



Problems on Capital

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PROBLEMS ON CAPITAL

1. Joe's widowed mother has a nest egg of \$100k to last her for 10 years, after which Joe expects to be ready to take care of her if need be. What yearly cash can she expect if:
 - a. She keeps it in a safe and prorates it over 10 years?
 - b. She declines to invade the principal while investing it at 9%?
 - c. She takes both principal and interest in a level annual payment, investing the unrecovered principal at 9%?
2. In the PV formula an increase of n by $x\%$ yields less than an $x\%$ increase of PV. Explain why:
 - a. Using a graph;
 - b. Using algebra, and calculus if necessary or desired;
 - c. Using numbers--what some call a "real" example.
3. You borrow 10k amortized over 40 years.
 - a. After 20 years, how much of the principal have you paid off?
 - b. In what year have you paid off, and the lender recovered, half the principal?
4. You buy a 60 month battery from Kragen's for \$45. After 38 months it dies. They refund you on a simple prorate basis for the missing 22 months.
 - a. What is your refund
 1. per month?
 2. total?
 - b. At 1.5%/mo. what is the monthly cost (m) of the battery to you? (e.g. if you paid over 60 months on the installment plan.)
 - c. What is the PV of m over 22 months?
 - d. How much of your capital of \$45 had you recovered after 38 months?
 - e. Are you going to argue with a clerk at Kragen's over this amount? Would he/she know what you are talking about anyway?
 - f. Is this a good deal for Kragen's? (in defense of Kragen's, my own battery failure was the fault of my car, not the battery.)

5. You sell a house and finance the buyer over 30 years at 11.5%. He makes payments only once a year.

- a. What share of the first payment represents amortization (which is capital recovery to you, and debt retirement to him.)
- b. What share of all payments is amortization?
- c. What share of payments 1-8 is amortization?

6. Learning through vocabulary expansion.

Define the following terms. Explain the use of each one in finance for managerial economics.

Amortization	Depreciation
Appreciation	Annuity
Level annuity	Compounding
Discounting	Present Value
Discounted Cash Flow (DCF)	Internal Rate of Return
Marginal Efficiency of Capital	
Yield to maturity	
Ep of PV w.r.t. n	Ep of PV w.r.t. i
Cash Flow	Com/simp ratio
Capital recovery	Increasing function
Economic life	PIPO
Capital-intensity	Equilibrium
Ln	Log derivative
Sensitivity analysis	Reciprocal
Rule of 72	Commensurability
Overhead	Redeemable bond
Fund and flow	Appraisal
Capital	Valuation
Collateral	Land
Stumpage	Self-liquidating
Service Flow	Rate of growth
Target Rate of Return	Asymptote
Real interest rate	The Secret of Donaldson Brown
Opportunity Cost	Nominal interest rate
Optimizing	Imputed interest
	Leverage

7. a. Derive the PV formula (flow into fund, not PIPO)

- b. From the PV formula, find:
 1. Installment Plan Formula
 2. Future Value of annuity
 3. Sinking Fund Formula
 4. Capitalization in perpetuity
 5. PV of periodic payment
 6. I.R.R. of annuity
 7. Payback time

c. Give an example of the use of each in managing capital and land.

8. A manager's job is always to manage her/his capital to synchronize cash flow with cash needs--thus to "meet the payroll", not to mention all his/her other bills. How does she/he use her/his knowledge of depreciation to achieve that end?

9. You believe that an orange grove will yield cash flow of \$210/acre for 15 years; then be sold to a developer for \$20,000/acre. Using $4\% = i$, how much would you pay for it today? Using 8% ?

How is the relative weight of the land resale element affected by using the higher i ?

10. An office building will yield net cash flow of \$10k/yr (in constant dollars) for 40 years, and can be built now for its gross DCF, discounting at a low 5% to allow for inflation.

- a. How does allowing for inflation mean using a low 5% ?
- b. Is using the low 5% consistent with expressing cash flow in constant \$?
- c. How much does the building depreciate in year 1?
- d. When will you get back half your capital?

--To answer that one first solve the PV formula for n , which is done with logs. Then plug in half of P for P , add sugar, stir well, and solve for n . Take advantage of the accuracy of your calculator out to 7 places or more, which lets you use logs with the speed of light--not like the old days when you had to interpolate using clumsy tables.

11. Forester Sven Bjornsen wants to plant some Douglas-fir seedlings on the western slope of the Coast Range in Oregon, for harvest in 50 years. He believes they will then be worth \$10,000/acre in constant dollars.

- a. How much can he spend planting them, to earn 5% ?
- b. How much less must he plant them for in order to earn 7% ?
- c. Tom Dooley, Farm and Forest Advisor, hangs down his head and tells Sven that borrowing and discounting at more than 7% is fatal because then the discounted value of the future stumpage will be negative. Is this possible?

12. Ollie Ollie has an oil leasehold to develop. The quicker he extracts the oil the more its present value. But to speed the flow requires more wells, which cost money. Let S = the total stock of oil, a fixed and known quantity; then S/n is the cash flow yearly. You may assume that K halves each time you double life, and vice versa. From those beginnings can you draw up a formula to help Ollie balance K with cash flow and maximize PV?

13. What is the value of land yielding cash flow of \$1:
- @8%?
 - @8% if net income is to rise by 2% each year?
 - If you borrow to buy the land at its correct PV, at 8%, how long will it take to retire the debt from the cash flow of the land?
 - How does the cash flow of the land differ from the income of the land?
 - When a banker lends to you to buy land, is this loan "self-liquidating"?
14. Find the PV of a 32-year cash flow that begins with \$1 and declines at 3%/year. Discount at 9%.
15. Sawdust Sally has 50 acres of trees. Her stand is "normalized" or staggered, which means the following: each acre was planted in a consecutive year, such that the oldest trees are 50 years old, the youngest are one year old, with one acre for every age in between. The investment value of each stand grows along a curve of compound interest, beginning from a planting cost of \$500/acre.
- What is the aggregate value of all 50 acres of timber? (This answer happens to come a little easier if you say the oldest trees are 49 years old and the youngest ones were just planted today, i.e. are 0 years old. You are now authorized to assume that.)

--To solve this one remember two things. First, the future value formula is the PV formula times I^n . Second, PV is the sum of a series beginning with I^{-1} . Multiply every element in that series by I^n and you get every column under the compound interest growth curve. The sum of those columns is the aggregate value being sought. You may be temporarily baffled because you have to reverse the order of the years, but you should unbaffle yourself as to that--a sum is a sum, regardless of the order in which you add.
 - What is the yearly interest on that aggregate value?
 - What is that interest as a share of the yearly output? (The trees are cut at age 50).
 - There is a "fallacy of the disappearing inventory" which goes like this. The product champion who likes to grow timber says that now that the forest stand is normalized he gets mature timber out each year at the same time that he puts new capital in. He has thus eliminated that lapse of time between planting and harvesting. Therefore, no interest should be charged. How would you answer this champion?

16. Doris Developer spots a corner site for a new office building. Street improvements are in; zoning is right. Her building would cost \$700k. Adding on interest on construction money during the period of building and leasing makes that \$800k.

