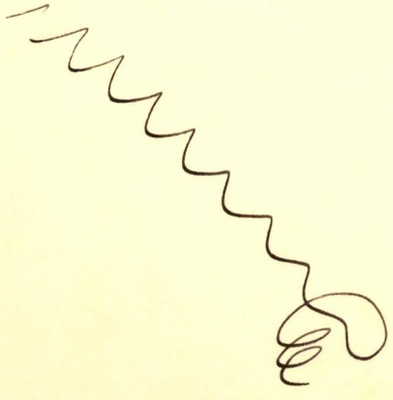
Winnie Chang 1:15 Fri.

Vivy Tungga " "

Susan's box 3382

Learning Ctr. 7436 Sarah Well

Hughes + Anderson



Ec. 102, W92, Current

Ec. 162, Mon., 23 Nov.

1. Complete survey of Capp.

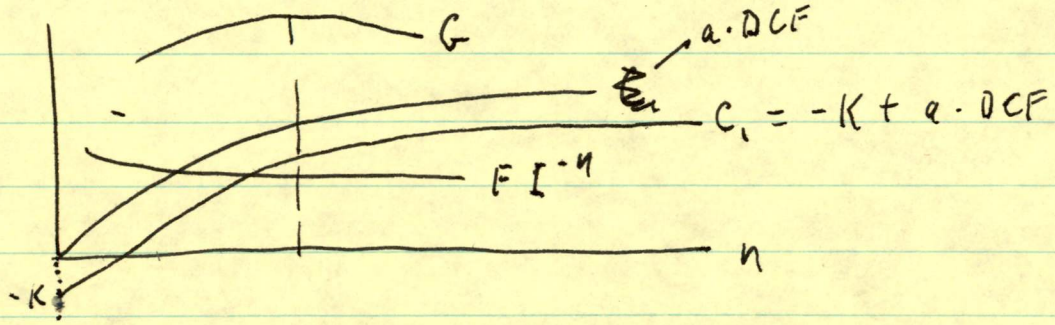
2. Deal w. more advanced problems:
   Non-repeating cycles
   Taxes.

Will use 2 models
   a) Modified Faustmann: orange grove model,
where $G = DCF C_t + F I^{-n}$ F = Future resale

   b) ROR -- redeemable bond model

3. Modified Faustmann:

$$G = C_1 + F I^{-n}$$



4. a decays. Modify DCF formula

$$C_1 = a \frac{1 - (I + d)^{-n}}{i + d} - K$$

d makes C_1 flatten out fast

Every maximizing solution is also a marginal equality.

5. $\Delta C_t^* = a(I+d)^{-n}$

$\Delta F I^{-n} = -F \frac{I^{-n}}{i} = F(-i I^{-n})$

(Check this on your calculators)

Omit

Δ	1.1^{-n}	$\Delta 1.1^{-n}$
1	.91	$-\frac{1}{1.1}$
2	.83	$-.08 = \frac{-1}{12.10} = \frac{-.1}{1.1 \times 1.1^2} = \frac{-i}{I^n}$
3	.75	$\frac{-1}{13.31} = \frac{-.1}{1.1 \times 1.1^3} = \frac{-i}{1.1^3}$
4	.68	$\frac{-.1}{1.464} = \frac{-i}{1.1^4}$

G max when $\Delta C^t = -\Delta F I^{-n}$

i.e., $a(I+d)^{-n} = F i I^{-n}$

or

$a(1+d)^{-n} \cong F i$

(rigorous - NB) (C.F. of def.) = (rent of challenger [(+ F_q)])

$(1+d)^{-n} = \frac{Fi}{a}$; $-n \ln(1+d) = \ln\left(\frac{Fi}{a}\right)$; $n = \frac{-\ln \frac{Fi}{a}}{\ln(1+d)}$

6. $\text{Max } G = -K + a\frac{1-(I+d)^{-n}}{i+d} + F I^{-n}$

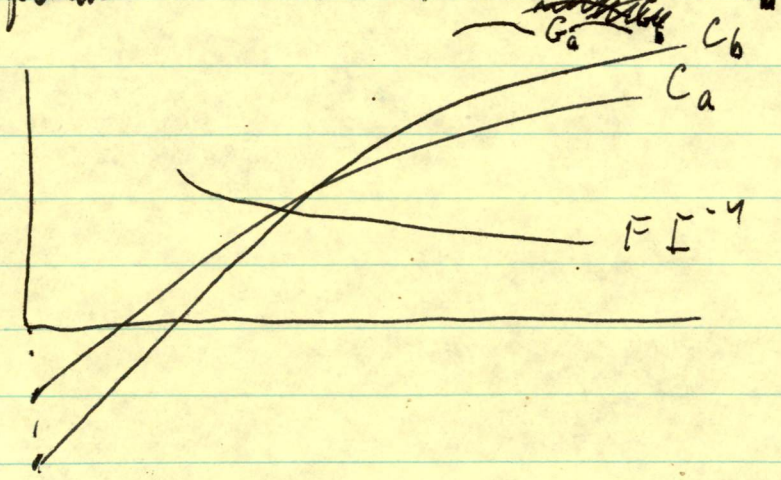
7. Mgr's job is to find
Best combo. of K and a and d
More K means higher a, and lower d, and higher n.

Ordy. analysis just looks at C_1. Thus, overstates benefit of higher K. Gives you all of future time to recover it.

In fact, must fit ~~the~~ C_1, a cycle of investing, recovering, into a time slot. The slot is stretchable, but only at a cost. Our model lets us handle this aspect

8. As you ΔK, will Δa, automatically lengthening $\underline{n}$. However, $\frac{\Delta a}{a} < \frac{\Delta K}{K}$.

If $\Delta a \cdot DCF\ (= PV\, \Delta a) > \Delta K$, raises C_1. Normally, however, ΔK means lower $\underline{d}$ and higher $\underline{n}$, to spread K over more ~~time~~ G_b



To recover higher K of C_1, must last longer. Here we must account for another cost: lower $F = I^{-\eta}$. G_b max is to right of G_a max; but may be higher or lower

9. We can get numerical answers, using ~~etc~~ algebra, + calc., + playing "what if?".

10. w/o taxes, here is basic eqn.

$$G = -K + a\frac{1-(1+d)^{-n}}{i+d} + FI^{-n}$$

11. Here are some cases where I have solved for n^* which gives G_{max}. Student problem: using this n, which gives higher G? You can solve for n: see #5. Result is $n^* \pm 1$, but max. is flat, so is ok.

$$n^* = \frac{-\ln \frac{F \cdot i}{a}}{\ln(1+d)}$$

Row No	K	a	i	d	F	n^*	G?	$-K + a \cdot DCF$	FI^{-n}
	9	10	.04	.15	100	6	(104.13)	28.1	76.0
2	27	20				11	(127.36)	67.3	60.1
3	27	15	.10			14	(121.8)	63.0	57.7
4	36	15	.075			19	(125.3)	79.6	45.6
5			.09			15	(116.5)	60.9	55.5
6			.08			17	122	70.8	52.4

Cf. Rows 2 + ~~3~~ 4.
2 is slightly better. 4 uses $\frac{1}{3}$ more K to drop d by 50% (and lower a by $\frac{1}{4}$). Raises DCF by $\frac{79.6}{67.3} - 1 = 18.3\%$ but lowers FI^{-n} by $\frac{45.6}{60.1} - 1 = 24\%$.

Cf. Rows 1 + 2. Triple K, to double a, + it pays off.

Cf. Rows 2 + 3. Sacrifice $\frac{1}{4}$ of a to lower d by $\frac{1}{3}$. It doesn't pay.

Cf. Rows 3 + 4. Raise K by $\frac{1}{3}$ to lower d by $\frac{1}{4}$. Effect on $(-K + a \cdot DCF)$ is + 16.6, but severely hurt as FI^{-n} falls by 12.1. Cf. 3 + 5, fails to pay off.

Extended Table on Sensitivity of G

$n = \frac{-\ln(\frac{F \cdot i}{a})}{\ln(1+i)}$

Row No.	K	a	i	d	F	n*	G	−K+a·DCF	F I^−n
7	36	15	.04	.09	100	17	116.3	64.9	51.3
8					200	7.7	182	30.4	152
11	9	10	.08	.15	100	1.6	92	3.3	88.4
12	27	20				6.6	98	38	60
13		15		.10		6.6	88.6	28.4	60.2
14	36			.075		8.7	84.3	33.1	51.2
21	45	40	.04	.15	100	16.5	206	154	52
24	90	60	.04	.075	100	37	446	422	23
34	90	60	.08	.075	100	28	302	290	12
31	45	40	.08	.15	100	12	154	114	40

Cf. 3 + 7, G falls altho (−K + a DCF) rises.
Cf 7 + 8, High F lowers n*; lowers weight of −K + a · DCF.
Cf 1 + 11. Hi i pumps n* to 1.6 years
Cf 2 + 12 Hi i " " to 6.6 "; lowers C_1
Cf 3 + 13 " " " " " 6.6 ", lowers C_1
Cf 4 + 14 " " " " " 8.7 " "

Effect of Taxes

$$\left[\frac{K}{x} + \frac{1 - I^{-x}}{i}\right]$$

$$\left[-K + a(1-t)\frac{1-(I+d)^{-n}}{i+d}\right] \qquad \left[(1-s)I^{-n}\right]$$

$$F(1-s)I^{-n}$$

Row No.	K	a	i	d	F	t	s	x	n*	G
41	9	10	.04	.15	100	.5	.2	3	4	92.5 6.2 4.2 66 .83

Cf. 41 and 1, effect of a is much reduced.
Effect of fast write-off is small.
Resale is big element.

44	36	15	.04	.075	100	.5	.2	9	9	4.7	15	56	.86

Cf. 44 + 4. Even w. higher a, and lower d, G is still dominated by resale, (and deduction of G at time of sale.)

Finals

Wed 8-11 Ec. 162

Ec. 120

Mon. 5-7 Review

2200 Speth

Ec. 162, Wed. 16 May '90 F - Rijnmij

Faustmann Factor when future generations are of higher value than G_1.

g = annual fractional growth of NPV.

$$G = G_1\left[1 + \left(\frac{1+g}{1+i}\right)^n + \left(\frac{1+g}{1+i}\right)^{2n} + \ldots + \left(\frac{1+g}{1+i}\right)^{\infty}\right]$$

To simplify, use this approximation:

$$\frac{1+g}{1+i} \cong 1+g-i \qquad \begin{cases} g < .10 \\ i < .10 \end{cases}$$

$$G = G_1 \frac{1}{1-(1+g-i)^n} = \frac{G_1}{1-(1+g-i)^n}$$

When $n = 1$

$$G = \frac{G_1}{i-g}$$

Economics 162
Quiz #8
November 20, 1992

Answer 3 of the 5 questions. Show all your work.

1. What is the price/earnings ratio of land when n=1? Show how you arrived at your answer.

2. In the Riverside market for income producing properties, the current yield is 15%. If the interest rate is 8.5% and the tax rate is 1%, what is the expected growth rate of the income stream?

3. A $1,000 bond (redemption value) with a coupon payment of $100 per year currently sells for $1511.50. If the bond matures is 25 years, what is the yield to maturity?

   a. 6.6%
   b. 10.2%
   c. 5.1%
   d. 6.0%
   e. 9.3%

Extra credit: What is the fractional difference between the yield to maturity an the current yield?

4. What is the difference between cash flow and income? For what asset are they equal? Explain.

5. How do growth rates and property tax rates affect the L.V.M.? Explain.

Fri, 20 Nov. 92 Ec. 162

1. $\frac{V}{a} = \frac{1}{(i-g)+t}$. i) If $g \to i$, only t holds bid on.

2) Rockefeller, i
You, j

3.) Rock. can bid more for any ~~site, s~~ a
Dim. MP of land — so Rock. doesn't get it all.

