

Bentick's analysis compared the present value of income streams produced by two, mutually exclusive development projects on a *single* land parcel before and after the tax is introduced. The tax was shown to be capable of reversing the order of the landowner's preference between the projects, and thus changing the use to which the parcel is put. Implicit in this approach is the assumption that introducing the tax does nothing to change the magnitude of income streams themselves. This is not objectionable where the tax is introduced for the parcel in question only. But if it is introduced through a tax jurisdiction where demand for projects is downward sloping, it will cause enough land to be shifted between projects to alter the income streams. The favored project will become more plentiful and the periodic income produced on land devoted to it will decline accordingly. The other project will become more scarce and its periodic income will increase.

This note extends the analysis of taxes on land value to include endogenous land-market adjustments precipitated by their imposition. While these adjustments do not eliminate the distortion Bentick described, they mitigate its effects and have considerable bearing on the proper measure of its resource cost. They also enable a conventional treatment of the incidence and burden of the tax.

I. A Model of the Land Market

Consider a tax jurisdiction with a fixed amount of homogeneous land, L , for the land market to allocate between two perpetual projects.⁴ The amount allocated to project 1 is x . If demand for projects is sufficiently great that no land is left undeveloped, the amount allocated to project 2 is $L-x$. Land ownership is diffuse and the land market atomistic. Following Bentick, we assume the first project is

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undertaken in year 0 and the second in year $T > 0$. Demand for each project is downward-sloping, so their incomes are a decreasing function of the amount of land devoted to them. After all inputs are purchased, the annual income per unit of land is $c_1(x)$ for project 1 and $c_2(x)$ for project 2, where:

$$dc_1/dx < 0, dc_2/dx > 0.$$

Both $c_1(x)$ and $c_2(x)$ are constant over time and begin in the year projects are undertaken. Land used for project 2 earns no income between 0 and T .

Land market equilibrium requires that the present value of each project be equal on a per unit of land basis. If the annual discount rate is r , the present value of a unit of land for project 1 is

$$V_1 = c_1(x)/r.$$

For project 2 it is

$$V_2 = e^{-rT}c_2(x)/r.$$

Thus in equilibrium, x must satisfy

$$c_1(x) = e^{-rT}c_2(x). \quad (1)$$

This is more than an equilibrium condition; it is also necessary and sufficient for an efficient land allocation. It implies that $c_2 > c_1$; since project 2 requires a gestation period without income, its annual income must exceed that of project 1 once it begins.

Now let us suppose that an annual tax on the income from land is introduced. If the tax rate is b and the discount rate does not change, the present value of a unit of land for project 1 is

$$V_1 = c_1(x)(1 - b)/r.$$

For project 2 it is

$$V_2 = e^{-rT}c_2(x)(1 - b)/r.$$

These must be equal in land-market equilibrium. Since this upholds (1), the land

allocation is the same as before. An annual tax on the *income* from land is neutral toward resource allocation.

II. The Tax on Land Value

Suppose instead that an annual tax on *land value* is introduced. Let the effective tax rate be b . In this case, the present value of a unit of land for project 1, V_1 , must solve

$$(c_1(x) - bV_1)/V_1 = r,$$

where annual property taxes are bV_1 . This means:

$$V_1 = c_1(x)/(r + b). \quad (2)$$

The present value of a unit of land for project 2, V_2 , is less easily derived. This is because the return to the owner of such land between 0 and T comes in the form of an increase in its market (and assessed) value rather than current income. Let the market value of a unit of land being held for project 2 at time t , $0 \leq t \leq T$, be $V_2(t)$. By the argument employed above to derive V_1 , we obtain

$$V_2(T) = c_2(x)/(r + b). \quad (3)$$

Between 0 and T , $V_2(t)$ must increase at a rate that provides the land-owner a rate-of-return (in the form of capital appreciation) of r after property taxes are paid. At t , property taxes are $bV_2(t)$, so this means that for $0 \leq t \leq T$,

$$\frac{\dot{V}_2(t) - bV_2(t)}{V_2(t)} = r,$$

or

$$\dot{V}_2(t) = (r + b)V_2(t).$$

This implies that $V_2(t)$ grows exponentially at the rate $r + b$, and that

$$V_2 = e^{-(r+b)t}V_2(t), \quad \text{for } 0 \leq t \leq T. \quad (4)$$

Solving (4) for $t = T$ using (3), we get

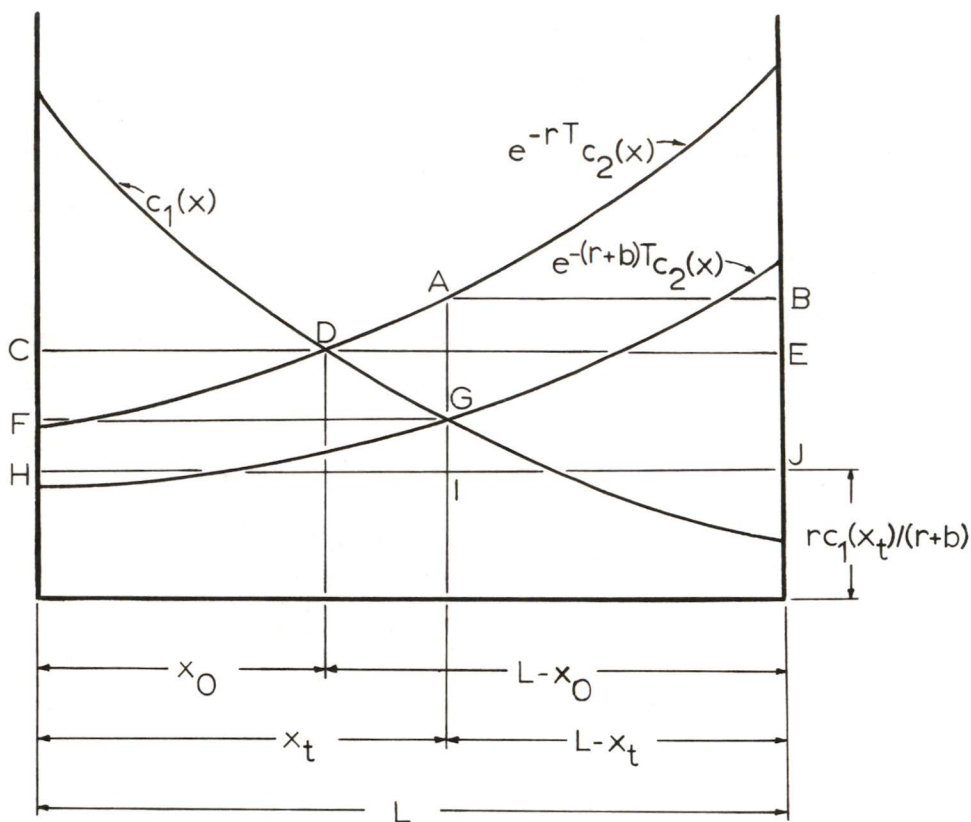


FIGURE 1

$$V_2 = e^{-(r+b)T} c_2(x) / (r+b). \quad (5)$$

In equilibrium, (2) and (5) must be equal. (1) is not upheld in this equilibrium, but instead we have