She believes cash will flow in at \$12k/mo at first, rising by 1.5%/yr, for 50 years. She can finance at 14%. The land seller wants \$220k. Is it worth it?

17. Discuss the gains and losses from spreading fixed capital over time, in the case of depreciating capital like machinery or buildings.

18. In maximizing the PV of a building investment, explain how you determine the optimal lifespan. Explain why this is a typical example of an optimizing problem.

19. You pay \$4550/acre for a kiwi farm. It yields cash of \$600/acre/year for 7 years. How much must a buyer pay you for it after that 7 years in order for you to have made 15% overall?

20. An office building will yield net cash of \$5k/year for 45 years, and can be built for its PV (using 6% for discounting).

a. How much does it depreciate in year #3?

b. When will you have recovered 85% of your capital?

--If it helps, here is a similar problem with the answer. If the yield is \$10k/yr for 50 years, PV is \$183k at 5%. In year one it depreciates by $1.05^{-50} \times \$10k = \872.04 . You get back 50% of your capital in 37.51 years.

21. Old Settler Sam has a gravel pit containing 10k tons of usable gravel. It sells for \$20/ton, which would seem to make it worth \$200k gross, that is if Sam could dig it all out right away for nothing.

It would, however, cost him \$900k in capital cost to install the capital capacity required to extract it all in one year. And then the capital would have no salvage value. He could slow down and do it in two years for half that; in four years for half of that again; and so on.

What is the optimal rate of extraction, i.e. that rate which maximizes DCF?

22. Fannie Farmerette can buy a new combine for \$80k. It will replace two workers @\$7k/year. Flush Fred Financier is ready to finance it at 11.5%. Fuel and maintenance figures to run \$3500/yr. How many years must Fannie keep the combine in service in order to make out financially?

23. Gianetta gets deeply in hock so her debts equal 90% of her gross worth, consisting of real assets pledged as collateral. Real interest rates rise by 30%. Should this make her banker happy, if:

- a. All her assets are self-liquidating within 6 months;
- b. Her asset is an old "cash cow" nearing end of life;
- c. Her asset is unripe real estate in a potential boom area.

24. Pamela Plunger is drooling over the prospect of snapping up \$400k of drilling pipe for her oil field supply business. The price is down because demand is dead and there are acres of pipes looking for buyers. But she believes she can sell 3/4 of the pipe for \$400k in 42 months and break even. "After that it's all profit", says she. But before plunging she asks your opinion. What is it? In this business, figure interest at 2%/mo.

25. Riverside borrowed \$80m at 12% to buy 1.79% of SONGS (San Onofre Nuclear Generating Station) units II and III. The engineering consultants, lawyers, and Wall Street underwriters in their dark 3-piece suits, oozing expertise and authority, assured all doubters that SONGS II would come on line in January, 1982. But, alas, in the event, it proved not so. In the spring of 1983 the embarrassed city fathers had to lift the curtain and admit that SONGS was yielding no cash to pay debt service (interest and amortization) at 1%/mo. Since that is almost all interest we'll just call it \$800k/mo. (How would you like to have the clock ticking at that rate?) What to do?

- a. Miss some interest payments?
- b. Repudiate the debt? (This is what Washington Public Power Supply System did, in part.)
- c. Raise electric rates to pay the interest? (Imagine how the voters would take that.)
- d. Borrow to pay the interest, and hope the voters don't notice?
- e. Sell our share of SONGS?

The City Council chose (d). SONGS II came on line (sort of) in August, 1983. The new bonds could be sold for 10%, and were. But how much was added to capital cost by capitalizing interest from January 1, 1982 through August 31, 1983?

(Note: In this sad, true story many details are omitted, such as ongoing cost overruns, the dates of Unit III, the high downtime rates of both units, and the inadequate provision for future waste disposal costs.)

26. Find the value of a 30 year cash flow that begins with \$1 and declines by 4%/year. Discount at 11%.

27. Find the value of a 30 year cash flow that begins with \$1 and declines by 10 cents every 5 years. Discount at 12.4%.

28. Find the value of a 15-year cash flow that begins with \$1 and rises by 5.15% yearly, terminating in scrapping the asset and selling its site for \$11.50. Discount at 10%.

29. Find the payback periods (n) in the following cases. Note that only P/a is given. Like the CIA we are working here on a need-to-know basis.

<u>P/a</u>	<u>i</u>
10	.09
5	.15
6	.12
20	.05
20	.00

30. "We call that one Rathole Mine," says B. S. Plenty, the p.r. man for Cutten Runn Enterprises. "We poured in \$1m and never recovered a nickel."

"How do you figure?" said the auditor. "Your books show a cash flow of \$150k for 18 years."

"Our target rate is 14%" replies B. S. Explain his reasoning. How sound is it?

31. Joe does custom work with his \$100k rig. Operating expenses are \$80/day, and he uses the rig 250 days/year.

- How much must he charge p.d. to recover his \$100k in 6 years at 15% interest?
- By working the rig on weekends and holidays he can reduce the required daily rate by what %? Operating expenses on these extra days rise to \$100/day; but he will not be able to pass this on and will charge the same daily rate.

32. Arlo has to choose between two new processes in his Euphonia plant. The Green Hornet Process costs \$3m and lasts 3 years. Its GPV over that time is \$4m, discounting at 14%. The Lavender McDade Process costs \$6m and lasts 9 years. Its GPV over that time is \$8m.

- Which process makes better use of the plant? (The plant itself is so longevous you may treat its life as infinite here.)
- The Lavender McDade process can be foreshortened to 6 years. This would reduce its K cost to 5.5, but at the cost of reducing GPV to 7.5. Does that change the ranking?
- Arlo retires and is replaced by Otis Lee. The new manager wants to know if the rankings would change if he ranked by GPV, and if so, how? Also by NPV? By plant rent? By plant value?
- Did you rank by plant value in perpetuity? If so, show how this relates to plant rent.

33. A new apartment complex will last so long that some investors treat the economic life as infinite. In this question you may do so, too.

Huggin Arms Apartments were expected to yield cash of \$6700 in 1980, rising at 10%/year. Interest was at 17%. What would you have paid in 1980 to buy Huggin Arms?

Now, interest rates are down to 12% or so, a drop of 29% from 17% (got that?)

- a. What is the % rise in PV with g constant at 10%?
- b. By what % must g drop to keep PV constant?
- c. If apartment PVs have in fact risen by a smaller factor (i.e. %) than apartment rents, 1980-84, what would you infer about real interest rates?

34. It costs \$100/acre to reforest cutover timberland to yield stumpage of \$12k after 60 years, in constant \$. What monthly rent from grazing would be of equal value, using a real $i = .06$?

35. Congress and the state legislatures routinely authorize and start public works projects without fully funding them. Each year they fight over appropriations of real money (as opposed to mere authorizations, which are as cheap as a politician's promise) to make progress on the dozens, hundreds, or thousands of works under construction. Thus each project is built out slowly over many years, as the annual appropriations are secured, rather than at a quick stroke in a few months.

The English economist Jevons observed that the French Canal du Midi took over 100 years to complete, and the same is true of the Tenn-Tom Canal which was finally completed last year in Alabama.