4. $cc = V(i+t) = a + V_g$
"Neg. c.f." — ~~a~~ personal c.f. in this case, net of cc.
$a - V(i+t) = V_g$ ~~Bought back V_g~~ or Buy V_g each yr., from outside savings. ("a" can also be avoided cost, i.e. avoided rent prints, aka service flow; or energy saving; etc.)

5. Self liq. loans.

a) Review $a = P\frac{i}{1-I^{-n}} > Pi$ ~~(wrong)~~

~~$a - aI^{-n} = Pi$~~
~~$a = Pi + aI^{-n}$~~

$\Rightarrow a = Pi + aI^{-n}$
~~$Pi + I^{-n}$~~ (?)

b) Land ($t=0$)

$a = V(i) - V_g < V_i$

c) Machine

$V = \frac{a}{i+d}$

$a = Vi + Vd$

(cont)

6) $V = \frac{a + V_g}{i}$. Your one cap.g growth expectations.

7) $Vt = \frac{t(a + V_g)}{i + t}$ (a) Prop. tax is tax on V_g (incl. infl.)
(b) Effective t depends on i.

8) $i - g = \frac{a}{V} - t$ Finding real i.r. (go where real $g = 0$)

9) Leverage of i on $V = \frac{a}{i-g}$

Leverage is where sensitivity is more than proportionate.

~~(5)~~ $Pi + X = P\frac{i}{1 - I^{-n}} = a$

$X = \frac{Pi}{1 - I^{-n}} - \frac{Pi(1 - I^{-n})}{1 - I^{-n}} = \frac{Pi\, I^{-n}}{1 - I^{-n}} = P\frac{i}{I^n - 1}$ $\quad aI^{-n}$

~~or~~

$X = a - Pi = a - ia\frac{1 - I^{-n}}{i} = a(1 - 1 + I^{-n}) = a\, I^{-n}$

°° Table 1: RRAT (%) with different values of x, with n=50

x	.3/x	PMT A	PMT B	PMT C	RRAT(%)
5	.060	.1200	.06	.88	7.92
10	.030	.0900	"	"	7.57
15	.020	.0800	"	"	7.32
18	.0167	.0767	"	"	7.20
28	.0107	.0707	"	"	6.91
50	.0060	.0660	"	"	6.57

Ec. 162 Wed. 18 Nov. 92

1. Notes use "V" for "G" : and "a" for "b".

2. V not a fund, K is a fund.

3. "usufruct." CF ≡ Income
(But must subtract (Mdg. depreciation.) (This is a problematic lot)

4. Current $ used, & nominal i.r., & nominal q

5. CC of land not deducted to get c.f. of land
(they are deducted to get c.f. of person owning land)
("The trouble w. women....")
Watch the context.

6. CC are pure time costs: int. + prop. taxes.

7. Diff. of ~~int~~ prop. tax & inc. tax.

8. V/a is like P/E value of stock. $= \frac{1}{i+t-g}$

V found by arbitrage. - is dependent variable.

9. An observer of mkt can infer invisible q

$$q = i + t - \frac{a}{V}$$

10. Tax capn. -- effect of t

11. $\frac{a}{V} = i - g + t$

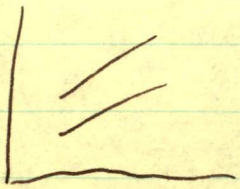
$\frac{V}{a} = \frac{1}{i - g + t} =$ ~~Exp~~. LVM

If you lack ~~tow~~ K, have a hi $\underline{i}$ — go where t is high (Wis.)
If loaded, less compn. where t is low.

9:30 Th. Alex B.

Ring problems this PM

162 Wed 12 Nov. 42

1.  Recap. Mudekovs Meggyi

2. Problem: $a_{1-4} = 10$

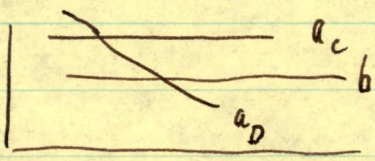
$a_{5-9} = 8$

$a_{10-14} = 6$

Find LV

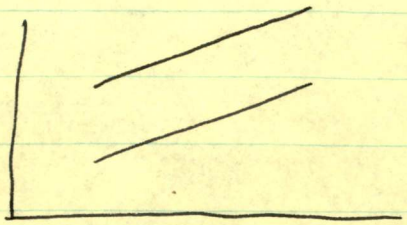
3. Recap. solutions, what to max.

4. When demolish old bldg.

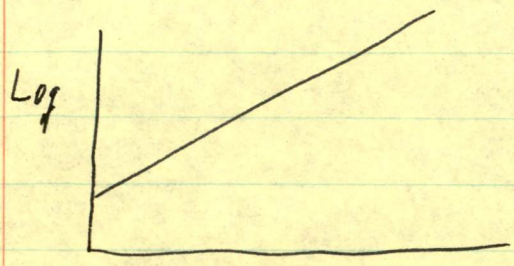


$b = -K_c \cdot IIF + a_c$

5. Red. Bond.

1. 

Are growth rates equal?
" " " "
constant?

2. 

Same qus.

3. Log derivative.

4. Recap of qu. 2.

a. A procedure. No understanding.

b. Finds u & b simul, to max b

c. $F' = F_i + b$

d. $b = G_i = \dfrac{-K + I - I^{-u}}{1 - I^{-u}}\, i$

5. Qu. 3 $\qquad G = C_i \cdot LVM$

$$\frac{C_i}{G} = \frac{1}{LVM} = 1 - I^{-u}$$

6. Freebie. Show graphically how max b.
7. When demolish old bldg?

Wed

10:30 A Steve W.
10 A Th. Kevin G

1. Review sessions, weekly.
Wed. ~~4000~~? 2 PM

2. $G = PVC_1 \times LVM$

If DCF, $PVC_1 = a\frac{1-I^{-n}}{i}$

$\therefore \quad G = \frac{a}{i}$

3. Const. a but recurring K.

$$PVC_1 = -K + a \cdot \frac{1-I^{-n}}{i}$$

$$\therefore G = \frac{-K}{1-I^{-n}} + \frac{a}{i}$$

Shows PV of recurring K (remodeling, replacement, etc.)

4. Recip. of LVM is Measure of G coming from C_1

a. Num.
b. Algebraic

5. Note how all fits together, mutually consistent.
That's how you know you're right.

6. $b = Gi = -K \cdot IPF + a$

$b < a$

~~II.~~ Reprise:

~~$b < a <$~~ $b < Y < a$

$Y = i(G + K)$

$a = Gi + K\frac{i}{1 - I^{-n}}$

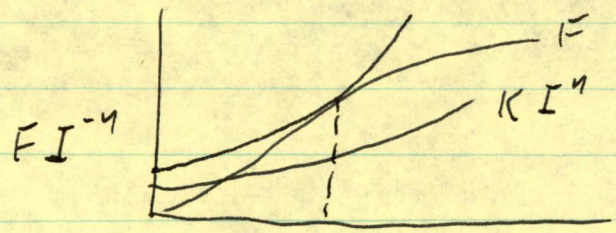
$b = Gi$

~~=~~

7. ([7, p. 36, is another approach. Leave to you)

8. Pp. 36-38 are there to show the fallacy in common principle of fuel. maturity: $\frac{F'}{F} = i$

a. The alleged principle



b. Fallacies

9. Probs.
Will give growth rates and G values.

162, Wed.

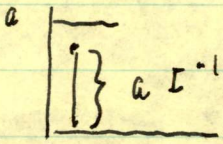
1. p. 23, par 2

2. p 23, Equ. (10)

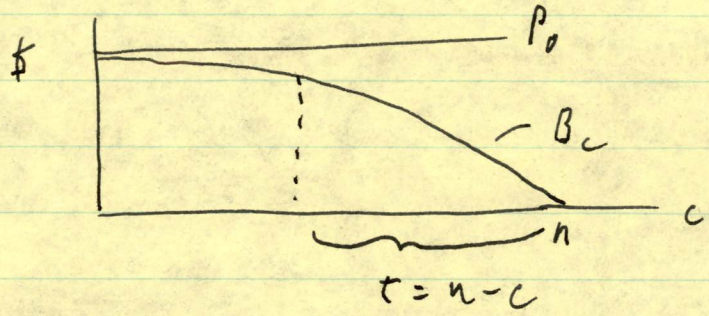
3. Com unpdd, n. 33

4. p. 24 c = current year
t = remaining term (n - c)
B_c = Balance unpaid in current year.

5. Last year, n - c = 1



6. B_c = PV of a over t years.



7. Balance:
Value of B_c when $c = \frac{n}{2}$ p. 25, T. 3

T. 3 shows $\frac{P - B}{P}$ when $c = \frac{n}{2}$

8. Half-life.

Brian Coleman, Fri.
Oct. 23 exc.

8. A) Solve for n

Looks harder than solving for i, but is easier bcs. only one soln.

B) X is t when $B = \frac{P}{2}$

$h = n - x$

c) p. 28 Values of $\frac{h}{n}$

9. p. 28 Linder's viewpoint.

10. Problems

Ec. 162 Monday Nov. 2

1. $F + G = (K + G)\, I^{-n}$

2. Solve for K

$$K + G = (F + G)\, I^{-n}$$

$$K = (F + G)\, I^{-n} - G$$
$$= F I^{-n} - G(1 - I^{-n})$$
$$= F I^{-n} - b\,\frac{1 - I^{-n}}{i}$$

(Note use of "b" instead of "ä".)

How much can you spend planting, if sheep wd. pay \$30/yr., trees grow 20 000, logging costs 12,000, in 55 yrs, at 7%. Walk thru.

E.g. is specific, but principle is generic.

3. Solve for i

Ask about effect of G.

of ΔG. p. 32

of $-\Delta G$. p. 33

4. $$n = \frac{\ln(F+G) - \ln(K+G)}{\ln I} = \frac{\ln \frac{F+G}{K+G}}{\ln I}$$

Higher G lowers num., lowers n required to earn i.

Probs for near future, + past.

A. ~~Flow-fund + fwtm.~~ IPF

1

B. PV

2, 7, 9, 12, 13, 14, 16, 21

C. Payback, depreca., CCA

3, 4, 5, 8, 10, 20, 22

D. PIPO

11

E. Normalizing

F. Replacement

17, 18

G. ROR

H. Redeemable Bond

Probs. on G, b, trees, etc.

Economics 132
Public Finance

Spring Quarter 1984
Mason Gaffney

SOME BASIC MATH. DEALING WITH APPRECIATION, DEPRECIATION AND TAXATION

1. To prove: expensing of investments in durable capital is tantamount to exemption from income taxation.

Let:

K_o = original capital cost
a = annual cash flow, assumed level over n years
n = number of years of cash flow (also called "economic life")
i = rate of return before tax
I = 1 + i (to simplify reading equations)
t = income tax rate
r = rate of return after tax
R = 1 + r (to simplify reading equations)

With no tax:

$$K_o = a \frac{1 - I^{-n}}{i} \qquad (1)$$

$$i = \frac{a}{K_o} (1 - I^{-n}) \qquad (2)$$

The rate of return is the value of i that satisfies (2). The solution is not as simple as the form of (2) suggests, because I = 1 + i and so i is on both sides, and you cannot collect terms. But clever mathematicians have long since developed various ways to speed to a solution by successive approximation; and with modern computers this is a snap. Even hand calculators will do it when programmed for it; and several commercial models are because this is a standard business calculation.

With tax, and expensing of K_o, the taxpayer's initial outlay (after deducting K_o from taxable income in year zero) is reduced by tK_o and becomes $K_o - tK_o$ or $K_o(1 - t)$.

Thereafter he has no more deductions deriving from K_o. He is taxed on the full cash flow (a) which is composed both of income and recovery of capital (K_o). After tax, his cash flow is reduced to $a(1 - t)$.