$$c_1(x) = e^{-(r+b)T} c_2(x). \quad (6)$$

(6) establishes the non-neutrality of the land value tax. By implicit differentiation, (6) implies that

$$\frac{dx}{db} = \frac{Tc_1(x)}{(e^{-(r+b)T} dc_2/dx - dc_1/dx)} > 0.$$

When a land value tax is introduced, or the tax rate increased, x increases. This means some land that would otherwise be used for project 2 is diverted to project 1.

The tax favors the project with the early-payoff income stream.

An intuitive explanation for this bias is supplied as follows. Both (2) and (5) indicate that the imposition of the tax has an effect on land value *equivalent* to raising the discount rate to $r + b$. This is so because, as Bentick says, landowners "will require a gross rate-of-return of $r + b$ in order to continue enjoying a net return of r " (p. 863). It is elementary that an increase in the discount rate would favor the project with the early-payoff income stream. Because the land value tax has an equivalent effect, it too distorts resource allocation.

This bias and its resource cost are illustrated in *Figure 1*. Units of land are plotted horizontally and x is plotted from

the left. The slopes of labeled curves reflect earlier assumptions. The c_1 curve represents the annual derived demand for project 1 land beginning in year 0. Thus, the present value of net benefits (regardless of their distribution) provided by x

units of such land are $\int_0^x (c_1(y)/r)dy$. The

c_2 curve (not shown) represents the annual derived demand for project 2 land beginning in year T . The present value of net benefits provided by this land is

$e^{-rt} \int_x^L (c_2(y)/r)dy$. The latter is equivalent

to having an annual derived demand for project 2 of $e^{-rt}c_2$, as shown in the figure, beginning in year 0.

This equivalence can be exploited to illustrate conditions (1), (6) and their respective land allocations. Then it can be used to demonstrate the incidence and burden of the land value tax in *annual-equivalent* terms.

With no tax on land, (1) holds (see D) and x_0 units of land are assigned by the market to project 1. The remainder is used for project 2. This means land is allocated so that the net benefits produced in both uses are equal at the margin. After the land value tax is introduced, (6) holds (see G) and project 1 gets x_t units of land where $x_t > x_0$. With the increase in x , net benefits are not equated at the margin; too much land goes to project 1.

Because of this, demanders of project 1 gain at the expense of project 2 demanders. In annual-equivalent terms, the gain to demanders of project 1 is CDGF; the loss to demanders of project 2 is ABED. Landowners also lose. Before the tax, their annual-equivalent income is $c_1(x_0)$ for each of the L units (again, see D). After the tax it is rV_1 , or from (2), $rc_1(x_t)/(r+b)$ for each unit of land (see I). Thus, their annual-equivalent loss is CEJH. The tax revenue gain from each unit of land is of course the difference between the revenue paid by demanders— $c_1(x_t)$ with project 1 and $e^{-rt}c_2(x_t)$ with project 2—and the after-tax income received by landowners. The annual-equivalent gain in tax revenues is thus FGIH + ABJI.

When the gains in this analysis are set against the losses, there is a net loss equal to AGD. This is an annual-equivalent measure of the resource cost of the site value tax in every year.

A rough notion of the empirical magnitude of this annual resource cost can be provided. AG in Figure 1 equals $(e^{bT} - 1)c_1(x_t)$, so the annual resource cost is approximately $(e^{bT} - 1)c_1(x_t)/2$ per unit of land diverted. If for instance $b = .02$ and $T = 5.5$ years, the annual resource cost per each ten units of land diverted equals $c_1(x_t)$ —the annual income from one unit of land used for project 1. The amount of land diverted depends of course on the magnitude of b , T , and the elasticities of the c functions.

III. Conclusion

The fact that real property taxation fosters inefficient land use by penalizing structures and improvements has been voiced by tax theorists for some time. It has been a major consideration among those who favor eventual displacement of the tax with a pure tax on site value. The chief virtue of the latter is held to be its neutrality. Daniel M. Holland, for instance, has said that:⁵

"supporters of a tax on site value as a *substitute for the property tax now in use* make their case by stressing the neutrality of the former. Unlike the tax on improvements, a tax on site value would be invariant with the development decision. What was the optimal development in the absence of the tax will remain optimal in its presence" (emphasis in original).

The neutrality claim can no longer be maintained in the case of a tax based on land value. (The claim is sustained for a tax on land-generated income, but administration of such a tax is fraught with formidable problems.) This should provoke a reconsideration of the presumption that a site value tax is more efficient toward resource allocation than a property tax. Granted, two distortions are operative in the latter: the traditional one penalizes capital-intensive projects and the other favors projects with early-payoff income streams. But if a property tax is to be re-

placed with a site value tax producing equal revenue, the tax rate applied to land value must rise significantly with the switch. While this eliminates the first distortion, it enlarges the second. It is therefore possible (although by no means certain) that the resource cost of the site value tax is actually greater than that of the property tax.

Whether this would occur is of course an empirical question. It is not likely to be one easily resolved. The first distortion above has been well understood for some time, yet empirical efforts to measure the magnitude of its effect have been of limited success.⁶ The intertemporal bias that is the subject of this note is considerably more subtle. Empirical measures of its importance will certainly be no easier to obtain.

FOOTNOTES

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¹For instance, Dick Netzer writes (*The Economics of the Property Tax*, Washington, Brookings, 1966):

"[T]he site value tax will be entirely neutral with regard to landowner's decisions, since no possible response to the tax can improve the situation. . . . [I]t is a lump-sum charge to the owner" (p. 205).

²Brian L. Bentsick, "The Impact of Taxation and Valuation Practices on the Timing and Efficiency of Land Use," *Journal of Political Economy*, V. 87, no. 4, (August 1979): 859-868.

³This result is not without precedent. Donald C. Shoup demonstrated the bias a decade ago in "The Optimal Timing of Urban Land Development," *Papers of the Regional Science Association*, V. 25 (1970): 33-44. He observed that a site value tax creates an effect on development-timing decisions that is identical to that of an increase in the discount rate:

"It is interesting that the difference between this optimal development timing condition and the one found previously in the absence of a land tax is one sense in which a pure site value tax may not be perfectly neutral in its effect on resource allocation, as is frequently claimed" (p. 39).