Sparkling Fork Dam is thus built out ever so slo-o-o-wly over 18 years. K is \$20m. What is the accumulated capital in Year 18 when SFD finally goes into service? Use $i = .12$ and constant \$.

36. Given the following data about stumpage values, what is the year of optimal harvest of redwood trees in Mendocino County? Let $i = .05$.

n:	25	30	35	40	45
S:	400	510	820	1030	1240

To speed your search and ease your task, first find the 5-year interval containing the right year; then do it year by year.

37. An orange grove yields \$ a for n years followed by resale to a developer at a future value, F .

- a. Choose values for the needed parameters and find the present value, P .
- b. Choose a value for P and find the IRR (Internal Rate of Return). If you can't handle all the repetitions, show how you would do it, if you had more time.

38. Compare naive payback time (zero-interest), full payback time, and half-life.

39. What is the elasticity of production of volume inside a cylinder w.r.t. the radius, the height, and the two together (assuming they are locked in fixed proportions, so $R=kH$)?

What is the E of substitution of height for radius, given constant volume?

40. Sludge from Hyperion

Los Angeles until very recently pumped 4.2 mgd (million gallons/day) of sludge through a 20" pipe (meaning 20" in diameter) from the Hyperion Treatment Plant out to the rim of the undersea Santa Monica Canyon (L.A. Times, 14 Dec. 87, Section CC II, p. 1).

But can you really get 4.2 mgd through a little pipe like that? How fast in cfs (cubic feet/second) must the sludge be travelling through the 20" pipe? (Note that in using mgd and cfs the custom is to omit any symbol for /, or "per", but don't be fooled, it is there in the math and the units.)

Let V = Velocity in feet/second

Q = Quantity of sludge in cf

Basic formula: flow in cfs is V in f/s x cross-section area of pipe.

$$(1) \quad F = V\pi r^2$$

Solving for V ,

$$(2) \quad V = F/\pi r^2$$

First, convert flow to cfs. Conversion factor: one cf = 7.481 gallons. (The hardest part of this problem is looking that up, and I've done it for you. But check it yourself, some day you'll need to find a conversion factor on your own.)

Activate calculator! Ready--you're on center stage!

$$F = \frac{4.2 \text{ mgd}}{7.481 \text{ g/cf}} \times \frac{1}{86,400 \text{ s/d}} = 6.498 \text{ cfs}$$

Plug in values:

$$V = \frac{6.498 \text{ cfs}}{\pi(5/6)^2 \text{ ft}^2} = \frac{6.498}{2.182} \text{ f/s} = 2.978 \text{ f/s}$$

(Supplemental problem: convert answer to mph. Onemph is 5280/3600 f/s or 1.467 f/s. Show that the answer is 2.031 mph.) **A**

41. Case of the southbound crude

A consortium of Chevron, Shell and Texaco is promoting an oil pipeline gathering crude from offshore Santa Barbara and from Kern County to refineries in L.A. County (L.A. Times, 14 Jan. 88, Part II, p. 7).

Pipe diameter is to be "nearly that of a hula hoop" -- say 2.5'.

Daily flow is to be 330k bbl/d.

Problem: How fast in mph must the oil flow through the 2.5' pipe?

Conversion factor: one bbl of petroleum = 42 gallons.

(Just for the record, a bbl is not always a bbl. One bbl of fermented beverages in only 31 gallons.)

$$42 \text{ gallons} = 5.614 \text{ cf.}$$

Basic formula: flow in cf/h = V in mph x Q per mile of pipe.

$$F = V\pi r^2 L, \text{ where } L = \text{one mile or } 5280'.$$

$$V = F/\pi r^2 L$$

$$F = 330k \text{ bbl/d} \times 42 \text{ cf/bbl} \div 24 \text{ h/d} = 77.196k \text{ cf/h}$$

$$V = \frac{77.196k \text{ cf/h}}{\pi(5/4)^2 \times 5280'} = 2.98 \text{ mph}$$

Check: Work this out to find V in cfs, be sure answer is equal to 2.98 mph.

42. Plant Manager Peter learns he can buy a new industrial robot for \$65,000. It will replace one line worker at \$12,000/year. Financing is at 10%. "Good deal," thinks Peter, "that comes to \$6500 year vs. \$12,000--no contest."

But life has taught him to check with Smart Sally the bookkeeper. "Wait a minute, dummy," says straight-talking Sally, "those things cost \$4,000/year to fuel and maintain; and only last 7 years."

So they refigure the deal. How does it come out?

43. Matilda Moonlighter wants energy independence. She plans to build a still in her offtime to produce fuel alcohol. Being realistic, she knows her spare time is limited, so it will take 5 years, investing each year \$5,000 of her time and \$5,000 in materials (it's to be a big still). Then it will produce alcohol for 5 years and become scrap. But the alcohol should bring \$13,000/year after expenses.

Matilda's small savings are in a money-market account at 9%. Is energy independence a worthy competitor for her capital?

44. Thoroughly Modern Millie laughs at corny Matilda. Millie uses her credit to get what she wants, when she wants it. She borrows \$100,000 at 13%, amortized over 20 years. Payments are annual.

- a. What is her first payment? What percentage of it is interest?
- b. In the third year, what is the payment, and what percentage is interest?

45. (Double Credit) In Oregon-Washington it takes 50 to over 100 years to mature a Douglas-fir tree, depending on the site, and the reforestation effort. In the Southeast, southern Yellow Pine matures in about 20 years.

- a. What is the annual value of land, given that you can plant an acre for \$500 and harvest out for \$4,000 (net of harvest cost) in 20 years? Select a rate of interest of 5%.
- b. Having formulated the annual value of land, what is the present value of land, assuming you can repeat the 20-year cycle in perpetuity?
- c. What percentage of the land value comes from the first 20-year cycle?
- d. How much can you justify adding to the \$500 planting cost to speed growth and advance the harvest by 3 years? (Yes, you can get seedlings off to a faster start by better and costlier efforts.)

46. Builder Benito has a slow period, so he decides to use it constructing an apartment building to serve as his retirement pension. Experience tells him that gross rents will be double the annual expenses for "O&M" (operation and maintenance).

He has plans ready for a building that will gross \$45,000 the first year. He figures the natural decline of rents with age will just be offset by general inflation, so he plans on no decline in dollar rents over 50 years. The nominal interest rate is 13%. He believes inflation will be at 6% over the period.

But he is approached by an admirable young carpenter wanting a job. Benito could add her to his crew and add 2 units, so the building would gross \$50,000. Labor is 20% of construction cost. What would be the value of her discounted net marginal product?

47. What is the elasticity of P (present value) w.r.t. (with respect to) n , using the formula for present value of an annuity?

- a. Define the concept.
- b. Give one numerical example (one number is all, not a whole column).

48. Refer to (47).

- a. How is elasticity w.r.t. $\underline{n}$ affected by use of a higher interest rate?
- b. What is elasticity w.r.t. $\underline{i}$?
- c. Give one numerical example.

49. The formula for the sum of an infinite geometric progression is simpler than that for a finite progression. It is simply:

$$(1) \quad S = \frac{a}{1-r}$$

where $\underline{a}$ is the first term and $\underline{r}$ is the rate of progression.

