



Annualizing with the Installment Plan Factor (IPF)

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INTRODUCTION: Solving DCFF for its different elements

In Reading III, D we saw how the Compound Growth Factor (CGF) may be solved for its different elements. We saw that equations do not show causation: causation must be inferred from other information. Equations do show necessary relations, so you can find the unknown from the known.

That was just a warmup. Here we begin the process of doing the same thing for the DCFF. Up to now, we have focused mainly on valuation, where present value (P) is the unknown. From here on we focus on new uses for the DCFF, where P is known, and we use it to find something else.

As a reminder, here is the DCF equation:

$$P = a \frac{1 - I^{-n}}{i} \quad (1)$$

Annualizing: solving for a

$$a = P \frac{i}{1 - I^{-n}} \quad (2)$$

The ratio on the right of (2) is the Installment Plan Factor, or IPF. (Some call it the "capital recovery factor"). This is the factor or coefficient by which you multiply P to get the equivalent a.

This is how you buy on time in level periodic installments. "a" paid regularly over n periods is equivalent to P. From the lender's viewpoint, he recovers both Principal and interest.

(2) is something akin to a mean per year. That is, it is akin to P/n. However, it is always higher, because it covers interest as well as Principal. It is just a little higher than P/n when n is low, and a lot higher when n is high.

Exercise: check the extreme values algebraically. Show that IPF=I when n=1, and IPF=i when n=∞.

Next, use the numerical, tabular method. Make four columns, showing n, 1/n, i, and IPF, for various values of n. Use i=.10 the first time, so we can check our results together. Work out

3-4 intermediate values of IPF: it will always be higher than either $1/n$, or i . (On exams, you never know what crazy value for i I will ask about, so don't groove on .10 permanently.)¹

Next, use the graphical method. Using your table, put your few points on a graph. Interpolate by eyeball to sketch out a smooth curve of the whole function, and visualize how these functions behave. Put n on the abscissa, and decimals from .10 to 1.10 on the ordinate.

Think of IPF as a combination of dividing by n , as in a simple yearly average; and multiplying by the interest rate, i . IPF approaches $1/n$ when n and i are both small; and approaches i when n and i are large, but it always remains above both $1/n$ and i .

When you retire or "amortize" a debt on the installment plan, using level annual payments, IPF is used to figure the annual payment. To convert to monthly payments, which are commonly used, multiply n by 12 and divide i by 12.

What if you want to convert to weekly, daily, hourly, or minutely payments? No problem, with your calculator. Play around with it, and you will end up rediscovering the magic number e , the base of natural logs. You will find it a lot easier than poor old John Napier, in the 1590s or so: he did it all by hand.² For more on this, see the next reading, "Napier and Continuous Compounding."

Just for fun, figure out how much you would have today if John had put aside a tuppence for you then, and no one had touched it or its income of 7.2% annually over those years. 2^{40} gives a quick and dirty estimate, and makes it fairly clear that no one has ever had a string of descendants with that kind of will power or luck.

¹After you have worked out 3-4 values, you will understand better why people write programs. You who know how to program your calculators may want to do so at this point, although it is optional. If you do, you can easily find many intermediate values, and get a detailed picture of the whole function.

² That was towards the end of the long reign of Queen Elizabeth I, as she was mulling over whether to make James Stuart her successor (she did, just before she died in 1603). There were no calculators easily available at that time. It was the year they settled Jamestown, 17 years before the Pilgrims landed at Plymouth Rock, if that helps you place it.

What if you know the monthly payments, and want to figure out what i they are charging? Patience, that comes later when we solve for i .

What if you've made your payments faithfully for a few years, and they tell you your unpaid balance is nearly as high as when you began? Is the bank robbing you? That comes later, too, under "Amortization."

The IPF with Zero Interest

An interesting feature of the IPF (and its 3 related forms) is what happens when $i=0$. At first glance it looks as though the IPF collapses into $0/0$, which you were taught is "indeterminate." To be sure, your first-year calculus book tells you about l'Hôpital's Rule for resolving such indeterminacy, but you probably overlooked or forgot it, as I did once. You may confirm the rule by using your calculator and plugging in very low values of i . You'll see the IPF $\rightarrow 1/n$ as $i \rightarrow 0$.

Similarly, the DCFF $\rightarrow n$ as $i \rightarrow 0$. This is intuitively obvious: if it were otherwise you would figure something was wrong. Review the graphical picture of DCFF from Reading III, F, "The Discounted Cash Flow," p.6. Watch the curve of exponential decay there (Doonesbury's Nose) flatten out as $i \rightarrow 0$, and the area under the curve $\rightarrow n$.