The bias is also described by Roger S. Smith, "The Effects of Land Taxes on Development Timing and Rates of Change in Land Prices," in *The Taxation of Urban Property in Less Developed Countries*, Roy W. Bahl, ed., Madison, Wisconsin, 1979.

⁴The homogeneity assumption means land parcels of equal size are perfect substitutes. This removes the possibility that some part of land rent is due to locational, topographical or other site-specific land features. To assume that parcels are imperfect substitutes would increase the model's generality but complicate the issue at hand unnecessarily. An alternative and more practical interpretation of the homogeneity assumption is to define a unit of such land using William S. Vickery's "standard state" basis ("Defining Land Value for Taxation Purposes," in *The Assessment of Land Value*, Daniel M. Holland, ed., Madison, Wisconsin, 1970). The two-project assumption is used merely to facilitate a graphical representation of land-market equilibrium; the analysis generalizes too many projects. The fixed-quantity assumption is a convenient way to incorporate the fact that land in the jurisdiction is scarce. If it is not, land rent is zero and tax distortions are irrelevant.

⁵P. 6, "Introduction," *The Assessment of Land Value*, Madison, Wisconsin, 1970: 3-8.

⁶This is admitted by Donald C. Shoup ("The Effect of Property Taxes on the Capital Intensity of Urban Land Development," *Metropolitan Financing and Growth Management Policies*, Madison, Wisconsin, 1978: 105-132), who makes a careful attempt with two case studies involving micro-data.

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Rising Rent and Its Effect on Land Values

P = Price of land

a = Annual rent, year zero

i = Rate of interest

g = Annual increase in rent, as a percentage

n = Number of years

Tool formula. The sum of a geometric progression is:

$$S = a \frac{1 - r^n}{1 - r} \quad (1)$$

where

a = first term

r = rate of progression

n = number of terms

P is the sum of discounted future values.

$$P = \frac{a}{1+i} + a \frac{(1+g)}{(1+i)^2} + \dots + a \frac{(1+g)^{n-1}}{(1+i)^n} \quad (2)$$

(2) is a progression.

The first term is $\frac{a}{1+i}$

The rate is $\frac{(1+g)}{(1+i)}$

$$P = \frac{a}{(1+i)} \left[\frac{1 - \left(\frac{1+g}{1+i} \right)^n}{1 - \frac{1+g}{1+i}} \right] = a \left[\frac{1 - \left(\frac{1+g}{1+i} \right)^n}{1+i - 1-g} \right] = a \left[\frac{1 - \left(\frac{1+g}{1+i} \right)^n}{i-g} \right] \quad (3)$$

For land, $n = \infty$, so (3) becomes:

$$P = \frac{a}{i-g} \quad (4)$$

If Smith, Brown and Jones have different credit ratings and borrowing rates, the effect of g is to exaggerate the effect of those differences when they enter the market for land. Table 1 shows the leverage effect of g .

T. Leman

Rising Rent...--page 2

Table 1.

<u>Farmer</u>	<u>His Own Interest Rate</u>	<u>Capitalization Rate i - g, where g = .06</u>	<u>Land Value, Derived from a = 100</u>
Smith	.08	.02	5,000
Brown	.09	.03	3,333
Jones	.13	.07	1,428

The advantage of the lower "own interest rate" (sometimes called "internal interest rate") in bidding for appreciating land is very great.

We gain further insight by juggling (4) around.

$$P_i - P_g = a$$

(5)

$$P = \frac{a + P_g}{i}$$

P_g is the annual increment of land value, so (5) says that we value land by capitalizing the sum of ordinary income, (a), plus the capital gain, (P_g).

But now note that the ordinary income is subject to regular income tax rates, while the gain is never taxed unless and until the farm is sold.

The annual gain in value of farm land over 1950-79 has about equaled the annual income, before taxes. After taxes, the P_g element carries more weight than the a element.

This is what gives so much weight to credit ratings and tax shelters in farming.

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August 12, 1981

Dear Nic:

Thoughts on your MS, up to p.6.

1. When land price is rising, a high tax rate on the price will exceed current rent. But, the high tax rate (anticipated) will reduce the annual increment absolutely and, if high enough, eliminate it entirely. The last is presumably our ideal goal. Yet, our theorizing and communicating is mostly premised on a moderate tax rate, where we must cope with the fact of the annual increment to price. Does this lead us into difficulties at any point? (I am thinking out loud, here.)

2. What is rent, where the future is infinite, irregular, unpredictable (with certainty), and (probably) rising in value? Is it different every year? Does it include price increments (dP)? Is it levelled over the life of each discrete lump of capital improvement? *See 14-16 below.*

These questions could lead us to address the question whether it is, as alleged by N. Ture et al., "double taxation" to tax future ordinary income after already taxing dP. This gets into some very fundamental unresolved issues, and broadens the topic and the audience, and deepens the import.

3. A (minor?) flaw in Bentick is to have the investment date (I-date) of G-1 fixed at $t=20$, unaffected by the land tax. Presumably the tax would, along with its other faults, advance the optimal I-date. This is the point made by Don Shoup, Roger Smith, Peter Mieszkowski, et al. *See 13, below.*

4. A useful device is to define "carrying costs", $\int (c.c.)$, equal to $V(m+r)$, and show them to be a constant, because $\log V$ and $\log (m+r)$ change equally and inversely. This works when future incomes are level; does it also work when they are rising? *more simply, because $V = \frac{a}{m+r}$*

5. As discussed, let's find another term than lump sum.

6. As discussed, I would not be hard on people who want to tax the income from land. The conventional pro-income tax argument is that the best use after tax is the same as that before tax. This argument is so strongly embedded that we would do better to use the belief in it, rather than confront it. The argument is used wrongly by Friedman re personal efforts: wrongly because much of the cost of producing and fielding a person is not deductible, so it is a tax on gross, not net income. But ^{re} a tax on land income, most and perhaps all of the costs are or could be deductible.

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The cost of entrepreneurship may, in some ultimate sense, be non-deductible, that is there is a kernel of ultimate owner responsibility that cannot be hired out. This argues for supplementing a land income tax with an independent tax based on assessed value. I certainly go along with this. But, in the context, we need a stronger argument on p.2. We must avoid any impression that we are reduced to nit-picking. There is a major point to make, and we should focus reader attention on that.