- a. Use the formula to derive the formula for the value of a perpetuity that grows at a rate g per year.
- b. A text says that the rate of return on "equity" (the ownership interest in a firm) is:

$$(2) \quad k = \frac{D}{P} + g$$

where K = rate of return
 D = dividend in year one
 P = Price of purchase (e.g. of common stock)
 g = expected growth rate of $\underline{D}$.

Show that (2) is a variant form of your answer to (a), and therefore consistent with it.

50. Decommissioning costs of a nuclear plant are expected to be $X\%$ of original construction costs, K_0 (the latter including capitalized interest during the construction period).

The plant is 100% financed at 11%, the loan amortized over 30 years. Decommissioning is to be handled by a sinking fund, and added to debt service as part of annual fixed cost.

- a. By what percentage will annual fixed cost be increased?
- b. Answer (a) if $X = 50$.

51. You believe that an orange grove will yield cash flow of \$210/acre for 15 years; then be sold to a developer for \$20,000/acre. How much would you pay for it today? Use 4%.

52. An office building will yield a net \$10,000/year for 40 years, and can be built for its PV (discounting at a low 5%, to allow for inflation).

- a. How much does it depreciate in year 1?
- b. When will you get back half your capital?

Can you:

1. Knowing the answer, explain it?
2. Find the answer to this and similar problems? It's done with logs, which your calculator lets you handle as easily as any other number. Solve the PV formula for n ; then find n when P_n is half of P_{50} . (To solve for n , remember the rule that $\ln a^y = y \ln a$.)

53. Oilie Olivia has an oil leasehold to develop. The quicker she can extract the oil, the more its present value. Speeding the flow is therefore like creating capital, from the same physical volume of oil. But to speed the flow requires more wells, spaced closer. They are capital, too, and when the oil is gone, the wells have no salvage value. Buffeted this way and that by opposing forces on life's stormy sea, what is a poor girl to do?

54. Land lasts forever, and yields a perpetual net income. What is the value of a perpetual annuity (or perpetuity) of \$1:

- a. At 8%?
- b. At 8%, if the net income will rise by 2% every year?
- c. In (b), how much does the land value rise each year?
 1. % wise?
 2. In \$?
- d. If you borrow at 8% to buy the land at its correct PV, how long will it take to retire the debt from the net income of the land?
 1. In case (a)?
 2. In case (b)?

55. Mom lends Sonny \$10,000 at 8% amortized over 7 years.
- In year one (the first year) how much is the principal reduced?
 - How much is it reduced in year 6?
 - After 7 years, what is Mom's capital recovery as a share of all the payments made?
56. a. Beginning from the PV formula, show how to find the PV of a perpetual succession of repeating cycles of investment and recovery.
- In the above, what is the land value multiplier? What does it do? Explain.
 - In (a), where does the initial capital cost appear, i.e. the investment that begins each cycle?
57. In the case of tree growth:
- Show why maximizing SI^{-n} , discounted present value of stumpage, corresponds to the marginal condition that $S'/S = i$.
 - FOR DOUBLE CREDIT, THIS PART IS OPTIONAL. Show why maximizing $S/(I^n - 1)$ corresponds to the marginal condition that $S'/s = i/(1 - I^{-n})$.
58. What is the value of land yielding cash flow of \$1:
- At 8%?
 - At 8% if cash flow is to rise at 3% per year?
 - If you borrow to buy the land at its correct PV, and pay 8% on the loan, how much will you owe the bank after one year? After 10 years?
59. Find the PV of a 25 year cash flow that begins with \$23.53 and declines at 4.658%/year. Discount at 7.452%. Make your answer accurate to 3 places.
60. You have to choose between two processes in your plant. The Red Process costs \$4m and lasts 5 years; its GPV over that time is \$6m, discounting at 13%. The Yellow Process costs \$7m and lasts 11 years. Its GPV over that time is \$8m. Which process makes better use of the plant? Give a numerical answer and show your work.
61. If costs \$125/acre to reforest cutover timberland to yield stumpage of \$14k after 55 years, in constant \$. What monthly rent from grazing would be of equal value, using a nominal interest rate of 15%? You forecast inflation over the period at 11%/year.

62. a. Corazón calls Fire and Ice Heating and Air Conditioning to help cut her cooling bills. They send Feanna Fenian who recommends a pre-cooler (an evaporative cooler that delivers cooler air to the vanes and coils of the basic air conditioner). Feanna guarantees a 25% drop in power use. But instead, Corazón runs the air conditioner 25% more. What happens to her power bill?
- b. If Corazón stuck with her original usage-time, and paid \$250 for the pre-cooler installed, and her original monthly power bill was 40% of the cost of the pre-cooler during the $3\frac{1}{2}$ month cooling season, and 35% of that bill went for air-conditioning, and her cost of money was 17%, and the CPI was rising at 5%, and power costs were rising 11%/year compounded for the foreseeable future, and there was rioting in Reykjavik, and the Rams went to the Super Bowl, and they caught the real Freeway Strangler under a 17-car pile-up on Mulholland Drive, and Nancy Reagan eloped with Price Charles, how long would it take Corazón to recover her \$250?
63. A rectangle of low symmetry has one value of ϵ for A, and another for H. Is the sum of those two higher or lower or the same as that for a square?
64. As Philo reached 30 he noticed that each passing year was a smaller percentage of his life. "Why was I ever in such a hurry?" he mused, "to own a home." "I'm not going to pay interest at 15% just to get a house a year earlier. A year is only 3.3% of my life, and growing less."

But his fiancée Philonia, also at 30, read up on Downs Syndrome and resolved to have no children after 40. She started reading Philo the poetry of Lovelace: "Had we but world enough, and time..."; and Omar Khayyam: "The bird of time has but a little way to flutter, and Lo! The bird is on the wing." "Holy Smoke!" said Philo, switching off the TV, "you mean!...a 3.3% gain in what's past is a 10% loss in what's left?" "Right, said Philonia, "and rising fast." "But look on the bright side. By giving up 3.3% of your idle youth, you can add 10% to the breeding years."

But Philo learned that, with amniocentesis, Philonia could safely bear until 45. While they are arguing over the correct number, and aging to 32, what is the relevance of this to explaining the forces determining interest rates? Why should you care?

65. Explain how economic returns to scale of buildings are affected by discounting peripheral space by distance from the center. How is the value of symmetry affected?
66. Central Administration has allocated a fixed number of graduate student slots to UCR, called Graduate Student FTE. Economics had 61. The number of slots filled at any one time, assuming the throughput is steady (Input = Output), is the throughput $\times$ the average period in the program: e.g. if there are 10 graduates each year, and each one takes 6 years, 60 slots will be required at any given time.

Turnover is the reciprocal of the average period. If the total FTE allocated to UCR is T, and turnover in Economics were made to rise by 35%:

- a. How many FTE slots would be released to the rest of UCR?
- b. How would our percentage loss compare to their percentage gain?

67. Due to the law of the cube, larger buildings provide space at less cost per unit than smaller ones do. That means capital is in Stage I, with increasing returns. The theory of production says that Stage I is irrational, because "the fixed factor, Y is so abundant in relation to the variable input that its marginal product is negative" (P&B). What is the fixed input here? Is its marginal product negative? What's going on?

