



Simple 'Capitalization in Perpetuity' and the DCFF

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Simple "Capitalization in perpetuity" and the DCF

These notes deal with a perpetual stream of net rent or other pure net income of "a" dollars per year. Think of it as the annual rent of land, a kind of pure net income. A usefully expressive and analogous concept from property law is "usufruct," meaning "the enjoyment of the income of an estate without impairment of its substance."

"Without impairment of its substance" - that's the key idea. It's an idea many theorists blind themselves to by fusing or lumping land and capital as though they were alike in all ways. Land endures, while capital rusts, molds, rots, obsolesces, cracks, and in a hundred other ways **depreciates**, while land endures and often **appreciates**.

To finesse many complications, think of this land as yielding cash flow without any buildings, and with minimal expenses: it could be a parking lot, space used for open storage, grazing, annual cropping, or other such use. Another reading, "Land Prices with a Series of Discrete, Progressively Higher Future Uses," addresses those complications (and, I believe, makes them fairly simple to understand and deal with).

The lump-sum up-front present Site Value is "S." S is often called the "capitalized" value -- not that it is capital, for it is not stored up by saving, nor can it be drawn down by dis-saving. But it is expressed in the same present-value terms that are customary with capital. "Capital value" is a common contraction of "capitalized value."

Converting a flow value to the equivalent point value is simple where the future flow is infinite. This mainly applies to land, so this point value is denoted in some later readings as "G" (for Ground).¹ Here, where we are simply getting our math straight, I will use P, for Present Value.²

¹G also can be remembered from the Greek Geo. It also works in German, using Grund. Why German? Because the use of K for Kapital is standard, so it is fitting to use a German word for land. Why not L? Because L is normally used for Labor. Why not T, for terra, tierra, terre, etc? Sometimes you just flip a coin: "G" won.

²P is useful because, in addition to Present value, it stands for Principal, Price, Point, Planting cost, Prime (as in Priming the Pump), Prior, Put up, Put out, and other things that seem fitting.

$$P = a/i$$

(1)

(1) is called simple capitalization in perpetuity. It is simple because a is assumed to continue the same, or "level." "In perpetuity" is legalese for "forever": this also keeps the math simpler. In practice, unless interest rates are **very** low, 75 years or so has nearly the same effect on "P" as "forever" does.³

The reasoning is, if your money in the bank earns i, then other cash flows will earn the same. The market process that makes them the same is "arbitrage." Arbitrage is buying and selling securities and land titles, pushing up P until rates of return (a/P) are equalized.

Some people have trouble getting used to this reasoning, and understandably so, because cause and effect are inverted. When you put money in the bank to earn interest, P is the cause and a is the effect. When capitalizing cash flows, however, a is the cause, and P the effect. The question of what causes a is left unanswered. Sometimes the answers are not very good, and have been papered over.⁴ In this course we take a-values as given, without endorsing all their origins.⁵

When a is growing you get fascinating variations on (1). See Reading on "Valuation of Rising Cash Flows."

The derivation of (1) is so simple we didn't even use the basic building block, the Compounding Factor. (The Compounding Factor is covered in Reading #8. Its use for discounting is covered in

³Of course, what people did 75 years ago affects and often dominates landownership today. The point here is, though, that today's effects had little influence on values then.

Remember, too, that we are assuming constant values for a forever. Some buyers 75 years ago anticipated rising values of a, which would offset the discount factor. This is covered in Reading #23, Valuation of Rising Cash Flows.

⁴Perhaps the original owner jumped a claim, shot the first squatters, massacred some Chinese and Indians nearby, bribed a land office agent using money earned in the slave trade, bought an election, and perjured himself to acquire the property. Things like that did happen in California, 19th Century.

⁵We will also take as given the reality of taxes on property and its income. These taxes compensate in part for injustices in the origins of property, or did until 1978 when Prop. 13 cut them by two-thirds.

Readings #12 and #15). To see how things fit together, let's derive (1) by using the Discount Factor, where a Present Value (P) of a given Future Value (F) is as follows

NOTE THAT I USE "I" TO MEAN (1+i) - saves a lot of clutter

$P = F_n I^{-n}$, where F_n is received at the end of year n . A level flow of future values of \$a, each received at the end of the year, is represented by substituting a for each F . The present value of a future flow of \$a is the sum of an infinite geometric series:

$$P = a[I^{-1} + I^{-2} + \dots + I^{-\infty}] \quad (2)$$

The sum (S) of an infinite geometric progression is:

$$S = f \frac{1}{1-r} \quad (3)$$

where S is the sum, f is the first term, and r is the rate of progression. (This is high-school algebra; check your old textbook.) In (2), f is $a[I^{-1}]$, and r is I^{-1} . Substituting those values in (3), we have (1) again:

$$P = I^{-1} \frac{a}{1-I^{-1}} = \frac{a}{I-1} = \frac{a}{i} \quad (1, \text{ rpt.})$$

(1) is the basis for valuation of all real estate; also most common stocks, which are shares in the assets of corporations whose permanence rests on their ownership of real estate. (1) is highly oversimplified, but that doesn't stop most practitioners from using it most of the time as the basis of most valuations. You should know both it, so you know what others are thinking; and its limitations, so you can do a better job.

You can, of course, solve (1) for any of its three elements. Thus, $a = Pi$, and $i = a/P$. Each of those forms is self-explanatory and familiar.

Only slightly less familiar is to solve for P/a , the "price/earnings ratio," or "P/e ratio." Note that our notation makes it the "P/a ratio."

$$P/a = 1/i \quad (5)$$

(5) tells us that the P/a ratio is the reciprocal of the interest rate: higher i , lower P. This is one reason why Wall St. and Main St. both keep such a sharp eye on what the Fed is doing.

The elasticity (ϵ) of P w.r.t. " $\underline{i}$ " is now minus one, meaning that an $x\%$ rise of $\underline{i}$ causes an $x\%$ drop in P . (Warning: this value of ϵ applies to small changes only! For large changes, work it out numerically. For example, a 100% rise of $\underline{i}$ causes a 50% fall of P .)⁶

Actually, (1) understates the sensitivity of P to changes in $\underline{i}$. This is brought out in later readings, which add more factors to the equations.⁷

Access to credit is uneven: a lucky few have no need to borrow, some borrow at the prime, some at higher rates, some from their friendly local Mafia juice man, and some not at all. This phenomenon is called "credit rationing": it is one of the major facts of life, often neglected in theory courses. Those with the lower "internal" rates can have their pick of the land for sale.

Those with poor credit can hardly enter the land market at all, except as renters. What looks like a small difference in the borrowing rate, $\underline{i}$, makes a vast difference in what the individual can bid for land.

While we're at it, what if n is finite? Then the high school algebra formula tells us that you subtract r^n from the numerator of (3). Since $r = I^{-1}$ we now have:

$$P/a = [1 - I^{-n}]/i \quad (4)$$

The right side of (4) is the Discounted Cash Flow Factor (DCFF), a generic rate of exchange between flow values (a) and point values (P), where " a " is level (i.e. constant). If " a " is rising at a steady fractional rate, g , just lower " i " by " g " (subtract g from i).

⁶Do not be misled by the ambiguity in writing of "an $x\%$ change of $\underline{i}$," because $\underline{i}$ is itself a percentage. A rise of $\underline{i}$ from .04 to .05 is a 25% rise, not a 1% rise.

⁷For some perverse reason, theorists when they add complications like to say they are "relaxing the strict conditions," which makes it sound as though they are lowering standards. There is nothing lax or low-standard about making theory more realistic; and there is nothing relaxing about adding more elements to an equation.