The Discounted Cash Flow (DCF) of Infinite and Finite Flows
By [Dr. Mason Gaffney]

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The Discounted Cash Flow (DCF) of Infinite and Finite Flows

Valuation: Converting Flow Values to Point Values

Finding the point value (or lump sum, or single-payment) that corresponds to a known flow (or "stream") of payments is called "capitalization." Valuation, appraisal, and assessment are also standard terms. (The converse operation is "annualization," which we defer for now.)

Point values are often called "funds," but it is a poorly chosen word. "Point value" is more general than fund. It means a value at a single point in time.[1] An ordinary text will tell you all values are either funds or flows, but the term "fund" is wrong because many point values are not funds. Land and corporate stock prices, for example, are point values, but not funds.[2] We will use point value, or just "point," rather than "fund."

Durable capital, likewise, is not a pure fund. Like a fund it is stored up, and it may be drawn down; but unlike a pure or prototypical fund, it can not be drawn down quickly, and not at a speed you can control or affect much. It is not on tap, like beer.[3] The value in it may be liquidated or recovered only at a rate predetermined by the way it was constructed. It is

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[1]In vulgar usage, e.g. by John Wesley Dean of Watergate fame, "point in time" is redundant, bureaucratic and affected. The present usage is expressive and functional.

[2]Land values are not created by storing-up, that is by saving and investing. Disinvesting is not possible, in the aggregate: they cannot be drawn upon like canned goods in the cupboard. One individual can sell to another, but this is a zero-sum transfer, not net social disinvestment or investment.

One could carry this further, and find another word instead of "capitalization." If I had a license to mint words, I would call it "pointilization." Discounting and cumulating are what is being done. These are valuation operations only: they do not transmute land into capital; they just make it commensurable with capital, to facilitate exchange.

However, to keep the parlance common, I will use the conventional term "capitalization."

[3]The German Lager means a store or stock. Auf Lager means on tap.

"illiquid," meaning the capital in it is frozen solid and you cannot thaw it at will: there is no universal microwave to handle this defrosting job. In a liquidity crisis (like now?) its future cash flows cannot be advanced to the present, however great the need for current cash, and however bleak the prospects of future demand.[4] In short, you're stuck with it.

The flow values are variously called cash flows, incomes, earnings, profits, and rents, each with a significantly different meaning. Many authors, sad to say, neglect to define the flows, paper over the differences among different words, and fall into errors. Let's you and I stay clean. Generally, "cash flow" is the right term, and will be used or implied unless otherwise specified.

The flows may be of service rather than cash, as in the case of houses and grounds, pleasure cars and yachts. "Service flow" is a standard term. Do not think they are nebulous or hypothetical. Service flows are capitalized into cash prices of residences, golf club memberships, pleasure craft, and so on.

Notation:

"a" is used for annual flow. The equations are kept simplest by assuming that a is received in one payment at the end of the year. P is value at the start of year zero, i.e. one year before the first payment.

My convention is always to use lower case for flow values, and Upper Case for point values. Dates are indicated by P, for present, and F, for future. This often avoids the use of cluttering subscripts; but where necessary, I use subscripts to show dates.

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[4]Theorists speak too blithely of "opportunity costs." Capital loses its short run opportunity cost the moment it is sunk into hard, or durable forms. Oddly land, the most durable asset, retains its opportunity cost because of limited quantity and constant demand for the same old land for new uses. The saying goes, "you can't unscramble an omelette." That is true of much capital, which is unit-constructed, but as to land, it is like the skillet, not the omelette. You clean it off and use it again.

Infinite Flows: "Capitalization in perpetuity"

Converting a flow value to the equivalent point value is simple where the future flow is infinite. This mainly applies to land, so this point value will later be denoted as "S" (for Site). Here, where we are simply getting our math straight, I will use P, for Present Value.[5]

$$P = a/i \qquad (1)$$

(1) is called simple capitalization in perpetuity. It is simple because a is assumed to continue the same, or "level." "In perpetuity" is, of course, legalese for "forever."[6] This also keeps the math simpler.

The reasoning is, if your money in the bank earns i, then other cash flows will earn the same. The market process that makes them the same is "arbitrage": buying and selling securities and titles, pushing up P until rates of return are equalized.

Some people have trouble getting used to this reasoning, and understandably so, because cause and effect are inverted. When you put money in the bank to earn interest, P is the cause and a is the effect. When capitalizing cash flows, however, a is the cause, and P the effect. The question of what causes a is left unanswered. Sometimes the answers are not very good, and have been papered over.[7] In this course we take a-values as given, without endorsing all their origins.[8]

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[5]P is useful because, in addition to Present value, it stands for Principal, Price, Point, Planting cost, Prime (as in Priming the Pump), Prior, Put up, Put out, and other things that seem fitting.

[6]If you're an old movie fan, it stands for "From Here to Eternity," starring Deborah Kerr, Burt Lancaster, Frank Sinatra, Montgomery Clift, Ernest Borgnine, et al.

[7]Perhaps the original owner jumped a claim, shot the first squatters, massacred some Chinese and Indians nearby, bribed a land office agent using money earned in the slave trade, bought an election, and perjured himself to acquire the property.

[8]We will also take as given the reality of taxes on property

When a is growing you get fascinating variations on (1). These come later.