68. Explain why the maximum AP of input Y implies a low MP of Input X.

- a. When X varies, and Y is constant.
- b. When Y varies, and X is constant.

69. For a rectangle, $A = BH$. Use that equation to exemplify:

- a. Increasing returns to scale;
- b. Diminishing returns;
- c. Substitution.

(If it helps, you may think of the perimeter as a fence, and A, the area of fenced pasture, as the output of usable pasture "produced" by the fence.)

70. You double the radius of a circle.

- a. By what percentage does the area increase?
- b. What is the ϵ of production of area w.r.t. the radius?
 - 1. For small increases?
 - 2. For the present example?
- c. Are they the same? Why, or why not?

71. Sharon Shapely has small means but big plans. She charms a susceptible loan officer and walks out with a loan of 8 times her own worth. She puts the bundle on a carload of redwood lumber she hears is going to be in short supply. How much must the price fall to wipe her out?

(P.S. And what then would happen to the poor loan officer?)

72. Bibulous Billie and Ernie Enophile aren't making it in wine any more, so they plant Eucalyptus for the firewood market. The trees are adding value at 25% a year when Billie and Ernie realize the land is worth 60% as much as the trees. What rate are they earning on their investment? How does the land element in the base affect their timing?

73. A wage contract is tied to the CPI. But electric rates rise twice as fast as the CPI, which rises at 7% a year. After 5 years, what has happened to electric rates in constant dollars?

74. The Ruritanian Ob, a foreign currency, drops by 60%, rises by 40%, drops by 8%, and finally rises by 28%. How has its value changed overall?
75. Output from a farm varies with acres⁷. Pastoral Pierre adds a 40 to a section. What is the effect on yields?
76. "There must be more powerful economies of scale in the barn than the fence, because the barn has 3 dimensions." True, false or maybe?
77. You pay \$4550/acre for a kiwi farm. It yields cash flow of \$600/acre for 7 years. How much must a buyer pay you for it after that 7 years, in order for you to have made 15% overall?
78. An office building will yield a net \$5,000/year for 45 years, and can be built for its present value (using 6% for discounting).
- a. How much does it depreciate in year #3?
 - b. When will you get back 85% of your capital?
79. Yields from an olive grove drop by 24%, but prices rise by 53%. What happens to the gross revenues?
80. A firm has 46 workers and adds 15. Output rises by 25%. What happens to "labor productivity" (output per worker)?
81. A small Department in a University is in danger of extinction because it has too few teachers to offer a full range of courses. What can you say about the elasticity of production w.r.t. teachers in that Department? How would it compare with the elasticity of production in a large Department in which the average productivity of teachers (assuming we can measure that in some legitimate way) is the same as that in the smaller Department?
82. A new improved lubricant cuts friction losses in a motor from 22% down to 16%. How much more output can be had with the same input of energy?
83. A country is found that is wise enough to try the Gaffney Plan of basing income taxes on disposable rather than gross income. How much higher would the tax rate have to be in order to raise the same revenue?
84. Identify and briefly explain at least four economies of scale. (Be sure you are identifying different principles, not just different examples of the same one principle.)
85. Ditto for diseconomies of scale.
86. Joe gets deeply in hock, so his debts are equal to 90% of his gross worth. But inflation pushes up all values by 20%. What happens to Joe's equity, assuming that he was meeting his interest payments right along?
87. Explain the meaning of returns to scale, returns to one variable, and substitution, explaining the difference.

88. In maximizing the Present Value of a building, how do you determine the optimal lifespan--i.e., the year of demolition and replacement? Explain why this is a typical example of an optimizing problem.

89. You are showing the relationship of output and 2 or more separate inputs. If one functional relationship is explicit, what is implicit?

90. Labor input rises 15%, output rises 9%, while overhead remains fixed.

a. What is M.P./A.P.?

b. Is the increased input a move toward lower overall efficiency? Yes, No, or Maybe, and Why?

91. (2 units) Sparkling Fork is a crystal stream pouring from Mt. Snowcone onto the land of Alluvium. 25 farmers have rights to take summer water in July and August, when it is badly needed. But once (it was in the Year of the Horse, I believe) came a severe drought. The normal July flow of 1,000 cfs (cubic feet/second) dropped to 600 cfs, and there was great wailing, gnashing of teeth and rending of garments at the distress.

The oldest settlers said they should of course get their full usual amount, leaving nothing for the newcomers, following the practice of the wondrous neighboring civilization in Alta Babylonia, the Golden Land of wise and foresighted leaders. But Gobernador Pardo, after taking counsel with his sibyl, ruled against Plan 1, invoking his emergency powers. "Fair is fair," quoth he; "Let every farmer cut back the same amount." (Plan 2)

"No!" cried the little guys in one voice, "you must mean the same percentage."

"Ah, perhaps I did," said Pardo. "We call this 'prorating'. (Plan 3) 40% should do it."

Then up spake Farmer H. "What about elasticity of production, chief? Mine is 83%, while Eye, Jay and Kay over there have only 51%, 42% and 35%. What about a fair shake for the higher elasticity people?"

"This is the thanks I get for trying to be a nice guy," thought Pardo. "Where is my sibyl now that I really need her?"

"Here I am" said the sibyl, "and H has a point, because his percentage loss out output will indeed be greater. However, my study of the equimarginal principle taught me that we should cut H back more, rather than less than the others, in order to use our limited water most economically and maximize aggregate output. This is the Age of Limits, after all."

"A clever phrase," said Pardo, "I must remember that. Now, what was that principle again?"

"The equimarginal principle," said the sibyl, "says that the marginal yield of an input should be the same in all its alternative uses. Remember it."

"I will make it my own," said Pardo. "H, I am afraid you've had it."

- a. Explain Pardo's reasoning.
- b. Explain the defects of Plans 1 and 2.
- c. For extra credit, show the most economical way to allocate limited water in a time of shortage. Call it Plan 4.

92. Calculate the following:

- a. The PV (present value) when:

<u>n</u>	<u>i</u>	<u>FV</u>
0	90%	85
1	15%	62
10	10%	1005
25	15%	365
80	0%	500

- b. A new kind of bond is offered for sale. It pays nothing at all for five years, and then pays \$100/year in perpetuity (forever). It is never redeemable (you can never turn it in to get back your principal). If the going rate of interest is 15%, how much is the bond worth at present?

- c. The annual rate of return when:

<u>n</u>	<u>PV</u>	<u>FV</u>
1	1	1.5
15	83	46
50	128	1563

93. (2 units) You can plant Christmas trees today on your land and get the following:

<u>Planting Cost (\$k)</u>	<u>Net sales per \$ of Planting Cost (\$)</u>
1	2.7
2	3.2
3	3.8
4	3.7
5	3.5
6	3.2
7	2.9

"Net sales" means we have netted out (deducted) all costs other than the original planting costs. Your land is a fixed amount. There is just one complication you should consider. You must wait five years for the trees to grow to salable size, and money costs you 13%/year. What is your optimal outlay for planting costs? Show your work.

(Helpful hint: solve in two stages. First, tabulate as though there were no 5-year wait. Second, make appropriate adjustments.)