The argument for the property tax rather than income tax approach to taxing land rent gets down to questions of how perfect can markets be assumed to be; how perfect do we want them to be; which tax will make them better? But in our context, these are secondary. The primary difference is that: a land income tax is based on ordinary income; a property tax also taxes increments or dP. Mills, I take it, recommends taxing only ordinary income. We, then, need to: A, congratulate Mills for coming this far; and B, show the need to tax dP as well, and the lack of bias in doing so. And, of course, clarify the definition of the property tax base which Mill & Bentick fumbled.

7. As discussed, the examples of perpetual improvements stack the deck against addressing some relevant points: interim uses; why the prior use (there presumably was one) was discontinued; the advantage of an earlier G-2 if we build the smaller improvement; the advantage of reduced obsolescence when you build less, and replace more often.

8. p.3, assuming a local tax and an open economy seems unnecessary here. A global land tax in a closed economy should also not change the required interest rate (except indirectly, by revving up investment demand.) *(But it would also increase saving rates.)*

9. A useful concept is "highest and best use of land," as basis of assessment. This is lingua franca.

10. A succinct statement of B&M's point is that deferred rents are discounted at the rate $r + m$, and should optimally be discounted at only r . Can we refute this succinct statement? *See 13, below.*

I have often fallen back on the weaker position that, in a world where buildings are taxed at m , it is neutral to tax land values at the same rate, for uniformity. But what if buildings are exempt?

We can easily show that B&M erred by assuming that land assessments would be frozen at the level of the lower use. But we could then try to state their case more strongly, and meet that challenge. Perhaps that is what you have done -- read on.

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11. To refute B&M, we should begin by pursuing their numerical example and refuting it in its own terms. You presently leave it hanging. I suggest showing how the tax would rise to more than the rent of the lower use, and showing what its PV would be after such a tax -- probably negative, although I haven't worked it out.

12. Re the elegance of using e , it introduces substantial inaccuracies at high discount rates. Consider:

$$1.22^{100} = 432.5 \times 10^6$$

$$e^{22} = 3584.9 \times 10^6 \quad \text{or } 8.3 \text{ times as much}$$

Monthly payments come closer to continuous ones:

$$(1 + .22/12)^{1200} = 2900^{37} \times 10^6 \quad \text{or } 82\% \text{ of } e^{22}$$

The relevance of this is that payments of rent and interest are often monthly, while tax payments are annual. This can reduce the effective tax rate substantially (more so, of course, where assessments lag.) In an inflationary period, with high nominal interest rates, this factor assumes a new importance.

This is only tangential to the instant case, but could make an article in itself later. For now, we could note that the use of e is an approximation, but it is done for ease of computation, and with recognition that it is not always appropriate.

13. Most taxes are levied on a base which is the value before tax. Property taxes are levied on a price derived from the income after tax -- we call this tax capitalization, which diverts attention from part of what is being done. This, as you know, calls for extremely high apparent rates in order to collect a large share of the rent.

These high rates, then, are used to discount deferred rents, and determine the growth path of speculative value before a site is put to its best future use.

Now, I was about to say that this in turn may lead to a distortion of the kind claimed by Shoup et al. But it just occurs to me that when the site value is growing at rate $r + m$, the m part of the growth rate is entirely a product of tax capitalization. So ripeness occurring when the rate equals $r + m$ is the same date that would be optimal with no tax, and rate equals r alone. This could be a simple way to expound our point -- if it is valid. I will think some more -- pray you do too, and advise me.

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14. The word "rent" originated in the annual cycles of farming (although farming has always had longer cycles, too.) We need to adapt it to discrete multi-annual cycles. It is a notional level value corresponding to a cycle of varying cash flows. But when the flows are secularly rising, and the discrete cycles are overlapping and commingled, what to do?

It is necessary I think to have a distinct figure each year, to call rent. If the tax rate is low, this figure is the sum of the ordinary income plus dP , for reasons we have discussed. But if the tax rate is high, there is no P , or virtually no P , and therefore virtually no dP . Thus, land taxes destroy the tax base even more drastically than in the case of a level perpetual income. However, this is not wrong. There is still a dP , but it accrues now to the fisc, it is the present value of anticipated future tax revenues. It is still part of rent, but it is being publicly appropriated -- not in current cash, but the PV of future tax claims. A formal model that made all this explicit and unambiguous and clearly internally consistent would be very valuable.

15. One conceivable solution to 14 would be a tax that was level in perpetuity. We would discount all future cash flows, and multiply this PV by m . Ah, but hold on, this tax would not be level, it would rise each year. But now, let us calculate a level tax of the same PV as this rising tax. We could levy such a tax. This would be carrying the concept of a level annual notional rent to an extreme. This tax would not participate in rising rent, but anticipate them. The landholder would build up an equity in early years, for a later payoff as current rents rose above taxes.

I think we would agree that this level tax is not desirable. Rather than a once-and-for-all leveling operation, we need a moving DCF, as our time horizon -- always infinite -- moves inexorably ahead. And this leads us back to where we started, a tax base which is the current DCF. It is perhaps by looking at alternatives to this that we begin to see how many problems it solves.

16. Another alternative is to base the tax on the capitalized rent before tax: the sum of the private equity plus the public equity. (Ben Smith, a retired engineer with a good mind but cranky tendencies, has pushed this one.) This is a good mind-stretcher, dramatizing the point that the land tax is the only tax whose impact is reduced by being based on the after-tax value.

17. On p.4, 2d para. begins too abruptly. Signal is needed that the mood changes: instead of rendering B&M, we are refuting them.

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18. What have you in mind, p.4, for my part? You mentioned a history of thought section on the phone. Is that it?

19. pp. 4,5. Value does not depend on "the" investor's r , but the r of the highest bidder.

20. p.5, Eqns. 5,6. This level of math suddenly screens out 90% of the NTJ readers. I admit that I must return to my math books to follow it.

One solution is to expand on it. Part of math. elegance is to compress and be elliptical, thus flattering your readers that they can supply the missing operations because of course, like all right-thinking people, they use differential equations daily. It may be great for mathematicians, but for most economists it is something of an affectation, one which has made many journals highly exclusionary. On the other extreme this creates a counter-culture of institutionalists who reject all math as inherently evil. NTJ is somewhere in between.

Another overlapping solution is to use the simpler simul. eqns. derivation to introduce what you are doing, then offer your solution as a generalization from it, for the present problem.

It might also help to set $t = 0$, thus eliminating the confusion of t and τ which adds clutter. It has also confused your typist, I believe, in the two lines after eqn. 3, where t should be τ .

21. "Millage rate" could be confusing. In common usage, millage means percentage times 10. (31 mills means 3.1%, etc.) Your meaning is evidently true percentage tax rate, where "true" means based on market value (rather than conventional fractional assessments.)

