Replacing Growing Capital
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Replacing Growing Capital

This fragment on financial maturity is modified from the earlier paper on "Introduction to Optimizing." A fuller treatment is in a later paper, "When to cut a tree."

A classic, simple example of optimizing is maximizing the present value of a growing asset like a tree, by discounting it. One does this by dividing the net harvest value of the tree, H, by the compounding factor, I^n. This forms a ratio, whose numerator and base are both increasing functions of time, n. The tree grows along an S-shaped ("sinusoidal") biological path, aka a "logistic" growth curve. The compounding factor follows a path of exponential growth: its fractional growth rate is always i.

Discounted Value is a max when the "fractional growth rates" of the top and the base of the ratio are equal. Be sure you understand that term, "fractional growth rate." ΔH, alone is a kind of "rate," like MPH, because it means growth per year. "Fractional rate," on the other hand, means $\Delta H/H$. Multiplied by 100, it is a percentage. It is a pure number, like the interest rate, with no dimension, because the top and the base are in the same units ($ of timber, in this case).

For example, if top and base each rise by 8%, that means you multiply each by 1.08 to get the next year's value. The two multipliers cancel each other out, leaving you with no change in the ratio. This is how you know you have reached the max. (assuming the growth path is smooth and regular).

The numerator is $\Delta H/H$; the base is always just i. The solution, then, is when:

$$\Delta H/H=i \qquad (1)$$

Now you have learned something new and interesting; it is tempting to quit. Most analysts do quit, right here. (1) is so simple, so self-explanatory, people love it, and keep passing it on to new generations of students. It says the tree's growth must be enough to earn interest on its own value; when it falls below that, you cut it. However, there is more to think about, to do the job right.

In general, whenever you work with ratios, watch the base. Beware of a widespread tendency to focus on numerators, ignoring denominators (bases). For starters, beware relapsing into thinking that ΔH itself, alone, is the "rate of growth." Review paragraph #3, above. "Growth rate," unless otherwise indicated, means the **fractional** rate, that is $\Delta H/H$. When I write

"fractional growth rate," I am being redundant, but with a purpose: to head off a common student error. "Growth rate" by itself means "fractional growth rate." Remember that.

In this case, though, H is not the whole base, because the capital in the tree (measured by H) is not the only asset on which tree-growth must earn a return. There is also the value of the Site, S, occupied and preempted[1] by the growing tree.

So, be sure to add site value (S) to the base of the numerator. The correct criterion for maturity is:

$$\Delta H/(H+S) = i \qquad (2)$$

This leads to an earlier max, because S is constant, which lowers the fractional growth rate of the numerator:

$$\Delta H/(H+S) < \Delta H/H \qquad (3)$$

Thus, in the year when $\Delta H/H=i$, $\Delta H/(H+S) <i$, indicating it is already past time to harvest.

In practice, there is even more to this problem, because the value of S is not a given (it is not "exogenous"). It has to be **endo**genously found in the very course of finding the optimal value of n. That is, optimal n is the n that maximizes S, at the same time it depends on the value of S. They must be found simultaneously. For now, be assured that it has all been worked out: that's why you learned to solve simultaneous equations in high school algebra. For those who are curious, or like a challenge, it is in the later, optional reading, "When to Cut a Tree."

Meantime, the bottom line for us is Eqn. (2).

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[1]The word "preempt" includes empty space reserved for a sapling so it can later spread out its growing branches.