94. Terry Tannenbaum has been growing Christmas trees on a 5-year cycle, and using Discounted Marginal Productivity analysis (DMP) to decide how much to invest in each planting. But his new partner, Noel Arbor, is not content. He wonders if it wouldn't work better to try a 4-year cycle, making a little less on each one, but getting more cycles. Since each tree will grow only to a smaller size, they can crowd more stems onto each acre. "Pretty risky," croaks an old-timer, "you'll be risking more money each time, and making less on each tree. How can you justify that? Besides, if God meant for trees to be cut in 4 years, he would have made them stop growing then."

Terry and Noel check with Howard Ruff, and he says "Stay on the sidelines until I give the signal to buy Krugerrands. Meantime, stock up your survival shelter." They consider this, but decide against dropping out. Instead, they get some data from the local forestry department office, and translate it into their situation. Here is what they get.

C_0 in \$k	$AP/\$$	
	$n-4$	$n-5$
4	2.18	2.38
5	2.30	2.50
6	2.30	2.50
7	2.23	2.43
8	2.13	2.31
9	2.03	2.19
10	1.94	2.06

They use $i = .05$, and put all data in constant dollars. (Alternatively, they might have put the data in forecasted current \$, and used a current nominal or market interest rate.)

- a. Find optimal C for $n = 4$ and $n = 5$. Assume that C can only increase in discrete steps of \$1k @, so that your answer will not be between two figures.

(Answer: When $n = 4$, opt. $C = 9$.
When $n = 5$, opt. $C = 8$.)

This is when the DMP is just greater than \$1k, and falls below it on the next step.)

- b. Find Net Present Value of G_1 for $n = 4$ and $n = 5$.

(Answer: When $n = 4$, $G_1 = -9 + 18.3/1.22 = 6.06$
 When $n = 5$, $G_1 = -8 + 18.5/1.28 = 6.50$)

- c. Find site value, S , when $n = 4$ and $n = 5$.

(Answer: When $n = 4$, $S = 6.06/5.64 = 34.15$
 When $n = 5$, $S = 6.50/4.62 = 30.03$)

What is, finally, the best combo of C and n ?

95. Find the payback periods (n) in the following cases:

<u>P/a</u>	<u>i</u>
10	.09
5	.15
6	.12
20	.05
20	.00

Answers will be supplied in due course. Note that only P/a is given, since only the ratio of P to a is relevant to the answer.

96. In question #95, above, what does payback period mean? Is it a fair representation of the time that your capital is actually tied down (or up) in a piece of capital?

97. Ruthie Rushalong has been promoted to head of Division A, and wants to put in a new line of machines which are state-of-the-art, off-the-shelf, and all those good things. They cost a lot because they will last for 20 years, and are priced accordingly. The central finance committee, sitting in splendid isolation in the top floor board-room, turns her down. "Against Company Policy," says the memo. "Maximum allowable payback is 15 years. Negative."

Ruthie calls Betty Friedan and gets ready to file suit under affirmative action, but before Betty returns her call, she has a brainstorm. "I'll show those mossbacks," she mutters as she perseveres. "Now look: suppose I have 20 of these machines, and replace one each year. Let each one yield a net cash flow of \$1/year, and be priced accordingly. My total cash flow is \$20. My yearly replacement cost is the present value of \$20 stretched over 20 years. From this, I can cleverly figure out my annual interest charges, which are just the excess of cash flow over replacement. If I divide this by the interest rate, that is the total value of my unrecovered capital at any time. Well, that is a relief, that saves me the trouble of integrating (finding the area under) the present value curve, which is a harder way of finding the sum of all the unrecovered capital.

But now, look what I can do, gee whiz, no hands, mom! If I know what my capital is, and I know how fast I am recovering it, then I know what the payback period is (which is what we started out to find, remember?) All I lack is the capital recovered, but that is a snap. Cash flow less interest is capital recovery. Let's write this down in mathematical notation before it all slips away.

P = Present Value

K = Capital stock

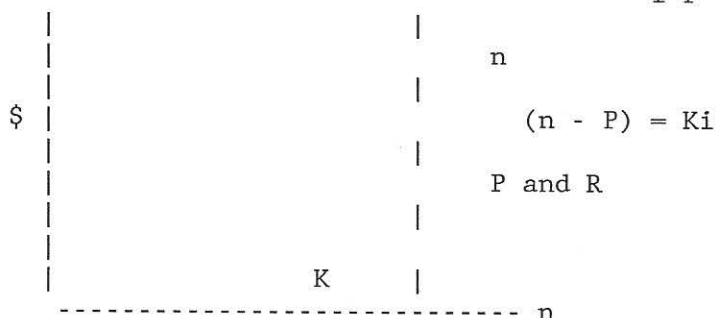
R = Recovery of capital in current year

$$Ki = n - P$$

$$K = (n - P)/i$$

$$R = n - Ki = n - (n - P) = P$$

$$\text{Period of payback} = K/R = (n - P)/iP = \frac{1}{i} \left(\frac{n}{P} - 1 \right)$$



Beautiful, except for one more hitch. It is now 1:30 a.m. and Ruthie has about had it, when she notes that there is an n in P , appearing as an exponent, and how will she ever find the value of n/P ? Fortunately she has learned about such problems, and has a calculator handy, so she takes one last gasp and works it out.

She finds that if $n = 20$, and $i = .05$, $n/P = 1.605$, and $K/R = 20 \times .605 = 12.10$ years! Hooray, that's under 15 years, she is well within the limit of company policy. She dives into bed for the sleep of the just.

The sad ending is that next morning she oversleeps and loses her job. They really are mossbacks. She should have had this worked out ahead of time. So, let's you work out some values. Tabulate values of K/R for $n = 2-40$, in steps of five or less, and at $i = 0, .02, .08, .15$, and $.25$. This is a sensitivity analysis. What does it tell you?

By the way, do you understand why the payback period is less than 20 years? It is because this is a mean payback period, taking account that you get much of your capital back before the last year. You could also work this out for one of these machines at a time, and you would get the same answer. For super extra credit, you may do this, but it is above and beyond the call of duty.

98. Solvent Sam inherited big bucks, so he never much worried about the cost of capital in his business. "After all," said he, "in the final analysis it's free, it didn't cost me anything." Sam always bought the best, and went first class.

But when the prime hit 21% he began to wonder how much it was costing him in foregone gain or opportunity cost to tie up all that capital.

He has a line of presses that cost him \$1M (million) each, and to which he can impute cash flow (net product) of \$100k @. He has 20 of these, and replaces them on a 20 year rotation, one each year. The whole 20 of them yield him \$2M/yr, and replacement is \$1M/yr.

What a money machine, exults Sam. It is like perpetual motion. All I needed was the brains and character to set it up in the first place, and now I double my money every year! I put in one and take out two. No wonder dad was rich; brains run in the family.

Hold on, says Andy Accountant, you're falling for the Fallacy of the Vanishing Inventory. That \$2M you take out each year is not earned by the current input of \$1M, but by the accumulated inputs of the last 20.

Are you telling me, says Sam, that I'm still living on dad? You can pick up your last paycheck on your way out.

What rate of return was Sam really earning?

If Sam had to borrow at 10%, how long would it take him to recover his capital from each machine?

If Sam had to borrow at 15%, what would be the slope of a curve showing the value of his unrecovered capital for different values of n ?