This is as far as I have gotten to date. Nic, I must drop this for a few days to finish another article with a deadline. But perhaps these comments will be something to chew on in the meantime. I have left many loose ends, hoping that perhaps you would apply your great conceptual ability to tying them up. It is easier of course to see problems than to find solutions, and I have just taken the easy way for the time being.

Sincerely,

Mare



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Aug 28, 1981

Dear Mason,

You will find enclosed a sample of what the math looks like in discrete terms and a proof that for a PIPO investment carrying cost is not independent of the property tax rate.

I look forward to talking with you.

Nic

10 AM

Tue, 15th

not the absolute size of either. Thus a gas station can sometimes compete with an apartment; though present value of revenues is less, so is construction cost. But the effect of building taxes varies with C , the tax base. As between two uses equally high and good, i.e., with an equal *difference* of PVR and C , the building tax intercedes in favor of the one of lower construction cost (C). Although its revenue is less, the gas station outbids the apartment because the apartment would have paid more building taxes.

This is a matter of leverage. A given percentage increase in cost cuts deeper into the residual land value afforded by the more intensive use, because its cost is higher relative to the land value. Let us give that some precision and generality.

We begin by converting the stream of future building taxes to a lump sum, their present value (PV). "Present value" of the stream means if you borrowed PV and paid it off on the installment plan over the life of the building, your annual payment would be the amount of your building tax. The PV of an annual payment of \$1 over sixty years is a lump sum of around \$13 (discounting future dollars at 7 percent per year compounded). So a property tax rate of 1 percent of building cost is equivalent to a present value of 13 percent of building cost.

This comes out of what a builder can bid for land. He reduces his bid by 13 percent of the cost of the planned building (C). The higher is C , the more the disadvantage.

Let us couch this in terms of the percentage drop in what competing uses can bid for a site. The absolute drop, for each 1 percent of tax rate, is:

$$(2) \quad -\Delta S = 0.13C.$$

That drop as a percentage of site value is:

$$(3) \quad -\Delta S/S = 0.13C/S.$$

C/S for a high-rise structure might run 8/1. Since $8 \times 0.13 = 104$ percent, the tax reduces the bid by more than 100 percent and so wipes out the site value.

For a \$70,000 gas station on a \$140,000 hot corner, C/S is $\frac{1}{2}$. Since $\frac{1}{2} \times 0.13 = 6\frac{1}{2}$ percent, the oil company can bid only $6\frac{1}{2}$ percent less than if there were no taxes. The effect of building taxes is to give the less intensive use a comparative advantage over the more intensive.

That does not mean the total abolition of high-rise buildings everywhere. This is not the way the world works. It means gas stations get more land, and better land. (They also spread out.) Apartments get less land, and worse. (They also are built shorter.) Gas stations move into the center; apartments

move outwards. This helps account for the anomaly of intensive uses popping up on poor land and mixed in with much lower uses, while low uses preempt much of the central land. In general there is a poorer matching of buildings and uses with sites.

The above is a picture of urban disintegration. Our ways of imposing taxes play a role in making cities sprawl. Sprawl, in turn, with attendant suburban enclavization, weakening of metropolitan community ties and facilities, and automobile dependency, spawns many of the problems of class conflict that concern us now.

More directly, the bias against uses with a high building-cost/site-value ratio (C/S) is a bias against the poor, who live crowded at much higher density than the rich and on land of lower unit value as a rule.

I noted earlier that the tax on buildings affected incentives somewhat as would a rise of interest rates. Here we reach the limits of that parallel. The building tax is more specifically targeted against intensive use than is the interest rate. In the extreme, on an unpaved parking lot yielding income with no building, the building tax does not lower its value a bit, while a higher interest rate would lower the value. More generally, in (1), building taxes are proportional to C , while higher interest rates have an effect which is proportional to PVR . Thus the artificial scarcity of capital caused by the building tax is more disruptive to the integrity of urban linkages than is a natural scarcity of capital reflected in high interest rates. Indeed, high interest rates would also make roads and allied infrastructure costlier, raising horizontal transportation costs and raising the premium on central location.

Ripening of Land for Higher Use

Criteria of ripeness. Under dynamic conditions, land is often in transit from one use to another and usually higher use. In anticipation of a move, it develops an "expectation value," or speculative value, that is higher than income from the current best use will support. When should the owner take the quantum jump and initiate the higher use? When is the land ripe for the change?

The choice of ripeness date (D) is difficult because a durable building, indivisible in construction, must be placed on the land to shift its use. As demand for the site grows with each succeeding year, the hypothetical optimal improvement that one would put up if he were going to build in that year changes. Each succeeding year's optimal building yields more net present value to the land.

To avoid premature, preclusive underimprovement or other irreversible error one postpones building — but not forever. R. T. Ely identified himself with the doctrine of "ripening costs" in which he argued the case for deferral,

but he failed to supply a criterion for ripeness. *D*-date (ripeness) arrives when the value imputed to the site by each succeeding year's hypothetical optimal building stops rising faster than the interest rate.⁷ (This is the same as selecting the date that maximizes present value of the land as of any fixed calendar year.) By not building in Year 1 you forgo — and thus in effect invest — the present value of site (S_1) realizable by building in Year 1 to gain S_2 in Year 2, or $S_3, S_4, \dots, S_n$. If any

$$(4) \quad S_n > S_1(1 + i)^{n-1}$$

then the value of holding the uncommitted site grows faster than money in the bank and is a good investment.

In addition, there is current site income (a_1) from the tag end of the prior use, or from some interim use. Adding these in, a site is not ripe so long as

$$(5) \quad (a_1 + \Delta S)/S_1 > i.$$

Note that ΔS is not just the yearly rise of land value in the market, but is $S_2 - S_1$ as defined in the previous paragraph. S is "use value." It is below market value until the optimal year of building, at which time use value has risen to equal market value. Market value is use value at *D*-date (ripeness) discounted, so market value grows at the investor's rate of interest. Use value (S) grows faster than that until ripeness, by definition of ripeness.

Taxing buildings affects ripeness. We have seen that taxing buildings reduces site values derived from buildings and applies leverage against intensive building. It follows that taxing buildings affects the growth rate of site values, assuming that the optimal building-cost/site-value ratio (C/S) changes with ripening. Thus taxing buildings affects the date of ripeness.

I shall show the following. As land ripens, the effect of taxes on the ripeness date depends on whether further waiting would lead to a higher or lower C/S ratio. Normally it leads to a lower ratio; C rises, but S rises by a higher rate. The effect of taxes is then to retard ripeness. They make one more disposed to sacrifice an earlier for a later use.