Solving for r:

$$r = \frac{a(1 - t)}{K_o(1 - t)} (1 - R^{-n}) \qquad (3)$$

But the $(1 - t)$ terms cancel, so (3) is the same as (2), and r = i, Q.E.D. There is no tax.

The taxpayer's initial investment has been reduced by t%. His cash flow is reduced by t% also. He gets a full untaxed return on his share. The Treasury gets only a return on its share.

The "taxpayer" is no longer a taxpayer, but a fully successful tax avoider. In effect, he is a partner of The Treasury (or "fisc," another inside term, which has the advantage of brevity). He is perhaps an involuntary partner, but on the other hand he has great gains. He gets a partner who advances about half of his front money; and who has no control over what he does with the investment. Things could be a lot worse!

A tax filer is not always a taxpayer, you see. We will thus use the expression Taxable Unit, or T.U. for those who file tax returns; they may or may not be real taxpayers. Actually they usually are--it is rather that selected activities, called tax shelters, are yielding untaxed income, usually in conjunction with taxable income from other sources. The shelter offsets some of the taxable income.

For a simpler example, see #3.B, below.

2. To prove: the annually accruing increment in value of appreciating land and common stock is current income (the Haig-Simons idea).

Let:

V = the present value† of a perpetuity (land or stock)
a = annual cash flow, first year.† (The asset never wears out so the cash flow is all income, although not all <u>of</u> income.)
i = the applicable interest rate
g = annual growth rate of <u>a</u>
I = 1 + i
G = 1 + g

<u>i</u> and <u>g</u> may be thought of as percentages, and expressed as such. (But mathematically they are the percentages/100. Thus, 3% = .03.)

With no growth:

$$V = \frac{a}{i} \qquad (4)$$

(4) is called "capitalization." I assume you understand it. (It is one of several meanings of "capitalization," so be alert to the context.)

---

† The convention is to assume <u>a</u> is all received at the end of the year; but V is the value at the beginning of the year. This is just because the formulas are simpler that way, and the inaccuracies are minor for our present purpose.

With growth:

$$V = a\left[\frac{1}{I} + \frac{G}{I^2} + \frac{G^2}{I^3} + \cdots + \frac{G^\infty}{I^\infty} \right] \qquad (5)$$

(5) contains an infinite geometric progression, in brackets. It collapses into:

$$V = a\left[\frac{1}{I} \cdot \frac{1}{1 - \frac{G}{I}} \right] = a \frac{1}{I - G} = \frac{a}{1 + i - 1 - g} = \frac{a}{i - g} \qquad (g < i)$$

$$V = \frac{a}{i - g} \qquad (6)$$

(Ask me if you need help on the geometric progression formula.)

Note in passing the logical impossibility of $g \geq i$. This says something about the effect of inflationary expectations on interest rates, and the role of land valuation in the process of adapting rates to expectations.

Now rewrite (6) in a form parallel to (4).

$$Vi - Vg = a$$

$$V = \frac{a + Vg}{i} \qquad (7)$$

(7) shows that the market value of land is found by capitalizing Vg in the same way that it capitalizes a, the cash flow.

Vg is ΔV each year. You can intuit this from (5) because each year the progression begins from one term further to the right, thus growing by gV. Or you can write it out, so:

$$\Delta V = V_1 - V_o = \frac{a(1 + g)}{i - g} - \frac{a}{i - g} = \frac{ga}{i - g} = gV_o \qquad (8)$$

Putting (6) another way:

$$Vi = a + Vg \qquad (9)$$

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4. To prove: deducting land from taxable income, at any time, achieves tax exemption in perpetuity.

Land is not depreciable for income tax because land does not (normally) wear out with time. Buyers often depreciate land covertly, however, by overallocating value to old buildings (or trees or vines, etc.) when they are dividing their "basis" (the purchase price) between depreciable capital and non-depreciable land. The tax filer is motivated to allocate† as much basis as possible to depreciable capital, to maximize his future deductions.

When you deduct land value, the "fisc" (Treasury) becomes your co-investor, just as in case #1, supra (expensing). But since the land never wears out, the advantage, once gained, lasts forever, just as the land does. In the case of capital you have to depreciate it fast, before it wears out, to gain an advantage. In the case of land, any time will do because you are looking forward to infinity, a goal that recedes as you approach it. Of course sooner is better for the individual tax filer because he wears out; but the land will always be there.

The landholder who sells land which he has depreciated is now subject to a tax on the gain over the basis, which basis he has reduced by depreciation. Thus there is some partial "recapture" by the fisc of the improper depreciation. But the recapture is on a capital gains basis. That means: a) only 40% of it is "recognized" as taxable; b) it occurs long after the deductions were enjoyed; and c) it may be further deferred by the devices discussed in class.

But then the new buyer may do the same thing all over again, beginning from the new basis. All he needs is an old "junker" on the land, to which he can overallocate basis. It is not uncommon for local tax assessors to be allocating half the value of a parcel of real estate to an old junker in the very year that it is junked.* The local tax assessor has nothing to do with the income tax except that the I.R.S. invites tax filers to use the assessor's allocation between land and buildings. This is one of the few cases where I.R.S. actually helps you avoid taxes.

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Economics 132
Public Finance

Spring Quarter 1984
Mason Gaffney

SOME BASIC MATH. DEALING WITH APPRECIATION, DEPRECIATION AND TAXATION

1. To prove: expensing of investments in durable capital is tantamount to exemption from income taxation.

Let:

K_o = original capital cost
a = annual cash flow, assumed level over n years
n = number of years of cash flow (also called "economic life")
i = rate of return before tax
I = 1 + i (to simplify reading equations)
t = income tax rate
r = rate of return after tax
R = 1 + r (to simplify reading equations)

With no tax:

$$K_o = a \frac{1 - I^{-n}}{i} \qquad (1)$$

$$i = \frac{a}{K_o} (1 - I^{-n}) \qquad (2)$$

The rate of return is the value of i that satisfies (2). The solution is not as simple as the form of (2) suggests, because $I = 1 + i$ and so i is on both sides, and you cannot collect terms. But clever mathematicians have long since developed various ways to speed to a solution by successive approximation; and with modern computers this is a snap. Even hand calculators will do it when programmed for it; and several commercial models are because this is a standard business calculation.

With tax, and expensing of K_o, the taxpayer's initial outlay (after deducting K_o from taxable income in year zero) is reduced by tK_o and becomes $K_o - tK_o$ or $K_o(1 - t)$.

Thereafter he has no more deductions deriving from K_o. He is taxed on the full cash flow (a) which is composed both of income and recovery of capital (K_o). After tax, his cash flow is reduced to $a(1 - t)$.

Solving for r:

$$r = \frac{a(1 - t)}{K_o(1 - t)} (1 - R^{-n}) \qquad (3)$$

But the $(1 - t)$ terms cancel, so (3) is the same as (2), and r = i, Q.E.D. There is no tax.

The taxpayer's initial investment has been reduced by t%. His cash flow is reduced by t% also. He gets a full untaxed return on his share. The Treasury gets only a return on its share.

The "taxpayer" is no longer a taxpayer, but a fully successful tax avoider. In effect, he is a partner of The Treasury (or "fisc," another inside term, which has the advantage of brevity). He is perhaps an involuntary partner, but on the other hand he has great gains. He gets a partner who advances about half of his front money; and who has no control over what he does with the investment. Things could be a lot worse!

A tax filer is not always a taxpayer, you see. We will thus use the expression Taxable Unit, or T.U. for those who file tax returns; they may or may not be real taxpayers. Actually they usually are--it is rather that selected activities, called tax shelters, are yielding untaxed income, usually in conjunction with taxable income from other sources. The shelter offsets some of the taxable income.

For a simpler example, see #3.B, below.

2. To prove: the annually accruing increment in value of appreciating land and common stock is current income (the Haig-Simons idea).

Let:

V = the present value† of a perpetuity (land or stock)
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i = the applicable interest rate
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I = 1 + i
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<u>i</u> and <u>g</u> may be thought of as percentages, and expressed as such. (But mathematically they are the percentages/100. Thus, 3% = .03.)

With no growth:

$$V = \frac{a}{i} \qquad (4)$$

(4) is called "capitalization." I assume you understand it. (It is one of several meanings of "capitalization," so be alert to the context.)

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† The convention is to assume <u>a</u> is all received at the end of the year; but V is the value at the beginning of the year. This is just because the formulas are simpler that way, and the inaccuracies are minor for our present purpose.

With growth:

$$V = a\left[\frac{1}{I} + \frac{G}{I^2} + \frac{G^2}{I^3} + \cdots + \frac{G^\infty}{I^\infty} \right] \tag{5}$$

(5) contains an infinite geometric progression, in brackets. It collapses into:

$$V = a\left[\frac{1}{I} \cdot \frac{1}{1 - \frac{G}{I}} \right] = a \frac{1}{I - G} = \frac{a}{1 + i - 1 - g} = \frac{a}{i - g} \qquad (g < i)$$

$$V = \frac{a}{i - g} \tag{6}$$

(Ask me if you need help on the geometric progression formula.)

Note in passing the logical impossibility of $g \geq i$. This says something about the effect of inflationary expectations on interest rates, and the role of land valuation in the process of adapting rates to expectations.

Now rewrite (6) in a form parallel to (4).

$$Vi - Vg = a$$

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4. To prove: deducting land from taxable income, at any time, achieves tax exemption in perpetuity.

Land is not depreciable for income tax because land does not (normally) wear out with time. Buyers often depreciate land covertly, however, by overallocating value to old buildings (or trees or vines, etc.) when they are dividing their "basis" (the purchase price) between depreciable capital and non-depreciable land. The tax filer is motivated to allocate† as much basis as possible to depreciable capital, to maximize his future deductions.

When you deduct land value, the "fisc" (Treasury) becomes your co-investor, just as in case #1, supra (expensing). But since the land never wears out, the advantage, once gained, lasts forever, just as the land does. In the case of capital you have to depreciate it fast, before it wears out, to gain an advantage. In the case of land, any time will do because you are looking forward to infinity, a goal that recedes as you approach it. Of course sooner is better for the individual tax filer because he wears out; but the land will always be there.

The landholder who sells land which he has depreciated is now subject to a tax on the gain over the basis, which basis he has reduced by depreciation. Thus there is some partial "recapture" by the fisc of the improper depreciation. But the recapture is on a capital gains basis. That means: a) only 40% of it is "recognized" as taxable; b) it occurs long after the deductions were enjoyed; and c) it may be further deferred by the devices discussed in class.

But then the new buyer may do the same thing all over again, beginning from the new basis. All he needs is an old "junker" on the land, to which he can overallocate basis. It is not uncommon for local tax assessors to be allocating half the value of a parcel of real estate to an old junker in the very year that it is junked.* The local tax assessor has nothing to do with the income tax except that the I.R.S. invites tax filers to use the assessor's allocation between land and buildings. This is one of the few cases where I.R.S. actually helps you avoid taxes.

* * * * *

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* In such a case the owner has unused depreciable basis outstanding. He could try for an abandonment deduction--I do not know if that flies. If anyone finds out let us know.

pw

Economics 132
Public Finance

Spring Quarter 1984
Mason Gaffney

SOME BASIC MATH. DEALING WITH APPRECIATION, DEPRECIATION AND TAXATION

1. To prove: expensing of investments in durable capital is tantamount to exemption from income taxation.

Let:

K_o = original capital cost
a = annual cash flow, assumed level over n years
n = number of years of cash flow (also called "economic life")
i = rate of return before tax
I = 1 + i (to simplify reading equations)
t = income tax rate
r = rate of return after tax
R = 1 + r (to simplify reading equations)

With no tax:

$$K_o = a \frac{1 - I^{-n}}{i} \qquad (1)$$

$$i = \frac{a}{K_o} (1 - I^{-n}) \qquad (2)$$

The rate of return is the value of i that satisfies (2). The solution is not as simple as the form of (2) suggests, because $I = 1 + i$ and so i is on both sides, and you cannot collect terms. But clever mathematicians have long since developed various ways to speed to a solution by successive approximation; and with modern computers this is a snap. Even hand calculators will do it when programmed for it; and several commercial models are because this is a standard business calculation.