99. A citrus grove yields a net current income of \$5k for 10 years; then it is expected to sell to an Orange County developer for \$500,000. At a 10% discount rate, what is the present value?

100. An acre of trees will probably bring \$10,000 when cut in 2061 A.D. Planting is free, and running costs are negligible. A sheepman offers you \$1/yr rent to use the same land. At 10% interest, which is the higher use of the land and the better deal for you?

101. You invest \$100 today to receive \$200 at the end of 3 years.

- a. What is the annual rate of return (ROR)?
- b. What is the monthly ROR?
- c. Is (b) just $1/12$ of (a)? Why, or why not?

102. Weld can buy Zuni Co. bonds for a deep discount. A \$100 bond maturing Dec. 1, 1990 costs \$50; the coupon paid every Oct. 30 is \$9. Weld figures that doubling his money in 10 years is worth 7.2%, using the rule of 72. To that he adds 9/100, or 9%, for a total yield to maturity of 16.2% before taxes. But his calculator gives a different figure. How is the calculator figuring? Who is right?

Weld's marginal tax bracket is 40%, adding federal and state together. His average tax bracket is 25%. How should he figure his YTM (yield to maturity) after taxes? Bear in mind that the coupon is "ordinary" income, subject to the full fury of the tax rates; but the gain on redemption (\$100 less \$50) receives so-called "capital gains" treatment, i.e., only 40% of it is taxable, and none of the accruing gain is recognized as taxable until redemption.

103. Given a credit crunch with high interest rates charged, you naturally want to economize on capital. Name several general ways to do it (do not repeat with several illustrations of the same idea).

104. Gadfry Co. has a large fixed plant, but runs short of working capital. List several methods for raising working capital from the fixed plant over 2 or 3 years.

105. Jane has \$100k of capital, and great energy and talent, and wants to run a bakery. Because of economies of scale, and the need for market identification, she requires a volume of \$1 million. How can she stretch her capital?

106. An office building will yield a net \$10,000/year for 50 years, and can be built for its PV of \$181,000 (discounting at a low 5%, to allow for inflation).

- a. How much does it depreciate in year 1?
- b. When will you get back half your capital?
(Answer: in 37.51 years.) Can you:
 1. Knowing the answer, explain it?
 2. Find the answer to this and similar problems? It's done with logs, which your calculator lets you handle as easily as any other number. Solve the PV formula for $\underline{n}$; then find $\underline{n}$ when P_n is half of P_{50} . (To solve for $\underline{n}$, remember the rule that $\ln a^y = y \ln a$).

107. Land lasts forever, and yields a perpetual net income. What is the value of a perpetual annuity (or perpetuity) of \$1:

- a. At 8%.
- b. At 8%, if the net income will rise by 2% every year?
- c. In (b), how much does the land value rise each year?
 1. % wise?
 2. In \$?
- d. If you borrow at 8% to buy the land at its correct PV, how long will it take to retire the debt from the net income of the land?
 1. In case (a)?
 2. In case (b)?

108. Find the value of a 15-year cash flow that begins with \$1 and rises by 5.15% yearly, terminating in scrapping the asset (a building) and selling the site for \$11.50.

109. Chris Crackfiller knows he must be sharp to get started. He has no family connections, no politicians or bankers in his pocket, no inheritance. He looks for unused resources and neglected opportunities.

He notices a vacant commercial site being held "for the rise," with several years wait ahead. "Hmm," he says, "why not buy it now and find an interim use for 10 years or so?"

He believes a fast-food service will cost \$50,000 to build, and return net cash of \$10,000/year for as long as he likes. He also believes a new railway grade separation is coming in just 10 years, after which the land will be worth \$100,000 for a higher use.

- a. When should he expect to terminate the interim fast-food use for a higher use? (He can get money at 12%).
- b. What is the maximum he can pay for the land?
- c. Is the interim use worth fooling with at all?
- d. How much is the whole parcel worth three years before the switch to the higher use?

110. A power line carries a varying load. It is at peak only 2% of the time. You can get away with overloading it for short periods, but this heats it up, which causes line losses (for 2% of the time). More serious, it shortens life. Manager Jean has three choices:

- a. Overload during peaks, and figure on replacing lines each 10 years.
- b. Oversize. Doubling capacity will cost 50% more, and extend life to 20 years.
- c. Manage the peak load by rationing customers during peaks.

What are the pros and cons of each choice?

111. Sally runs a hardware chain. She is offered a steal on brass clamps, 10,000 for \$50,000. She figures she can sell 2,000 a year, at \$12.50 each. "Payback time of just 2 years" claims the seller. Why might Sally hesitate?

112. The concept of cash flow lets you juggle more variables than you may realize. Say you are sizing a building on a fixed site. When you increase building size, you may figure that operating and maintenance expenses will also increase in future years ("o&m" for short). Those are "complementary" or "associated" future costs. You automatically deduct those from gross rents to find cash flow. If I called this "net revenues after complementary costs and appropriate adjustments mutatis mutandis" you would think that an impossible notion to grasp; but that is just what cash flow means. Every successful investor uses this concept routinely in linking big investments to the resulting income which he can credit to them. Cash flow is the net product resulting from and credited to a capital (K_0) outlay which causes it. K_0 is the explicit variable. Cash flow contains the implicit assumption that you adjust operating expenses, in quantity and in kind, so as to maximize the net return to any given K_0 , whatever its quantity and kind. This covers a huge variety of possible combinations, to model which formally would cover reams of paper with curves and tables. It is tedious and unnecessary to go through all that, if we can assume that you operate any facility at a reasonable level of efficiency and rationality, to maximize its net cash flow. All economic modeling depends on this kind of assumption: the implicit variables are handled rationally and reasonably, with at least an average level of skill and care.

We don't have to assume that future expenses are complementary with capital outlays, either. Some K_0 increments substitute for future expenses. E.g., fuel use tomorrow declines with insulation applied today; future electrical repairs decline with better wiring today; etc. Here, the increased net cash flow results from reduced future o&m rather than increased gross revenues.

If cash flow is net revenue, then marginal cash flow is marginal net revenue. In sizing a building, you determine the marginal net revenue for each dk_0 (i.e., each increment to K_0). You also discount these future cash flows. The result is "marginal discounted cash flow," MDCF, a concept in

such common use in business that it may be recognized by its initials. This is the end result of a long series of calculations. You should understand every step; but, once you understand what it is all about, you can take the end product as a single, unitary number to weigh in the balance with K_0 , or dK_0 .

Having these ideas in hand, we can go back another step to value the land itself. Before the building comes the land. To find the value of land, deduct the K_0 of the optimal improvement ("highest and best use") from its DCF. This is how people determine how much they can pay for building lots. (We will omit here the future salvage value of the land, which will add more or less to the present value, depending on how far in the future it lies.)

In sizing your land parcel, you will want to balance the marginal cost of extra land against the marginal increment to $(DCF - K_0)$.

Before the land come the public improvements needed to make land usable. When can we justify extending water supply to desert lands? The water engineers do not try to forecast all the details of what people will do there. They look at land values, which are their proximate (i.e. immediate) product. Will the water supply raise land values by a sum equal to its cost? If so, will the landowners be willing and able to pay, or will someone else? Here we get into some fancy public and distributive questions which we will finesse for now. It just shows you that the bottom line of one set of calculations becomes the top line for the next stage of calculations.