This is the kind of elusive relationship that algebra was invented to nail down. To simplify, assume no current income from a prior or interim use. To simplify the notation, the present value of the stream of future net revenues,

⁷Mason Gaffney, "Replacement of Individual by Mass System," *Proceedings of American Real Estate and Urban Economics Association*, vol. 4 (1969), pp. 21-68. This piece also treats the effect on ripeness of later generations of use, a point omitted here.

PVR in equation (1), is denoted simply as R . The growth rate of use value is:

$$(6) \quad \frac{\Delta S}{S} = \frac{\Delta R - \Delta C}{R - C}.$$

Let T be the present value of future building taxes, expressed as a percentage of C . (We previously illustrated T as 13 percent for a 1 percent tax about 15-40 percent is normal, depending on the tax rate and the discount rate.) After taxes, assumed to lodge in lower site values:

$$(7) \quad \frac{\Delta S}{S} = \frac{\Delta R - \Delta C(1 + T)}{R - C(1 + T)}.$$

Taxes proportional to ΔC reduce ΔS , because ΔC comes out of ΔS . But also reduce S , the denominator of the rate. Which factor prevails? The rules of algebra provide an answer.

The effect of taxes is to raise the growth rate of S if:

$$(8) \quad \frac{\Delta R - \Delta C(1 + T)}{R - C(1 + T)} > \frac{\Delta R - \Delta C}{R - C}.$$

By cancellation (or inspection), (8) implies

$$(9) \quad \frac{\Delta C}{C} < \frac{\Delta R}{R}.$$

By inspection, (9) is true if C/R is falling. I shall call C/R the cost/benefit ratio. From (1), R is $(C + S)$, the sum of building and land value, so:

$$(10) \quad \frac{C}{R} = \frac{C}{C + S}.$$

Thus C/R is simply the share of building cost in real estate value. Falling C/R implies falling C/S as well.

Intuitively we think that land ripens into higher intensity, but intuitively is a blind guide here because it compares incommensurables: building value and land area. Economic intensity compares commensurables: building value and land value. Our C/S and C/R are proper indices to economic intensity.

For several reasons, I believe ripening usually entails a drop in C/R . C rises, but R rises at a higher rate, so C/R falls. These reasons are developed in section II of the Appendix. So the usual effect of taxing buildings is to retard ripeness by raising the percentage growth rate of the use value of the site. I conclude that the paramount effect of building taxes on ripeness is to defer it.

The demonstration above must be tempered to allow for the evasive action of the taxpayer faced with the threat of a tax on buildings. This modification is in section III of the Appendix.

Occasionally, ripening would lead to a higher C/R . Then the effect of building taxes is to advance ripeness by making the later use relatively less attractive. The point of consistency is that intensity loses either way. Taxing buildings makes landholders more patient to wait for lower uses, but less patient to wait for higher uses. But section II of the Appendix shows that excessive patience is the rule.

Neighborhood effects add to the retarding influence of building taxation. Part of ripening is not waiting so much for greater demand but for greater certainty. Certainty means waiting for neighbors to commit themselves. But Alphonse waiting for Gaston simply perpetuates uncertainty when Gaston is waiting for Alphonse. Much of the rationale for ripening is a hyperindividualistic one that does not bear examination from a social viewpoint and can only be painted a social good by committing the fallacy of composition. "We have no plans," said a San Francisco land speculator, "we're waiting for other people's plans." In such a context, whoever leads off ripens his neighbor's land and shortens the sterile downtime of land between major improvements. Building taxes that retard the improvement of one site thus retard the ripeness of neighboring complementary sites by generating uncertainty. Uncertainty of this kind in a highly complementary urban neighborhood is an external nuisance every bit as noxious as odors, fumes, noises, and shadows.

I remarked earlier that the property tax on buildings affects investor behavior somewhat as would a rise of the interest rate. In respect to ripening that is not true. A higher interest rate would also require the use value of sites (S) to grow faster to remain unripe; but a higher building tax rate has no such effect. Indeed, the macro nationwide effect of having buildings taxed in all jurisdictions is to lower the level of interest rates that investors require land to earn.

Turning to land taxes, they would be largely neutral if credit markets were costless. It is widely believed that they speed up ripening, but the belief has been wrongly rationalized. It rests mostly on assuming that land taxes are piled on top of interest costs of holding land. But land taxes are capitalized into lower values, and thereby supplant interest costs rather than supplement

them, as already noted. The reason that land taxes hasten ripening is that ripening land is mostly held by strong hands whose comparative advantage lies in holding assets where the main cost is paying interest on loans and foregoing interest on equity. Hastening the ripening of such land is simply an aspect of the transfer from rich to poor that land taxation effects via differential capitalization (see Appendix, section I).

Frequently the date of ripeness is outside the owner's direct control and depends on when public works are extended. Today, in many suburban areas, sewers are controlling. Here, land taxes cannot speed ripening until sewers are built. But they can then speed private building to match public building and effect great savings on public capital of all kinds.

Land taxes also have important distributive effects. Future sewers have a present value to landowners. Values rise above farm levels — not once and for all, but incrementally along a line wiggling around a basic compound interest growth curve. This annual accrual of value is a current income, in the true economic sense, just as depreciation is a current cost. Land taxes levied *ad valorem* on the base of this selling value are a way of tapping this accruing income for the public. Appreciation is proportional to value; the tax is proportional to value; therefore the tax is proportional to the appreciation.

Arguments against taxing appreciating land do not therefore hold water on the grounds on which they are usually presented — i.e., that ripening land yields no income. They do, however, make sense to the local interests whose welfare is the bottom line of the municipal enterprise. Land taxes redistribute income from landowners to other voters and immigrants. From the parochial Mercantilist view this is bad. From the national view, where the welfare of migrants and labor are a greater concern, it might be valued more positively.

It is traditional to blame premature building and sprawl on *ad valorem* assessment of ripening land. Premature extension of public works is more guilty, coupled with postmature conversion of ripe land close-in, made unripe or submarginal by taxes on building.

Aside from wealth effects, land taxes are neutral in their effect on date of conversion, so long as they are not contingent on the date selected. Noncontingency is the same principle that makes land taxes neutral towards other land use choices. Suppose someone were panicked by rising land taxes into premature urbanization of farmland, as is sometimes feared. What would he gain? Either he would overimprove and lose money the first few years; or he would gauge his building to the slim early market, and in a few years be locked into an underimprovement while his land assessment and taxes kept on rising.