With tax, and expensing of K_o, the taxpayer's initial outlay (after deducting K_o from taxable income in year zero) is reduced by tK_o and becomes $K_o - tK_o$ or $K_o(1 - t)$.

Thereafter he has no more deductions deriving from K_o. He is taxed on the full cash flow (a) which is composed both of income and recovery of capital (K_o). After tax, his cash flow is reduced to $a(1 - t)$.

Solving for r:

$$r = \frac{a(1 - t)}{K_o(1 - t)} (1 - R^{-n}) \qquad (3)$$

But the $(1 - t)$ terms cancel, so (3) is the same as (2), and r = i, Q.E.D. There is no tax.

The taxpayer's initial investment has been reduced by t%. His cash flow is reduced by t% also. He gets a full untaxed return on his share. The Treasury gets only a return on its share.

The "taxpayer" is no longer a taxpayer, but a fully successful tax avoider. In effect, he is a partner of The Treasury (or "fisc," another inside term, which has the advantage of brevity). He is perhaps an involuntary partner, but on the other hand he has great gains. He gets a partner who advances about half of his front money; and who has no control over what he does with the investment. Things could be a lot worse!

A tax filer is not always a taxpayer, you see. We will thus use the expression Taxable Unit, or T.U. for those who file tax returns; they may or may not be real taxpayers. Actually they usually are--it is rather that selected activities, called tax shelters, are yielding untaxed income, usually in conjunction with taxable income from other sources. The shelter offsets some of the taxable income.

For a simpler example, see #3.B, below.

2. To prove: the annually accruing increment in value of appreciating land and common stock is current income (the Haig-Simons idea).

Let:

V = the present value† of a perpetuity (land or stock)
a = annual cash flow, first year.† (The asset never wears out so the cash flow is all income, although not all <u>of</u> income.)
i = the applicable interest rate
g = annual growth rate of <u>a</u>
I = 1 + i
G = 1 + g

<u>i</u> and <u>g</u> may be thought of as percentages, and expressed as such. (But mathematically they are the percentages/100. Thus, 3% = .03.)

With no growth:

$$V = \frac{a}{i} \qquad (4)$$

(4) is called "capitalization." I assume you understand it. (It is one of several meanings of "capitalization," so be alert to the context.)

---

† The convention is to assume <u>a</u> is all received at the end of the year; but V is the value at the beginning of the year. This is just because the formulas are simpler that way, and the inaccuracies are minor for our present purpose.

With growth:

$$V = a\left[\frac{1}{I} + \frac{G}{I^2} + \frac{G^2}{I^3} + \cdots + \frac{G^\infty}{I^\infty}\right] \tag{5}$$

(5) contains an infinite geometric progression, in brackets. It collapses into:

$$V = a\left[\frac{1}{I} \cdot \frac{1}{1 - \frac{G}{I}}\right] = a\,\frac{1}{I - G} = \frac{a}{1 + i - 1 - g} = \frac{a}{i - g} \qquad (g < i)$$

$$V = \frac{a}{i - g} \tag{6}$$

(Ask me if you need help on the geometric progression formula.)

Note in passing the logical impossibility of $g \geq i$. This says something about the effect of inflationary expectations on interest rates, and the role of land valuation in the process of adapting rates to expectations.

Now rewrite (6) in a form parallel to (4).

$$Vi - Vg = a$$

$$V = \frac{a + Vg}{i} \tag{7}$$

(7) shows that the market value of land is found by capitalizing Vg in the same way that it capitalizes a, the cash flow.

Vg is ΔV each year. You can intuit this from (5) because each year the progression begins from one term further to the right, thus growing by gV. Or you can write it out, so:

$$\Delta V = V_1 - V_o = \frac{a(1 + g)}{i - g} - \frac{a}{i - g} = \frac{ga}{i - g} = gV_o \tag{8}$$

Putting (6) another way:

$$Vi = a + Vg \tag{9}$$

(9) says that the sum of cash income (a) and unrealized accrual (Vg) is equal to the interest carrying cost (Vi) of the land. Thus the market bids up land price (V) to the same level as though Vg were a cash payment. And investors bear a cash or other carrying cost higher than can be justified by the ordinary income, a. The premium they pay is Vg.

3. A) To prove: given an asset yielding a level annual cash flow (a) for n years, the depreciation in any year is $a(1+i)^{-n}$, where n is the years of life remaining.

Let $a = 1$
$I = 1 + i$

$$V_o = \frac{1 - I^{-n}}{i} \tag{10}$$

(10) is the basic formula for present value of an annuity. I assume you already know that.

$$-\Delta V = \frac{1 - I^{-n}}{i} - \frac{1 - I^{-(n-1)}}{i} = \frac{I^{(1-n)} - I^{-n}}{i} = \frac{I^{-n}(I - 1)}{i}$$

$$= \frac{I^{-n}(1 + i - 1)}{i} = I^{-n} \tag{11}$$

$$-\Delta V = I^{-n}$$

An industrial asset of high durability and long life--say a factory building--is somewhat like that level annuity. It yields a more-or-less level cash flow over several decades. It may well taper off toward the end, but for simplicity we now overlook that and assume it reaches end-of-life abruptly.

Let's compare tax depreciation in year one with "true" depreciation in (11). Under straight-line (SL) depreciation, and tax life = economic life = n, tax depreciation or "write-off" (w) is:

$$w = \frac{Vo}{n} = \frac{1 - I^{-n}}{ni} \tag{12}$$

w as a multiple of $-\Delta V$ is:

$$\frac{w}{-\Delta V} = \frac{1 - I^{-n}}{ni\ I^{-n}} = \frac{I^n - 1}{ni} \tag{13}$$

(13) is the ratio of compound interest to simple interest (com/simp), something we encounter often in finance. Table 1 gives a few values of (13) to give you an idea.

Table 1: Values of Com/Simp (13)

n	i = .05	.10	.20
5	1.11	1.22	1.49
10	1.26	1.59	2.60
20	1.65	2.86	9.33
40	3.02	11.06	183.60
80	12.14	255.93	135,014.22

Thus for $n = 20$ and $i = .10$, SL gives you 2.86 times as much tax depreciation in year one as there is true depreciation.

If you take "double-declining balance" ("DDB") you get double that, in year one.

If you cut tax life to 10 you get quadruple that. $4 \times 2.86 = 11.45$.

If you borrow much of the needed capital you can multiply this advantage further. In the limit, assume you borrow all the capital. The amortization of your debt follows the line of true depreciation, because that is precisely how debts are amortized in fact. Thus your net wealth remains constant, at zero. Your cash flow (which we assumed is one) just covers your debt service. Your true income from the asset is zero (which is as it should be since you have no money of your own in it).

But your taxable income is negative: i.e., you have a tax shelter. You deduct most of your debt service as interest; while most of your cash flow is offset by depreciation, meaning it is treated as a recovery of capital rather than as taxable income.

After a few years this catches up with you. Your depreciation slows down, and your amortization speeds up. If there were no way out of the later years of the investment you would still come out ahead because you would have gained early write-offs and lost later ones of equal amount. But it is better yet for the tax filer, who normally sells out after 7 years or so, taking capital gains treatment on the excess of sale price over basis. The buyer begins all over with a new basis, a restored tax shelter.

Those of you who like to build economic models, or dream about growing rich, or some combination, may have great fun playing that game. I'll leave it for you.

B) Some assets like trees, livestock, cheese and liquor do not depreciate, but gain value unto maturity. The rate of return is the value of i that satisfies (14):

$$K_o(1 + i)^n = R_n \tag{14}$$

where R_o is net sales value in year n. Solving for i:

$$i = \left(\frac{R_n}{K_o} \right)^{1/n} - 1 \tag{14A}$$

Expensing K_o, and taxing R_n at n, results in no tax:

$$r = \left[\frac{R_n(1 - t)}{K_o(1 - t)} \right]^{1/n} - 1 \tag{15}$$

The $(1 - t)$ values cancel and $r = i$, Q.E.D.

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pw

Ec. 162, Mon., 4. March.

1. Carter's Cut. – & politics.

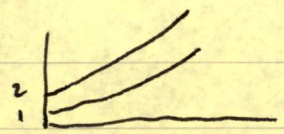
2. Declining CE model

3. Banned again
   Hi i.r., go to low invy.
   Hi w.r., " " hi "

   Invy. is stand-in for bldgs & machines, too.

4. How do you Δ labor w/o Δ in K?

   a. Free



$$\frac{2}{2\bar{L}^{\eta}} = \frac{1}{\bar{L}^{\eta}}$$

   b. Machine.



$$b = -K_{\Delta}^{\cdot IPF} + a$$

Raise K, also raise K-IPF

Only way: lower $\underline{u}$.

This raises $\frac{E\,low}{E\,mid}$ ratio.

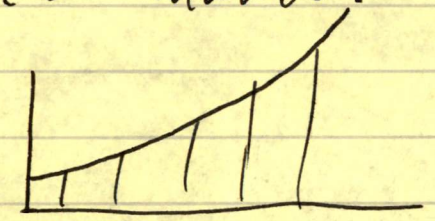
5. Concept of a mean pd.

$$P = \frac{W}{E} \quad ; \quad T = \frac{E}{K}$$ p. 3

E aka "volume".

6. Low model

8. Tree model.

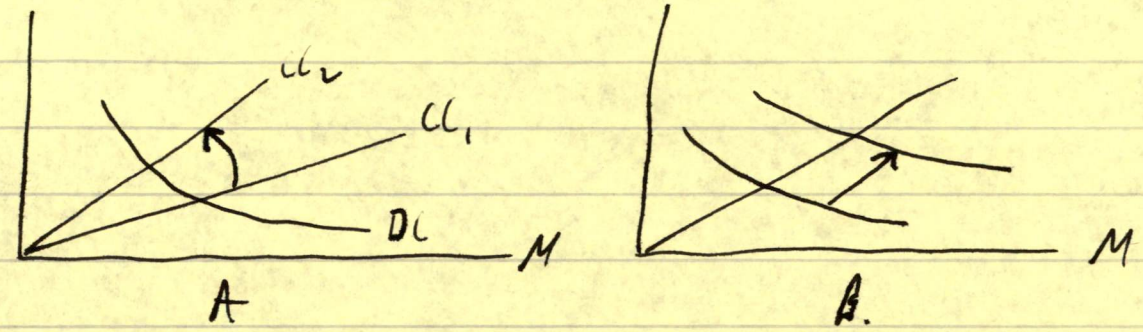


7. Thumb thru apps.

Wed. March 11

How adapt to Δi and Δ w.r.

1.



Panel A is intuitive? Δi means:

a) Less M

b) Higher DC! Subst. DC for CC to min. TC.

Apply greatly. M is K in stock; DC is cost of producing it.

If there is Δi, means ~~more~~ less M. Less M means more X. Each $ spends less time in storage. Shorten pd. of invt.