For a problem, let's go back to the man sizing his land parcel. You have a 20 acre site. Another 5 acres are available at \$200,000. Should you add them to your parcel? Line in sequence the steps needed to put it all together. I'll get you started.

Step 1: Gross revenues less expenses = cash flow.

Step 2: ?

113. You are buying a chemical that degrades with age, losing function at 2%/month. Interest is 1.5%/month. What is the present value of a gallon for use after 24 months? (It will be worth \$20 for use at that time, if at full strength.)

Whoops! We forgot you have to store this stuff, and that costs \$.25/gallon/month. Now what is the present value?

114. Susan's house needs more water. Its $\frac{1}{2}$ " pipes don't deliver it fast enough. She could:

- a. Install a twin $\frac{1}{2}$ " system now.
- b. Adjust the pressure regulator to increase the pressure. This won't cost her any cash, but will cause leaks and ruptures. The pipes are under the slab, and it costs dearly to fix them. She figures that the leaks will begin in 5 years. At that time she could:

1. Install 3/4" pipes, or
 2. Keep up the high pressure and incur regular repair costs equal to $1/5$ the K_0 of the 3/4" system.
- c. Cut down on her peak demands for water.
- d. Install a 3/4" system now, get it over with, and enjoy it for a full 20 years. (20 years is the remaining life of her house.)

She is not the greatest credit risk, but her local usurer will accommodate her needs at 21%. What is her best move?

115. You are planning a water system for Boomtown. A 12" main will do for 5 years; then you will have to twin it. Or, for 50% more, you could lay an 18" main, with 125% more capacity.

- a. Boomtown borrows at 12%. Which is the better choice?
- b. At what interest rate would it be a toss-up?
- c. Were the leaders who oversized such things as water mains in the past "foresighted," as claimed, or "spendthrift"? What parameters determine the wisdom or folly of oversizing today to meet possible future needs?

116. Sid Citrus can net \$400/acre from his orange grove each year. Gross is 4,000 but expenses for o&m run 3600.

Sid anticipates selling to a developer in 15 years, for 18,000/acre. He want to "milk" his citrus by neglecting maintenance. Thus he cuts expenses by \$600/year. But that means a decline in production of 5% a year. He asks you how this change will affect the DCF of his grove. What do you tell him? Use an 8% interest rate.

117. Junto del Rio is a city which owns its own electric power system, including a share of one nuclear plant. The Nuclear Regulatory Commission (NRC) rules that they must accumulate \$150m by the end of plant life to cover costs of disposing of spent fuel. They have to raise rates now to cover the annual input to a sinking fund.

"No problem" says Andy Analyst. "Using a 30-year life and 12% interest, here is your figure." But the true life turns out to be 11 years, and the proper interest rate 6%.

- a. By how much has Andy missed the correct annuity?
 1. In absolute terms?
 2. % wise?
- b. By how much has he missed the target of \$150m?

118. Chris Crusader believes that people should buy appliances based on "life-cycle" costs, not just today's apparent price. Refrigerator A costs 10% less than B, but uses 15% more kilowatt hours (kWh) to run each year; and the kWh cost of running A is, at current rates, 18% of its price. Both have 20-year lives, with financing at 19%.

- a. Chris, naturally, pushes B. Is its life-cycle cost really lower?
- b. How would your findings differ if rates per kWh are to rise by 8%/year?

119. "Be Prepared" is the motto of Phil Foresyte. "Plan Ahead." His plumbing supply business has grown at 6% a year for the last 24 years, and he assumes this will continue (even though this might be called "Planning Behind," which we overlook for now). He has overcrowded and worn out his present factory-warehouse and is raising a fund to replace it. He plans to build in space for future expansion, because the marginal capital cost of building in added square feet during construction is only 60% of the average cost; whereas adding it later, one piece at a time, will be 110% of average cost.

Flushed with pride and success, he applies for a 10% loan to construct a new building with enough space for 24 years growth at 6%. The loan officer raises an eyebrow and suggests 12 years. Who is closer to correct?

120. Solvent Cissie knows she lives by turnover. She has always dealt with fashion inventories. She works on an annual cycle, buying in the spring and clearing out in a year. That way, she has learned, she has half her capital back by midyear, so her average capital need is only half the peak.

Tired of renting, she buys her own shop using her savings which could otherwise earn 10%. A friend figures that her cash flow is enough to return her investment in 30 years, with 10% interest. Cissie figures she will recover, thus, half her capital in 15 years. How nearly right is she?

121. Until recently, you could "write off" the K_0 of a building for income tax purposes in a number of years based on its estimated economic life. Taxpayers might shade their estimates a bit, so a 50-year building would be written off in, say, 35 years. That means that you deduct $\frac{1}{35} K_0$ each year from taxable income, reducing your taxes by $\frac{tK_0}{35}$ (where t = tax rate).

Reagan pushed through a new law allowing 15 year write-off for all buildings. How much more would you bid for a building site as a result of this change? The cash flow is b , and $i = .07$.

122. Joe agrees to pay 22% for a loan. To find the monthly rate, he divides 22 by 12. There is no payback, the principal remains constant. When he makes the first monthly payment, he realizes he could have used that money in his business for a 22% gain.

- a. What rate paid annually would be the same as 22%/12 paid monthly?
- b. What monthly payment should Joe have stipulated, to correspond to 22% annual?

123. You can increase your money 25 times over 50 years. What is the annual ROR?

124. a. You can plant southern yellow pine for 3k and expect to sell it for 30k after 18 years. The land will cost you 4k, and you do not expect any change in its value. What is the ROR (rate of return)?
- b. How much more, in percentage terms, would ROR be if the land cost you nothing, and remains worth nothing?
- c. What is the ROR if you get the land for nothing, but sell it at the end for 4k?
- d. What is the RRAT (ROR after taxes) in case (a) if $t = .4$ and $s = .16$? (s is the rate that applies to gains of land value.) Note: these rates are hypothetical.
- e. You buy the land for 4k and sell it for 36. Now what is RRAT?

125. A farm manager put 5m into buying a herd of cattle, and sold it 7 years later for 10m. After putting a value on the land, he told Absentee Ann, the owner, that the RRBT (ROR before taxes) was 9.05% after allowing for the value of the land.

- a. What value was he putting on the land?
- b. Ann suspected the manager was overstating the true RRBT by undervaluing the land. She said it was really worth 3 times what he was assuming. What RRBT did she then say he had earned?

126. Motormouth Mike the stock broker has a deal for you. It is a growth stock that pays 5% current yield but is guaranteed to double in value in 12 years. Mike points out that to the sophisticated investor like you, yield consists of two parts, the current dividend and the appreciation. The first is 5% and the second is 5.95%, as you can show by taking the 12th root of 2. The sum is 10.95%, nearly 11%. "How can you go wrong?" says Mike.

Is the claimed rate correct?

127. An investment of 134k will yield 19k for 21 years, with no salvage value. Which, if any, of these RRBT is correct? 8.92%; 0%; 13.11%, 15.67%, or 12.17%?