The derivation of (1) is so simple we didn't even use the basic building block, the Compounding Factor. To see how things fit together, however, let's derive (1) by using the PIPO discounting formula. The Discounted Cash Flow of a future flow of $a is the sum of an infinite geometric series:

$$P = a[I^{-1} + I^{-2} + \ldots + I^{-\infty}] \qquad (2)$$

The sum of an infinite geometric progression is:

$$S = f \frac{1}{1-r} \qquad (3)$$

where S is the sum, f is the first term, and r is the rate of progression. (This is high-school algebra; check your old textbook.) In (2), f is $a[I^{-1}]$, and r is I^{-1}. Substituting those values in (3), we have (1) again:

$$P = I^{-1} \frac{a}{1-I^{-1}} = \frac{a}{I-1} = \frac{a}{i} \qquad (1, \text{ rpt.})$$

(1) is the basis for valuation of all real estate; also most common stocks, which are shares in the assets of corporations whose permanence rests on their ownership of real estate. (1) is highly oversimplified, but that doesn't stop most practitioners from using it most of the time as the basis of most valuations. You should know both it, so you know what others are thinking; and its limitations, so you can do a better job.

Present Value of Finite Flows

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and its income. These taxes compensate in part for injustices in the origins of property, or did until 1978 when Prop. 13 cut them by two-thirds.

Here, we build on (1). (1) gives the present value (PV) of all future time. Now we divide all future time into two parts. Part one is from now to n, where n is the number of years (the "term") of the finite flow. That is what we want to find. Part two is from there (n) to eternity. We find part one by subtracting the PV of part two from a/i (2). The PV of part two is its point value (which is always a/i) discounted back to the present over n years:

$$P = a/i - (a/i)I^{-n} = a\,\frac{1 - I^{-n}}{i} \qquad (4)$$

The more common way to derive (4) is using the algebraic formula for the sum of a geometric progression, which is:

$$S = f\,\frac{1 - r^{n}}{1 - r} \qquad (5)$$

where S is the sum, f = the first term, r = rate of progression, and n is the number of terms in the progression. In this case, f is aI^{-1}, and r is I^{-1}. Substituting in those values we have (4) again:

$$P = aI^{-1}\,\frac{1-I^{-n}}{1-I^{-1}} = a\,\frac{1-I^{-n}}{I-1} = a\,\frac{1-I^{-n}}{i} \qquad (4,\ \text{rpt.})$$

The ratio to the right of a is the Discounted Cash Flow Factor, or DCFF. Learn it. Find a way to remember and retain it. Either memorize it, or remember how to derive it. It is your HOME BASE. It is both a launching pad and a shelter from the storm: most of what we do from here on is derived from it; and you may find all the other factors, even if you forget them, by starting with this DCFF and rearranging it a little. REMEMBER THE DCFF!!!!!

You may program it into a programmable calculator if you like - that is a legal form of cribbing. You will need it from here on: it is another building block. This "block" is not like a brick so much as a LEGO part: it will be standing on its head and turning inside out to serve you, so long as you are ready to be served.

It devolves into a/i (1) when n is infinite; and into a/I when n=1. It applies strictly only to level cash flows that suddenly terminate after n years,[9] but can be adjusted (with added complexity) to handle flows that decline in various regular ways.

Isn't it remarkable that a factor derived by the first method of (4), where we subtracted one infinite series from another, is the same as that derived by the second method of (4), where we chugged along prosaically one year at a time. Can these findings really be the same? Check it out on your calculator, which has a cumulative memory (on simple calculators, use the M+ button). Add I^{-1}, I^{-2}, ... out to I^{-n}. You choose the values of n and i. Use the full power of your calculator, out to 7 or 8 decimal places. Store or record your answer. Now calculate the DCFF from (4). You should get the same answer, to the next to last digit,[10] and be convinced.

To visualize all that, retrieve the graph you drew earlier when we studied the PIPO Discount Factor, (1,B). This is the graph of Cyrano's nose (or Doonesbury's), facing to your right. You shaded in the first five columns; the shaded area is the Discounted Cash Flow of all five together. This is the value that the DCFF gives you, numerically, precisely.

You also drew a cume curve. Each point on the cume curve is numerically equal to the area under the discount curve out to that value of n. Extend that cume curve now out to n=50 and n=60. Note how it flattens out: life after 60 doesn't add much to Discounted Cash Flow at time zero.[11]

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[9]People still call this the "One-Hoss Shay Model" because of its resemblance to a vehicle described whimsically in *The Deacon's Masterpiece*, a poem by Oliver Wendell Holmes. The real masterpiece was Holmes' work itself, which is as droll today as when it was new, and filled with cultural allusions as well: painless learning. But a good metaphor for hurried moderns is a light bulb, which gives service at a constant level until it suddenly pops. Call this the "Light-Bulb Model."

[10]The last digit is rounded.

[11]Reasoning thus, one might follow ex-Gov. Lamm of Colorado, who invited all folks over 60 to move over, pull the plug, and let him scatter their ashes over the Rockies. However, in year 60

Test your mathematical ingenuity. Divide the DCF of a 60-year cash flow into three parts of 20 years @. Show how much, and what share, of DCF comes from each reach of 20 years. Now how would you go about that? Hmmmm - over to you. Think![12]

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itself, life after 60 has a much higher PV than it does at time zero. It is also possible that the annual service flow may be higher, depending on individual cases. Gov. Lamm may also be right about some folks (other than me, of course).

[12]A call of nature at this time is not avoidance, but a sign your body knows you are really getting into it. A sudden urge to visit the beach, on the other hand, is avoidance.