This is like a normalized model

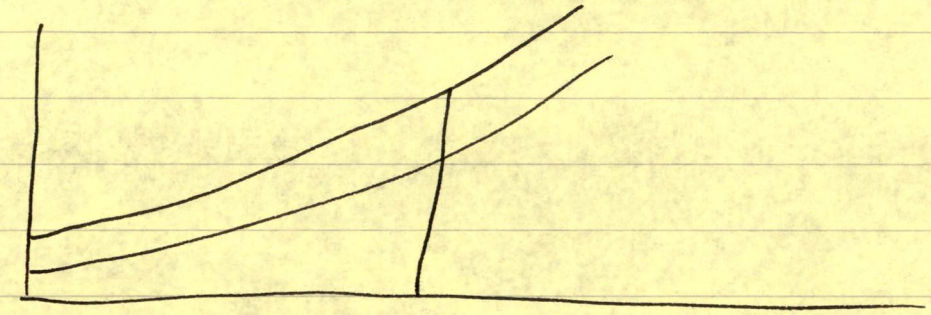
Panel B. Δ w.r. means more M

a) More M – less X. Apply labor less often.

~~b) – also more overall cost? a symmetry.~~

Fails to sub. K for labor

2. PIPO Model: Δ w.r. does not ~~decrease~~ change share going to K. Requires more K; same ratio.

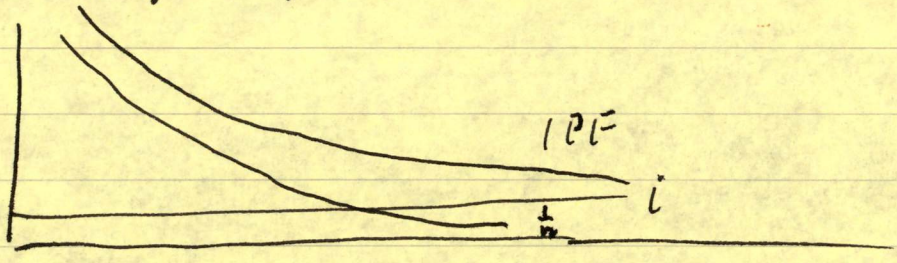


~~To reduce share of cap., must shorten u.~~ sub. K for labor, must lengthen $\underline{u}$. Lower $\underline{b}$ is what makes that happen. In normalized model, ~~lower life means~~ higher $\underline{u}$ means more K in stock.

3. Flowing model

$$b = -K \cdot IPF + a$$

Effect of Δi on b rises with $\underline{u}$



So Δi makes you invest less K ~~to keep~~ to lengthen $\underline{u}$: or if $\underline{u}$ is long.

Paradox: ~~If~~ si also leaves b, as genetic after-life.
The empty milk bottle effect.

Master Copy App. II-5

Minimizing total cost (TC) of a fund of cash (or anything else)

Notes by Mason Gaffney, 17 May 87, rev. 2/92

Notation taken from Hall and Taylor, pp. 296-98

W = monthly cash receipt (W is evidently from "wage")
M = mean amount of money held during month
x = number of transfers into cash holding during month (this is not explicit in H&T, but making it explicit makes things clearer)
k = cost per transfer
R = opportunity interest rate (I omit their subscript "o")
T = amount of money per deposit

$T = W/x$

$M = T/2 = W/2x$

Deposit Cost (DC) = $kx = (kW)/(2M)$

Carrying Cost (CC) = RM

Total Cost (TC) = DC + CC = $(kW)/(2M) + RM$ (1) [1]

Rather than follow their algebraic footnote, I assume you understand something about calculus, and will differentiate (1), and set its derivative equal to zero.

$d(TC)/dM = -kW/2M^2 + R = 0$ (2)

$R = kW/(2M^2)$

$M = [kW/2R]^{.5}$ (3)

(3) is ordinarily called the "square root rule." Let's not be ordinary: that is giving too much emphasis to the square root aspect, which is an artifact of the particular assumptions here. These are arbitrary and specific to the example. The important thing is the sensitivity of optimal M to k and R. M is an increasing function of k, and a decreasing function of R.

---

[1](NOTE TERRIBLE TYPO IN H&T, p. 298, where they use a "-" instead of a "+". They continue the error in the top line of the footnote; then correct it in the 2nd line, which however makes their algebra wrong. This gives you an idea of how carelessly some textbooks are edited. You have to be alert.)

Figure 1 shows graphically how this works. M is the independent variable here. As M rises, CC rise directly with it, and DC fall. You add these together to get the top curve, TC, which first falls and then rises, giving us a minimum.

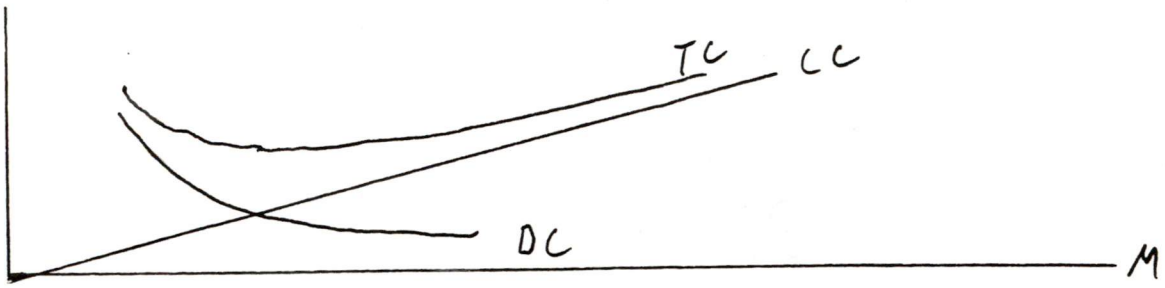


Figure 1: Minimizing TC by adding rising CC and falling DC

TC is minimized where and because the slopes of CC and DC are equal and opposite. It is of incidental interest that their ordinates are equal at this point -- i.e., it is where they cross Let's see if M has its cost-minimizing value when CC = DC.

$CC = DC$

$RM = kW/2M$

$M^2 = kW/2R$

$M = [kW/2R]^{.5}$ [as in (3)]

That is handy to remember, but also dangerous because you could easily get careless and think that the curves' crossing is a <u>cause</u> of minimizing TC. Not so: beware of this mistake. Like calling it "the square root rule," it is a case of confusing the incidental with the essential. The crossing is the minimum TC in this particular example; not in general.

Now use Figure 1 to observe the effect of R on the optimal holding of cash. Higher R means the CC curve is steeper, moving the min TC to the left, reducing the optimal cash holding.

Eqn. (3) tells you the same thing algebraically: the stock of M that minimizes TC falls as R rises.

Use Fig. 1 to observe the effect of k on the optimal holding of cash. Higher k moves the entire DC curve farther away from the origin (up and to the right), moving the crossover to the right, increasing the optimal cash holding.

Eqn. (3) tells you the same thing algebraically: the stock of M which minimizes TC rises as k rises.

What have we shown? If your opportunity cost of money is low, and the value of your time is high, you keep a large cash balance

on hand. That is an application of Baumol's inventory model to cash management. However, it is much more general.

SUPPLEMENTAL KNOWLEDGE

Having found optimal M, you can also find optimal x and T, just by substituting the value of optimal M for M. Do this as an exercise. (Where a square root sign covers several terms, just raise each one instead to the .5 power, then combine terms by adding exponents, watching your signs of course as you go.)

You'll find that optimal x is an increasing function of R and a decreasing function of k. This of course is because x, the frequency of trips to the bank, is inversely related to M, the mean stock of money on hand. Learn this well: it has much wider applications, as we will see.

1/x is the period of time between trips. Does that ring a bell? We've seen an analogous principle before, when finding the amount of capital in a pipeline. The frequency and the period of anything are always reciprocals.

You'll find that optimal T rises as k rises, and falls as R rises. That is, when you want to economize on your time you do things in large slugs rather than small dribbles; and let the results sit for a long time (1/x) before acting again.

WIDER APPLICATIONS OF THIS MODEL

If we were only interested in determining an optimal stock of cash we wouldn't lean so heavily on this model. But, as H&T mention, this is also a way to determine an optimal inventory of real capital. Just let M stand for the mean stock of goods in the store; and k for the cost of ordering, delivering and receiving a batch of goods.

But it's more general than that: much more general. Let M stand for the mean life of an item of durable capital; and k stand for the cost of producing same. Now we are moving toward a model of the turnover of all the capital in the nation, not just the inventories.

For now, just note that the incentive to substitute capital (M) for labor (k, in this model) rises as labor costs more. But this does not take the form of a rise in the flow of investment, but the reverse, because x falls as k rises.

Ec. 162. Wed. 25 Feb.
App. II-3. Rising Rents

1. $g \equiv$

2. i and g in nominal terms.

3. Evaluate $b = -K_1 \frac{i}{i - I^{-n}} + a$ when $n = 1$

$$\frac{i}{1 - \frac{1}{1+i}} = 1 + i$$

$$b = -K(1+i) + a$$

Or, when $K = 0$, $b = a$

To go from 1 yr. cycles to $n > 1$, we have necy equipment, above.

4. P. 2 various uses of cash flow. p. 3, too

5. CC defined, p. 2

6. $V = \frac{a}{i + c - g}$

---

1. Valuation.

$\frac{V}{a} =$ "$\frac{P}{E}$ ratio"

2. Arbitrage.

3. Reversal of cause & effect.

4. All future unknown, esp. g. (p. 4)

Value of old bldgs. limited by cost of new. No such limit on land.

5. Leave out $\pm$, at first.

6. Capn. of growth expectations.

$$V = \frac{a + gV}{i}$$ (3, a) – p. 4

(gV is annual rise of lv.) (This is how they sell shares, & r.e. & oriental rugs.)

~~Cf. depreca. ~~

~~$V = CF \cdot CRF$~~

Result: hi $\frac{V}{a}$ ratio, $= \frac{1}{i - g}$

If g is neg. (depreca.) $\frac{V}{a} = \frac{1}{i + d}$

E.g., $d = 10\%$, $\frac{V}{a} = \frac{1}{1 + i}$

~~Not sure~~ (also, $V = \frac{a - dV}{i}$) Relates to

~~$V_i = a$ CF = a depr.~~

$V_i + \text{depr} = a$; $CF = V_i + \text{depr} = V \times CRF$

Importance:

a. In gen'l, V of rising land highly sensitive to i

b. Buying land is major barrier to entry. Machinery easier to finance.

c. Debts to inv. in K are <u>self-liq</u>: $V_i = a - dV$

7. $CC > CF$ (for rising land) $< a = CF$

$V_i = a + gV$ p. 6. $V_i \geq CF$

These debts are non-self-liquidating. They are self-aggravating.

8. Effect of lower i

$$iV = i\frac{a}{i-g} = \frac{a}{1-\frac{g}{i}}$$

iV is a <u>decreasing</u> fn. of i (If $g = 0$, $V = \frac{a}{i}$, $V_i = a$)

9. $$T = tV = a\frac{t}{i+t} = a\frac{1}{\frac{i}{t}+1}$$

~~Exhaustible natural resource~~

an Δg fn of t

10. $$T = tV = a\frac{t}{i+t-g} = a\frac{1}{1+\frac{i-g}{t}}$$

11. p. 11 New defn. of cash flow! $\equiv NCF$

a) $NCF = a - Vi = -gV$ (100% fncd.)

b) $NCF = a - V(1-s)i = a - Vi + sVi$

but $a - Vi = -gV$

So $NCF = -gV + sVi = V(si - g)$

12. t as upper limit on V, when $g \rightleftharpoons i$

Wed. 5 Feb. Ec. 162

1. Go over exam.

2. $G = \dfrac{-K + FI^{-\eta}}{1 - I^{-\eta}}$ (16, G)

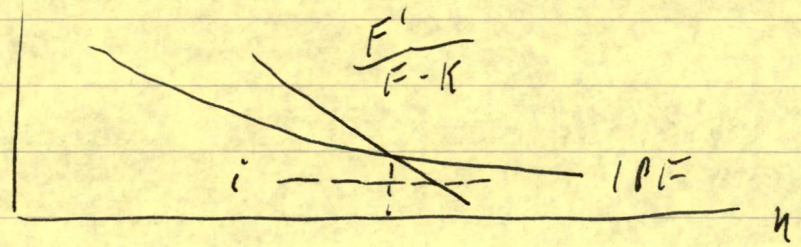
3. $\underline{b} = Gi = [-K + FI^{-\eta}]\ IPF$ (17)

$\underline{b}$ is a uniform performance standard, can cf. invts. of diff. $\underline{y}$, w/o bias.