If he behaves rationally he will not convert land sooner because of tax carrying costs. The time permanently to convert land use, with or without

taxes, is when the rapid rise of value begins to taper off. Then the land is ripe. This happens when the city has grown out to abut the land in question. Land taxes should tend to help conversion be less disorderly than now by equalizing carrying costs among owners of different credit ratings. Land assessments should also be given a quantum jump when public works are supplied, triggering action, prompting owners to fill in compactly and developers to sell quickly, lest the public investment in the works be sterile.

Land taxes would encourage premature conversion if the assessor maladministered them, i.e., if he raised them until the owner improved the land, then locked the assessment at that level. Under that system, the owner could buy himself a low assessment by early underdevelopment. Some assessors do maladminister like that. The solution is the building-residual method of assessment: assess the land at what it would bring if vacant; then assign the building the residual value.

Aggregate Local Effect of Property Taxes on Supply of Buildings

Building taxes. The overall effect of taxing buildings is to reduce the service flow from a municipality of given area. We have seen this in several aspects.

Intensity falls, in terms of quality, durability, and height especially. There is some compensatory tendency toward increasing lot coverage, the strength of it depending on whether people have somewhere else to go. If they do, lots get larger as buildings ramble.

Site renewal slows down greatly. This slowdown, coupled with less durable buildings, creates slums and out-of-service land. Each old building robs neighboring sites of their renewability, and the extreme result is nonrenewable, abandoned neighborhoods.

Building taxes magnify the motive to withhold land for ripening. The virtue of avoiding premature commitment is distorted into the vice of postmature commitment. Within a neighborhood the delay can be indefinite, as each separate owner, waiting for certainty, imposes uncertainty on others. In the still larger scene of the entire metropolis, postmature building in truly ripe areas disperses demand outwards. Development takes place in outer areas that are made to look ripe from a local viewpoint, even after taxes, but are grossly premature in the regional plan. This is the more likely when central city tax rates become substantially higher than suburban.

Taxing buildings tends to favor larger parcels and discourage subdivision and apartments.

Last, taxing buildings weakens the relative bidding power of more intensive uses in competing for land, and changes the structure of cities. Gas

stations and parking lots push high buildings out of the center. Out-of-service lands break up complementary clusters, and cause urban sprawl.

The combined effect is to reduce the service from any given amount of land and to diffuse demand over a wider area than is necessary, economical, or socially desirable. This result is a kind of forced consumption of land, plus forced consumption of capital, as sprawl inflates the infrastructure costs of urbanization. This heavy capital cost dashes the hope that moving out to cheap land in the old frontier tradition may open up land for housing the mass market.

Land taxes. The overall effect of taxing land is toward equalizing the intensity of use between rich and poor by displacing regressive interest costs with the tax. That means intensifying the use of land, because most land is held by the rich; but it means more land per family for most families, because the median family is far below the mean in income and even lower in wealth.

Taxing land is redistributive, the more so when the proceeds go to finance schools and welfare. So long as localities were infected with a wish to grow, and an egalitarian philosophy, land taxes found strong support. In today's mood of local Mercantilism, which questions growth and leans to elitism while still seeking local efficiency and community, tension and ambivalence surround the local taxation of land.

EFFICIENCY, EQUITY, AND POLICY

Policies of forced consumption of land and capital achieve their ends at high cost in social efficiency. They reduce density below what sovereign consumers evidently desire, running up heavy public costs in the process. They retard renewal and create slums. They frustrate the mass market desire for cheap land and cheap housing.

They make cities disintegrate in several ways. Building taxes weaken the relative power of intensive uses to compete for the most accessible land. Underemphasis on land taxes, as by underassessment and zoning dodges, causes the most rentable land to go to the strongest hands with superior financial power, rather than to the most productive use. A weak seller far out often looks better to a builder than a strong holdout close in. Zoning as we practice it is self-defeating, because a builder's biggest profit comes from breaking the zoning rather than following the rules and paying a high price for land already zoned to his needs. Long extension of the ripening period creates too many niches for shoddy interim uses that disfigure the American city.

$$\text{Ratio of Carrying Costs } \frac{c_p}{c_r} = \frac{0.08 - g + t}{0.05 - g + t}$$

$t \backslash g$	0	0.02	0.04
0	8/5	6/3	4/1
0.02	10/7	8/5	6/3
0.04	12/9	10/7	8/5
0.06	14/11	12/9	10/7
0.08	16/13	14/11	12/9
0.10	18/15	16/13	14/11

II. Why Land Generally Ripens into a Lower Intensity

Let C = capital cost of building

R = present value of future net revenues or cash flow from building

S = use value of site = $R - C$

$$\frac{C}{R} = \frac{C}{C + S} = \text{cost/benefit ratio } (C/R).$$

1. There is a hierarchy of land uses, qualitatively different. While each one is of variable intensity, they are discrete, with a quantum jump from one step to the next, as from grazing to row crops, row crops to orchards, acreage to lots, singles to walk-ups, walk-ups to high-rise, and so on.

Demand is generally rising, and land succeeds from one generation of use to another. Each use has an inner margin where it is the lower use, less intensive than its inner rival; and an outer margin where demand is weaker, or land is less suited, and it is the higher use, more intensive than its outer rival.

"Marginal" land connotes low intensity, but the connotation is misleading. It is based on a physical or per acre concept, while economics is concerned with values. The essence of economic marginality is that $C/R \rightarrow 1$, the cost/benefit ratio approaches unity. In these terms, marginal land is the *most* intensively used. As demand rises and costs fall, land yields a surplus. As the rings in the hierarchy of land uses expand, and a site shifts from the outer to the inner margin of its ring, it becomes less marginal and more rentable, and C/R falls.

2. Land ripens because of falling building costs as well as increasing demand. Obsolescence is a continuing expectation, an ineluctable factor in all decisions (independent of inflation and overstated fatalism about union wage rates, not at issue here). Of course, falling C means lower C/R .

Another general reason to expect falling C is economy of scale. As demand grows, a larger building is appropriate. There are great economies of scale in building.

Land can also ripen into a new generation of use at lower density. A quarter or more of many central business districts has ripened into parking lots, for example, and many a house has been knocked down to expand a gas station apron.

Another kind of falling cost generally to be expected is lower borrowing rates of interest. This results from better creditworthiness of land as the proposed use becomes less marginal, innovative, or experimental in an area. As the land ripens into a better credit rating, R rises because it is the discounted value of future revenues, and a lower discount rate is relevant.

This last factor is partly circular, but that is the nature of credit ratings. It depends on the cumulative process of forming conventional opinion among lenders, few of whom supply much venture capital.