Max PV of Cycle I is not.

4. $\dfrac{d\frac{u}{v}}{dx} = \dfrac{u'v - v'u}{v^2} = 0$ when $u'v = v'u$

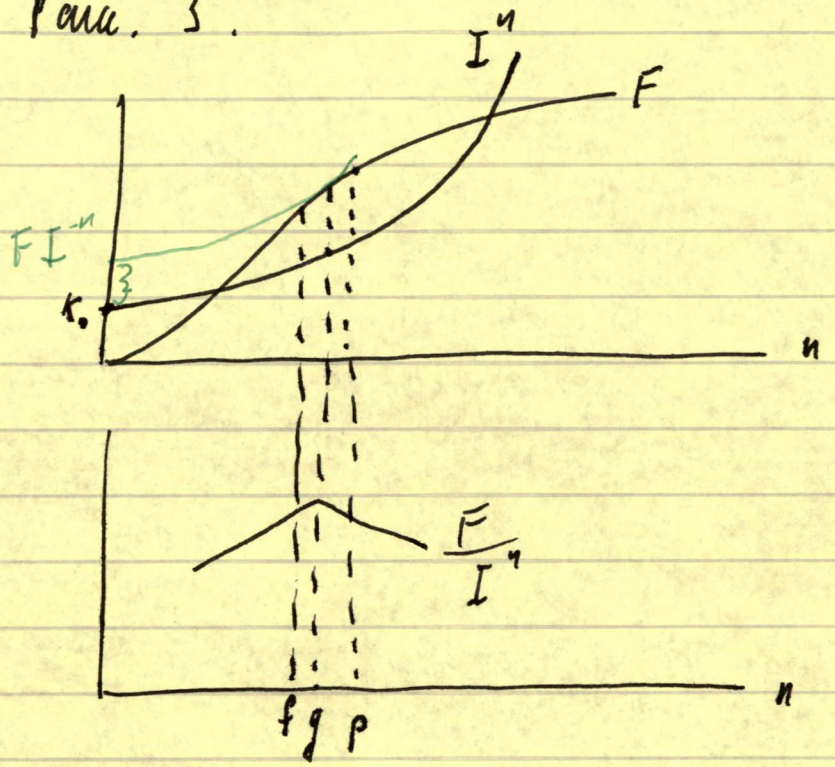
$\dfrac{u'}{u} = \dfrac{v'}{v}$



5. $F' = Fi + b.$

Econ 162. Mon., 10 Feb. 92

I. Refer to Sect. I, p. 10, on optimizing. Para. 3.



$n = p$: curves are ||
$n = g$: growth rates are =
$n = f$: Faustmann Value is max.

II. $n = g$ is where $\frac{F}{I^n}$ is max.

Ratio is max when ~~$\frac{F}{I^n} = \frac{F'I^n - nFI^{n-1}}{I^{2n}}$~~ $\frac{u'}{u} = \frac{v'}{v}$

E.g., $\frac{u \cancel{*} (1+.15)}{v \cancel{*} (1+.15)} = \frac{F}{I^n}$

This is standard, but too easy.

~~III.~~ a. Answer is the same for all K_0.

b. PV of $F > K_0$. This says you may become infinitely rich.

Actually, $[F I^{-n} - K_0]$ is PV of C_1.

To convert that to G:

$$G = \frac{-K_0 + F I^{-n}}{1 - I^{-n}}$$

III Now, max G. – p. 37.

A. $$G = \frac{-K I^n + i}{I^n - 1}$$

(I use this form because is easier to work w. I^n than I^{-n})

B. Δ of I^n, when $\Delta n = 1$, is $i I^n$

C. Use $\frac{u'}{u} = \frac{v'}{v}$

$$u' = -K i I^n + F'$$
$$v' = i I^n$$

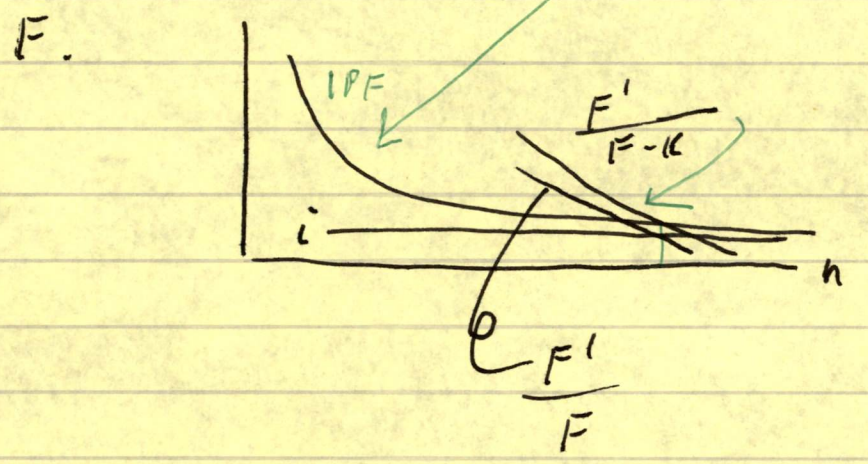
D. $$\frac{-K i I^n + F'}{-K I^n + i} = \frac{i I^n}{I^n - 1}$$

E. X-mult: (next page)

$$-K_i I^{-n} + K_i I^{n} + F' I^{-n} - F' = -K_i I^{-n} + F_i I^{n}$$

$$F'\left(I^{n}-1\right) = I^{n}\left(F_i - K_i\right)$$

$$\frac{F'}{F-K} = \frac{i\, I^{n}}{I^{n}-1} = \frac{i}{1-I^{-n}} = \text{IPF}$$



1. Answer not same for all K.
   Higher K means higher $\frac{F'}{F-K}$, later cut

2. Excess of $F I^{-n} > K_0$ now accted. for.

3. Result is much less sensitive to i.r.

Wed Feb. 12 Ec. 162

1. Trouble w. 18, C — its role as a unqual. solution is not self-evident. All max. solutions are also unqual. over.
2. MC of time = MB of time.
   Can derive it that way.

$$F' = Fi + b$$

or

$$F' = Fi + Gi$$

$$\frac{F'}{F+G} = i$$

3. Once you figure out value of G, this is simplest way to look at it.

4. With old bldgs, tear down when $CF < b$ ($Fi = 0$)

Exam

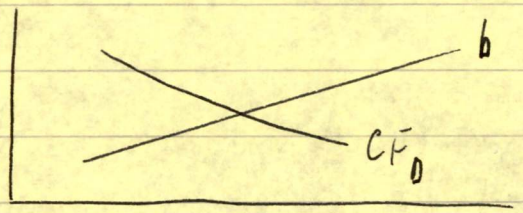
Ec. 162. Wed. 19 Feb. 92
Mon. 24 " "

I. End of bldg. life:

MR of time = CF of defender.
MC of time = Gi
Gi, or b, comes from challenger.

$$b = \left[-K + \sum_1^n \left(R_n I^{-n}\right)\right] PVF$$

(R_n is cash flow)



If R_n level, $\Sigma = R\frac{1-I^{-n}}{i}$,

$b = -K \cdot PVF + R$

II. Ripenus. Max G.

III. Redeemable Bond

$$P = a \cdot PVF + D I^{-n}$$

$$i = a\frac{1-I^{-n}}{P - D I^{-n}}$$

If $P = D$, $i = \frac{a}{P}$

IV Carter's Cut

$$i = \frac{a}{P} + \left(\frac{D}{P}\right)^{\frac{1}{n}} - 1$$

Wrong! But is based on correct idea; helps your understanding.

V. Land & Capl. This allows for appreca. (K that yields CF much appr.)

$$G + K = a\frac{1 - I^{-n}}{i} + GI^{-n}$$

Solve for G, generalize Faustman

$$G(1 - I^{-n}) = -K + a\frac{1 - I^{-n}}{i}$$

$$G = C_1 \times LVM \qquad (22, B)$$

$$= -K \cdot LVM + \frac{a}{i}$$

or,

$$b = -K \cdot PF + a$$

~~VI. Solve for Rot, from 22, a~~

How use? Shows how much ΔK is warranted to earn Δa; & how much ΔK to earn Δa.

$$a > K \cdot PF + b$$

VI. Solve for i

$$i = \frac{-K + a \cdot PVF}{G_0 - G_n I^{-n}}$$

$$G_i = -K \cdot PVF + a$$

$$i = \frac{-K PVF + a}{G}$$

or

$$G(1 - I^{-n}) + K = a \frac{1 - I^{-n}}{i}$$

$$G + \frac{K}{1 - I^{-n}} = \frac{a}{i}$$

$$i = \frac{a}{G + K \cdot LVM}$$

✓ If $G_n > G_0$

$$G_0 - G_n I^{-n} = a\, PVF - K$$

$$G_0 - G_n I^{-n} + K = a \frac{1 - I^{-n}}{i}$$

$$i = a \frac{1 - I^{-n}}{G_0 - G_n I^{-n} + K}$$

1. $P = a\frac{1 - I^{-n}}{i} + \frac{R I^{-n}}{}$

$P - RI^{-n} = a \frac{\cdot\cdot}{\cdot\cdot}$

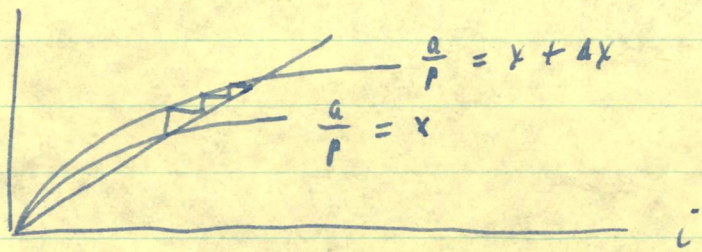
$i = a \frac{1 - I^{-n}}{P - RI^{-n}}$

(Value of programming)

2. $i < \frac{a}{P}$ when $i = a\frac{1 - I^{-n}}{P}$

i may be negative

3.



p. 21

---

4. Payback
$n > \frac{P}{a}$ bcs. int. comes off the top.

City Charter.
Foreclosure; "subordinating."

5. Defns, pp. 22-23

6. p. 23, year one.

$$\frac{1-L^{-q}}{1} - \frac{1-L^{-p}}{1} = \frac{1}{1}\left(L^{-p} - L^{-q}\right)$$

$$= \frac{1}{1}\left(L^{-p}\right)\left(1-L^{-1}\right) = \frac{1}{1}L^{-p}\left(\frac{1}{1}\right) = L^{-p}$$

Econ. 162, F85, Answers to Study Questions, Sets 1-3

Set #1. Answers have been previously distributed in cryptic form. This is a supplement for #2,3,6, and 7.

2. This is a ratio where the top is the value of the $US and the bottom is the value of the $C. The top remains constant while the bottom drops .2. The change in the ratio is therefore (0- -.2)/(1-.2)=+.2/.8= .25. The $US rises 25%.

Check. The $C now contains 80c US. Or, $1C=.8$US

1/.8$C=1$US

1.25$C=1$US

3. Here we can measure values in terms of some fixed third item. Gold content is a good standard measure. The gold content of the mark drops .10; the gold content of the franc rises .18. The top of the ratio here is the value of the mark, measured in gold content; and the bottom is the same for the franc. The change in the ratio is (-.10-.18)/1.18 = -.237.

Check. The new ratio divided by the old ratio is (1-.10)/1.18=.763=1-.237.

6. Use the product rule. R=PY, so r=p+y+py = -.70 -.45+.315 =-.835.

Check. The new product divided by the old product is .3x.55=.165=1-.835.

7. Rates of return are fractional increases from a base of one. Let r be the real rate of interest; i=nominal rate; and f=inflation rate. Then:

1+r=(1+i)/(1+f)

Solving for r,

r=(i-f)/1+f)

But in this problem we know r and want to find i. Solving for i:

i=r+f+rf=.064+2+.128=2.192

Answers to Set #2.

8. A. MP = 100-10x5=50

B. MP = 105 - Marginal Complementary Costs (MCC)

MP-MCC is called "Marginal Net Product".

In equilibrium, and when increments are small, MNP=MP. This is on the assumption that the costs of the new hired workers equal the marginal productivities of the internal workers who would otherwise have been reassigned from the first 10 acres. That is, it would be the equivalent of 50 bushels of corn.

The MNP of 105 is 5 acres more than the MP of 100. That extra 5 bushels is not caused by an economy of scale, in this example, but by avoiding a loss caused by unbalancing the inputs. Note, however, that when you expand in balanced fashion you avoid diminishing returns caused by unbalancing the proportions. This is when you find out if there are any scale economies, and if there are you can keep expanding.

9 and 11. Answer is consolidated.

WORKERS	AP	TP	MP	GAIN/WORKER	GAIN
		(Value figures in $000)			
7	20	140	--	6	42
8	22	176	36	8	64
9	24	216	40	10	90
10	25	250	34	11	110
11	24	264	14	10	110
12	22	264	0	8	96
13	20	260	-4	6	78
14	17	238	-22	3	42

Extra. You can also find MP from AP and Workers by using the product rule with fractional changes. Going from 7 workers to 8, the MP is (1/7 + 1/10 + 1/7x1/10)x140 = (18/70)x140 = 36.

10. "Productivity" usually is used to mean the AP of one input. It ignores the cost and value of the other inputs, and maximizing just one sets the MP of the others at zero. A correct procedure will maximize the residual return to the others, by setting the MP of the variable input at its MC.

11. "Profit" equals TP - $14,000n, where n is number of workers. But TP = APn, so profit = n(AP-$14,000), as in the table in 9, above, q.v.

12. The relationship is "inverse", using that term loosely, meaning that one rises as the other falls. You have learned to show this graphically; and also from the assumption that the sum of the Es of the inputs is equal to one, (an assumption indicating that returns to scale are constant).

13.

Stage	E
I	E>1
II	1>E>0
III	E<0

E=MP/AP. Relate your answers to a graph showing the relationship of MP and AP through the stages of productin.

14

15. Because incremental adjustments involve taking land entirely in our out of production -- there is no room for change at the intensive margin of production.

16, 17. --

18. Repeats 12

19. a. E=2, because 2 is the exponent of the production function.

b. Use your ingenuity here. What is the purpose of a fence?

20. Cf. qn. 8.

21. Deduct the MCC of the truck and the share of the dock used.

22. Simple productivity is partial and unbalanced. Cf. #10.

A better measure is net product. It is better because it accounts for the value of all inputs used, not just one.

23. The 40% cutback violates the equimarginal principle of resource allocation, unless all farms are identical, which they never are. Aa better system would charge every farmer an equal price, so the marginal cost would be the same to all.

24. Land increases by 1/16. E is .6, so output increases by .6 of 1/16, or 3.75%. The new ratio of output to land is 1.0375/1.0625 times the original ratio.

Econ. 162, F85, Answers to Problem Set #2, Questions 25-31

25. Find the E of the volume w.r.t. the base; and of the surfaces w.r.t. the base. Divide the first by the second, to get the E of the volume w.r.t. the surfaces. The first E is 3, since that is the exponent of the function relating volume to base. The second E is 2. 3/2=1.5.

For a fuller answer you could prove that the exponent is the E, as we did in class. The proof should be in your notes.

For a better answer yet you would note that production functions, which are what the question asks for, relate economic values to economic values, that is costs to value of output. The simple assumption is that all cubic feet have the same value, and all surface square feet have the same cost. In fact that is not quite true. Cf. #26.

26. The value per cubic foot (pcf) of rooms falls, after a certain moderate size is reached, as they get larger. The purpose of rooms and buildings is privacy, protection, and control over space for persons or groups. Rooms lose "unit value" (i.e., value pcf) as they get beyond "human scale". This is most obvious as to height. But horizontal movement is also costly as the distance increases from wall to wall. There is also diminishing marginal utility of the furnishings added to rooms after the essentials (e.g. a desk, a bed, a dining table, a sink and stove and fridge) have been accommodated. Meeting rooms are no more productive than the groups that meet there, and Peters and Waterman doubt the productivity of working groups larger than 15 people. If the meetings are infrequent the rooms are less productive than the groups. All that is common sense and you could add to it indefinitely.

The unit cost of surfaces also rises with size. Geometry may argue for larger cubes; gravity argues for smaller ones. Longer ceiling spans call for stronger, costlier construction. Higher buildings need more stairs and elevators, and stronger materials and foundations. Again, that is common sense, and you could add to it.

27, 28. These two questions go together. There was some malcommunication between me and the typist, resulting in excessive inclusions. You are responsible only for things we discussed in class. There is also one typo. The value of S when n=30 is 610, not 510.

The excessive inclusion is the last row, labelled "IPF". Instead of that, just use the interest rate, 5%, which remains constant for all values of n.

Now, as we discussed in class, optimal harvest time is when S'/S=i=.05. This is somewhere between n=35 and n=40 (rather than between 30 and 35, as stated in the Problem Set Qn. 28).

Then you can go ahead and follow the same procedure to nail it down to the individual year. It is between 35 and 36, which is close enough.

As you gather from the foregoing, the rule applied above is only a first approximation to what is generally correct. Before the second midterm we will discuss the use of the IPF in preference to the interest rate. Then the discussion of broilers will become meaningful. For the first midterm, you may overlook that.

29. The easiest way to find this elasticity is to use the rule I gave you which lets you use the approximation:

$$e_{in} = (1+i)^n$$

Now you can proceed as follows:

Check this by numerical example, with i=.12 and F=1

n	P	P	P/P	n	n/n	E
20	.0907					
21	.0805	-.0102	-.1129	1	.05	-2.2581

-ni=-20x.12=-2.4, which is slightly smaller (in absolute value) than -2.2581. This difference is of course Berkeley's ghost again. The higher figure is called "arc elasticity"; the lower one is "point elasticity". You can make them nearly identical by making the increase in n very small, and getting your results out to many significant figures. E.g. when you from 20 to 20.1, the arc elasticity works out to -2.3900

But now let's get back to the rule we just discovered. It tells us that the % sensitivity of P to % changes in n increases with n. This figures, since we know that the % sensitivity of P to absolute increases of n is always constant, equal to 100i. And of course as n gets larger it takes a higher absolute rise of n to constitute any given % rise of n.

But the same reasoning can be applied where i is the variable, because i and n have parallel roles in the formula. The E of P w.r.t. i is also -in. This immediately tells us something of enormous practical value: present values are more sensitive to interest rates when the future values being discounted are farther in the future. Work out some numerical examples to persuade yourself of this. Question 30 helps you visualize it.

30.

31. Most of the terms have been defined in class, in the text, or in the present notes. Only two items need comment here.

a. The notation for elasticity. The questions use "ep", which is short for epsilon, the Greek letter conventionally used for elasticity. I have no epsilon on my printer. However, I am now using "E" for elasticity, rather than "ep".

b. "Leverage". We have not gotten to this, it will not be on midterm.

Ecs. 162, F85, Answers to Set #3 of Study Questions

1. Not enough information is given for you to give a firm answer without adding some assumptions of your own. Part of a good answer is for you to see that, and make that point.

I will assume two things: that street costs are proportional to street area; and that lot values do not decline at all in step with lot areas (although they do decline some).

When lot size goes from 10 down to 1/8 acre the usable land per 100 acres falls from 97 down to 84 acres, and the number of lots per 100 acres rises from 9.7 (97/10=9.7) up to 672.0 (84/1/8=672.0). 672/9.7=69.3, so the increase in product (measured in lots) is 69.3-1 or 68.2 times. The increase in cost is (16/3)-1 or 4.3 times. The elasticity of production of lots with respect to street costs is thus much greater than one. That puts you in Stage I.

The value per lot would have to decline by a very large fraction to keep the % increase in the product from exceeding the % increase in costs. We have assumed it only declines a little.

You can also show this is Stage I by showing that the AP of street costs is rising in terms of lots produced, as you put more land and money into streets. The number of lots per acre in streets rises from 3.2 (9.7/3=3.2) up to 42.0 (672/16=42.0).

Extra credit. You would be astute to point out that we are looking only at two points on the curves. They may well straddle the dividing line between Stage I and Stage II.

2. Functional distribution is imputing shares of the product to the inputs responsible for increasing it, and is used in determining how much of each input to hire. Personal distribution is paying off the owners of the inputs, and is used in determining profits, which are the return to the ownership interest. Profits depend entirely on what is supplied by the owner, and what is hired from outside. At one extreme capital may all be borrowed and land all rented by a worker-owner. In such a case "profits" are the owner's wages and salary, basically. At the other extreme the owner may use his own land and capital and hire all the workers, including

the manager. In yet other cases the workers' pension fund grows and the nominal owners' equity shrinks until the pension fund absorbs all the equity and the pensioners become the owners. There are all possible mixes of ownership interests, which change personal distribution and the calculation of profit. Functional distribution is unaffected by such changes (except indirectly).

3. When land is open to all comers (x) the perceived marginal product of each is the average product of all. To see this geometrically derive the MP of x as the change in TP when TP is defined by a rectangle determined by a point on the AP curve. Draw two such rectangles, determined by two adjacent points on the AP curve. The change in the rectangle's area is now a column on the right less a row on the top. The column is the AP; the row is the decline in the AP of all the previous users.

With open access, newcomers enter the land attracted by the AP, all of which they keep, paying no rent. No one subtracts from his returns any amount corresponding to the decline in the AP of all the early entrants. Each perceives the AP as his MP. But this attracts more and more entrants so long as the AP>MC. Crowding continues until AP=MC. At this point there is no more rent, because rent is (AP-MC)x (where x is the numberf units of the variable input). This is the "dissipation of rent", as related to the average and marginal products of x.

To see this non-geometrically think of examples of open access and overcrowding: fishing beds; open range; freeways; parks; underground pools of water, gas and oil; etc.

4. Let p stand for pi, a symbol not on my printer.

$$C=2pr$$

$$A=pr^2=C^2/4p$$

$$dA/dC=2C/4p=C/2p=2pr/2p=r$$

$$\frac{dA/dC}{A} = 1/pr$$

$$E= \frac{1/pr}{1/C} = 2pr/pr = 2$$

4, (cont). K is proportional to C, so the E of production of A w.r.t. K is the same as that w.r.t. C, which is 2. But the E of cost with respect to capacity is the reciprocal of the E of capacity w.r.t. cost, so this E=1/2.

5. $V=pr^2h$ (1)

$S=2pr^2 + 2prh$ (2)

Hold V constant and substitute in (2)

$$S=2pr^2 + 2V/r$$

Differentiate w.r.t. r and set derivative = 0.

$$dS/dr = 4pr - 2V/r^2 = 0$$

$$4pr^3 = 2V = 2pr^2h$$

$$2r = h$$

This is a pure case of substitution, because output (V) is held constant while we substitute r for h.

6. Show by graph or table the residual return to the fixed factor while you vary the other input. Show that the residual return is maximized when MP=MC.

7. Defer until after midterm.

8-10. Easy. Over to you.

Last 3 Quizzes, Ec. 162

A. Weeks 1 & 2

1.

a. ~~Graphs~~ The going i.r. is 5%. Prices & costs were unjusted, so all PDO rents, of whatever duration, earn 5%,

Graph same log.