3. Empirical studies generally show that C/R is lower on better lands. It is very low in Champaign County, Illinois, or Benton County, Indiana, the best grain land. It is very low in Kuwait, the best oil land. It is quite low in Manhattan, the best urban land; generally lower on retail land than residential; lower in central cities than fringes; lower in rich suburbs than poor ones. It is lower for Site I timber lands on the western slope of the Coast Range in Oregon than for Site VI lands in the high Cascades. To some extent that may reflect underutilization of better land due to cartel behavior, the soporific effect of surplus income on owners, and other institutional factors, but it probably is more basic than that — basic as that is!

4. Time generally brings public works, paid by others, to lower costs. This is one factor in increasing demand, already postulated, but it is also a factor in lower costs. Thus extension of city sewer and water precludes septic tank and well.

Figure 1 is a schematic representation of these relationships.

III. Effects of Building Tax on Intensity and Ripeness Combined

In equations (7) and (8) in the text, I proceeded for simplicity as though C and R in each year would be the same despite the threat of building taxes. Actually, the landowner would turn to less-intensive alternatives so C/R would fall. This would almost certainly entail an absolute decline in both, as well.

Thus, consider all cases where

$$(1) \quad C(1 + T)/R > 1.$$

That means

$$(2) \quad R - C(1 + T) < 0.$$

i.e., the use value is negative, taxes have made the particular use of land submarginal. But they have not made the land itself submarginal. The owner can find a lower use whose

$$(3) \quad C(1 + T)/R < 1.$$

Now posit a base intensity-path, without taxes, following the course charted in Appendix II, with C/R falling; apply building taxes, and let the

taxpayer change his path, to avoid taxes optimally. He will lower C/R where it is highest, i.e., at the beginning of the path. Thus C/R will be lower at the start and will drop less along the path. This, in turn, reduces the high after-tax growth rate of S , and reduces the retarding effect on ripening.

The net effect of all this is a combination of two evasive wiggles. In general, he builds less, later. The emphasis depends on particulars. He may build much less and a little later; or a little less much later; or any combination.

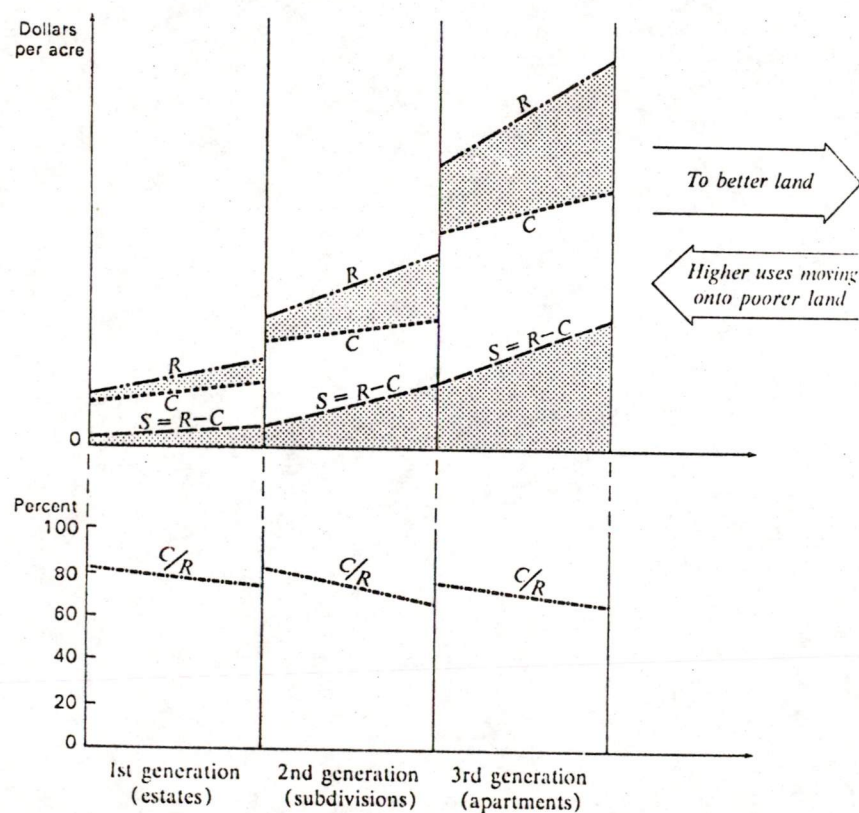


Figure 1. Costs, revenues, use values of sites, and intensities (or cost-benefit ratios), comparing succeeding discrete generations of uses as land ripens into higher uses.

Benjamin F. Rhodes, 1943. "The Thirsty Land." Ph.D. dissn., UCB.

Notes by M.G., from microfiche

pp.59-60 (1887) Modesto I.D. formed by vote of 700 to 156. Of the 156 nays, 150 owned 70,000/108,000 acres in I.D. They grew wheat.

p.108 1901-02, after "dismal 90s," construction revived on water works. Land sales perked up. 1903, water first arrives.

p.115 Alfalfa first replaced wheat. Alf gave quicker return than trees and vines.

p.122 Later, vines and groves came on, after alfalfa.

pp.119-20 Rural settlement proceeds fast. Ca. 10%/yr. (This seems to apply to 1903-07.) Land divided into small farms.

City of Modesto pop. rise followed county, with a lag.

p.124 1908-09, irrigated acres in M.I.D. rose 20%/yr. (after tax shift)

p.127 1904-10, acres in wheat fell fast.

p.128 No. of farms in Stan Cty. and Calif.

Date	No of farms	% rise	No of farms?	% rise	City of Modesto	% rise
	Stan Cnty		CA		Pop	
1900	951	Xx	Xx		2024	Xx
1910	2687	183	73k		4034	99
1920	4556	70	118k	62	9241	129

✓ Thus, no. of farms in Stanislaus County nearly quintupled, 1900-20. Likewise, the pop. of the City of Modesto. (No. of farms in Calif. also rose fast, 1910-20, as irrigation districts spread like wildfire. This trend continued up to 1930, but Rhodes' data stop with 1920.

✓ California trends bucked national trends, for nationwide the no. of farms was falling, and their ave. acreage was rising.)

✓ p.56 "Wheat ... is an industry freed from detail ... It has none of the petty incidents ... where each month has its duties. ... The cutting, thrashing (sic) and sacking of grain at a single operation is spectacular as well as effective. ... In cowboy land the irrigation ditch has always been regarded with disfavor because it is the badge and symbol of a despised occupation. The same feeling ... has prevailed in the wheat-growing districts of California, and for much the same reason."

---- Elwood Mead, Report of Irrigation Investigations in California, Bulletin No. 100,

p.32.

(Cf. psych. profile of Magnus Derrick, a character in *The Octopus* by Frank Norris, ca. 1903.)