Give value of $1 instant after 10, 20 & 30 yrs.

b. The i.r. rises to 6%.

What is the PV of the future values you calc. in (a).

2. 2 parts, what is int.

B. Weeks #6 (chance vars.)

C. Cow problem

#5

$i = .05$
$a = 800$
$F_1 = 25k \quad n = 10$
$F_2 = 18k \quad n = 15$

$G_1 = 800 \times 7.72$ or $6,177$
$+ 25k \times .614$ or $15,348$
$= 21,525$

$G_2 = 800 \times 9.62$ or 7696
$+ 18k \times .48$ or 8658
$= 16,354$

$$\frac{21,525 - 16,354}{21,525} = 1 - \frac{16,354}{21,525} = .24$$

6.

b. $200 \times \left(\frac{1.08}{1.16}\right)^9 = 200 \cdot 1.183 = 236.64$

$200 \cdot \left(\frac{1.08}{1.06}\right)^{18} = 200 \cdot 1.40 = 280$

Econ 162, F92, Final Examination
Wednesday, 9 Dec 92, 8-11 AM

As announced, the standard time expectation for this final examination is one hour. It is weighted the same as three quizzes. You may take more than one hour if desired. I do not recommend taking less than one hour.

If you are making up quizzes, remember to allow time for that.

Answer any ~~six~~ 5 questions.

1. Find GAT (the present value of ground after income tax) given the following parameters.

K = 25
a = 13
i = .058
d = .11
n = 12
t = .35
s = .45
x = 8
F = 55

Show separate values for four components of your answer, viz:

a. The discounted cash flow after tax, less K.
b. The discounted value of the tax-depreciation (or write-off) of K, over its tax life (x).
c. The discounted value of F, less the tax on sale of F.
d. The value of writing off G at time n. (Remember, this part involves dividing by something, rather than adding something, because we had to collect terms algebraically in order to solve for G. Your answer should show the number you divide by, and how you found it.)

2. Given the same numbers as in #1, except t = 0, s = 0, and x is not relevant, and you must find optimal n:

a. What is optimal n? Show how you find it.
b. What is G? Show the two components of your answer, viz:
   i. DCF, less K;
   ii. The discounted value of F.

3. If you answered #2, you may answer this one. In #2, change the interest rate to i = .08, and change the decay rate to d = .13.

a. Find optimal n and max G.
b. Show the two components of your answer, as in #2.
c. Proceeding systematically and carefully, make a table marshaling all the following:

i. The values of n, G, and the two components of G. Put these in columns, with the headings in a row across the top. Label the first row of data "Case #2," and the next row "Case #3."

ii. Changes in those values, as you move from the values of #2 to those of #3. Remember to show the sign of each change. Put these answers in a row under the Case #3 row. Round these numbers off, to prepare for the next sub-question. Remember, your calculator does not do this automatically if you retain numbers in memory, even though the display may be rounding them off.)

iii. The fractional changes (i.e., divide each change by the base from which it started). Put these data in the next row down. (For this exam, do not express these answers as percentages, but as decimal fractions only.)

d. For extra credit, add another row showing the elasticity of the changes w.r.t. i. This means, divide the fractional change in each dependent variable by the fractional change in i, which caused the change. (Strictly, we should hold d constant to get a true elasticity, but you don't have time now to go back and do that.)

4. DOUBLE CREDIT

a. What if G is known, and you want to find RORBT (Rate of Return Before Taxes). RORBT means, set t and s equal to zero, and solve for i. Show the equation you would use. (Note that x drops out.)

b. (a) was just to warm you up. Now, put t and s back in, and solve for i. (Remember, you must now put x back in, too.) This value of i is RORAT (Rate of return after taxes). Tough? That's why this is for double credit. If this appalls you, do not attempt this question. I never walked you through this in class.

c. Finally, solve for RORAT given the values in #1, except that n=9, F=55, G=48, and i is unknown. It is one of the following: .065, .074, or .094.

5. Angela's citrus grove grosses $4,000/acre per year, on the average. Her expenses run $3200/acre, and it cost $8,000 to plant and start up the grove originally. She had been expecting to sell to a developer from Beverly Hills for $25,000/acre in December, 2002, but he went belly-up in the S&L crash so she has lowered her expected sales price to $18,000/acre in December, 2007. By what fraction has the present value of her citrus grove dropped as a result of this change?

6. Using a semi-log scale, draw a curve showing the locus of all

points that yield 8% on a PIPO investment of $200. Part of a correct answer is to label your axes fully and correctly. Show the scale on the ordinate, remembering that equal distances represent equal fractional increases. Also show the scale on the abscissa.

Select two points on the curve, at n=9 and n=18.

a. What are their respective present values when discounted at 8%? Which is higher?

b. Show their present values when discounted at 6%. Which is now higher?

c. Generalizing from (b), what does it tell you about the sensitivity of discounted values to discount rates?

7. To determine financial maturity (optimal harvest time) of a PIPO investment, one proposed method is to select the time when the Rate of Return on planting costs (K) is a maximum.

a. Show graphically how this solution is found: you rotate a certain curve upwards from its y-intercept. What curve? What makes it rotate upwards?

b. Show algebraically what is being maximized.

c. What reasons have been given for rejecting this solution?

8. You are urged to install a water-saving john, costing $250. The plumber explains how it will save $50/year in water bills, paying for itself in 5 years. He will let you pay with your credit card, on which your interest is figured at 19%.

a. What is the true full payout period?

b. How much of the capital will you have recovered at the halfway mark, i.e. when $\underline{n}$ is half the full payout period?

c. What is the mean payout period (when your savings will have paid off half the debt, if devoted to that purpose)?

9. You are approved for a fully amortized loan at 11%, with a term of 20 years. Payments are annual.

a. What is your first payment, as a fraction of the principal?

b. What share (fraction) of the principal do you retire with your first payment?

10. You can raise timber worth $50,000 at the loading dock by

spending $1500 today to plant it. Money up front costs you 10%/year. You will sell the timber in 36 years. Oh, yes, harvest cost will be $20,000.

On the other hand, you could graze the land and gross $500/year, with expenses of $200/year.

a. Which is the better deal?

b. What gross grazing rental would make the two deals equally attractive?

11. What kinds of loans are "self-liquidating"? What kinds are not? Weave "negative cash flow" into your answer. Use algebra to show the ratio of price to cash flow for different kinds of assets.

12. Your choice: ask and answer any question you like. Answers will be graded on the quality of the question, as well as the answer. If you choose this question, a higher quality standard will be applied in the grading, for obvious reasons.

13. The national debt is growing at $300b/year. If no adjustments are made in revenues or spending, the annual deficit (deficit means addition to debt) must grow by the amount of interest on the added debt. If this goes on for 20 more years, at 8% interest, how much will be added to the national debt? (Sorry to play the Grinch, but someone has to spill the bad news: have a Merry Christmas anyway.)

14. The arrow diagram shows that sensitivity of present values to interest rates rises with futurity. Given that n = 15, the DCF factor is either more, or less, or no more or less sensitive to $\underline{i}$ than the PIPO discount factor.

a. Which is it, more, less, or the same?

b. Explain why, considering the lesson of the arrow diagram.

15.

a. An income tax rate of 50% is not likely to lower the present value of land by 50%. Explain why not.

b. A property tax rate of 2% might, in the right circumstances, lower the value of the taxed property by more than 50%. Demonstrate what those circumstances might be.

Econ 162, F92, Final Examination
Wednesday, 9 Dec 92, 8-11 AM

As announced, the standard time expectation for this final examination is one hour. It is weighted the same as three quizzes. You may take more than one hour if desired. I do not recommend taking less than one hour.

If you are making up quizzes, remember to allow time for that.

Answer any ~~six~~ five questions.

1. Find GAT (the present value of ground after income tax) given the following parameters.

K = 25
a = 13
i = .058
d = .11
n = 12
t = .35
s = .45
x = 8
F = 55

Show separate values for four components of your answer, viz:

a. The discounted cash flow after tax, less K.
b. The discounted value of the tax-depreciation (or write-off) of K, over its tax life (x).
c. The discounted value of F, less the tax on sale of F.
d. The value of writing off G at time n. (Remember, this part involves dividing by something, rather than adding something, because we had to collect terms algebraically in order to solve for G. Your answer should show the number you divide by, and how you found it.)

2. Given the same numbers as in #1, except t = 0, s = 0, and x is not relevant, and you must find optimal n:

a. What is optimal n? Show how you find it.
b. What is G? Show the two components of your answer, viz:
    i. DCF, less K;
    ii. The discounted value of F.

3. If you answered #2, you may answer this one. In #2, change the interest rate to i = .08, and change the decay rate to d = .13.

a. Find optimal n and max G.
b. Show the two components of your answer, as in #2.
c. Proceeding systematically and carefully, make a table marshaling all the following:

i. The values of n, G, and the two components of G. Put these in columns, with the headings in a row across the top. Label the first row of data "Case #2," and the next row "Case #3."

ii. Changes in those values, as you move from the values of #2 to those of #3. Remember to show the sign of each change. Put these answers in a row under the Case #3 row. Round these numbers off, to prepare for the next sub-question. Remember, your calculator does not do this automatically if you retain numbers in memory, even though the display may be rounding them off.)

iii. The fractional changes (i.e., divide each change by the base from which it started). Put these data in the next row down. (For this exam, do not express these answers as percentages, but as decimal fractions only.)

d. For extra credit, add another row showing the elasticity of the changes w.r.t. i. This means, divide the fractional change in each dependent variable by the fractional change in i, which caused the change. (Strictly, we should hold d constant to get a true elasticity, but you don't have time now to go back and do that.)

4. DOUBLE CREDIT

a. What if G is known, and you want to find RORBT (Rate of Return Before Taxes). RORBT means, set t and s equal to zero, and solve for i. Show the equation you would use. (Note that x drops out.)

b. (a) was just to warm you up. Now, put t and s back in, and solve for i. (Remember, you must now put x back in, too.) This value of i is RORAT (Rate of return after taxes). Tough? That's why this is for double credit. If this appalls you, do not attempt this question. I never walked you through this in class.

c. Finally, solve for RORAT given the values in #1, except that n=9, F=55, G=48, and i is unknown. It is one of the following: .065, .074, or .094.

R=4000-3200 K=8000 18,000
n=10 n=15

5. Angela's citrus grove grosses $4,000/acre per year, on the average. Her expenses run $3200/acre, and it cost $8,000 to plant and start up the grove originally. She had been expecting to sell to a developer from Beverly Hills for $25,000/acre in December, 2002, but he went belly-up in the S&L crash so she has lowered her expected sales price to $18,000/acre in December, 2007. By what fraction has the present value of her citrus grove dropped as a result of this change?

6. Using a semi-log scale, draw a curve showing the locus of all

points that yield 8% on a PIPO investment of $200. Part of a correct answer is to label your axes fully and correctly. Show the scale on the ordinate, remembering that equal distances represent equal fractional increases. Also show the scale on the abscissa.

Select two points on the curve, at n=9 and n=18.

a. What are their respective present values when discounted at 8%? Which is higher?

b. Show their present values when discounted at 6%. Which is now higher?

c. Generalizing from (b), what does it tell you about the sensitivity of discounted values to discount rates?

7. To determine financial maturity (optimal harvest time) of a PIPO investment, one proposed method is to select the time when the Rate of Return on planting costs (K) is a maximum.

a. Show graphically how this solution is found: you rotate a certain curve upwards from its y-intercept. What curve? What makes it rotate upwards?

b. Show algebraically what is being maximized.

c. What reasons have been given for rejecting this solution?

8. You are urged to install a water-saving john, costing $250. The plumber explains how it will save $50/year in water bills, paying for itself in 5 years. He will let you pay with your credit card, on which your interest is figured at 19%.

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