

Phantom Profits; Indexing Article Current

W

Growth of Wealth with Inflation & Income Tax (All income reinvested.)

$$r = \text{real growth rate} = .03$$

$$g = \text{inflation rate} = .09$$

$$t = \text{tax rate} = .31$$

$$P_0 = \$1.$$

$$n = 40$$

$$CPI_{40} = 1.09^{40} = 31.41$$

$$i = r + g + rg = .123$$

$$I = 1 + i = 1.123$$

$$a = \text{annual cash flow}$$

Savings Account

$$a) g=0, t=0 \quad 1.03^{40} = 3.262$$

$$b) g=.09, t=0 \quad 1.03 \cdot 1.09 = 1.123 \\ 1.123^{40} = 102.5$$

$$c) g=0, t=.31 \quad [1 + r(1-t)]^n = \\ [1 + .03 \cdot .69]^n = 1.021^{40} = 2.269$$

$$d) (c)/(a) \quad 2.269/3.262 = .696$$

$$e) g=.09, t=0. [(1+r)(1+g)]^n = (1.03 \cdot 1.09)^{40} = 102.5$$

$$f) g=.09, t=.31 \quad [1 + (r+g+rg)(1-t)]^n = \\ (1 + .123 \cdot .69)^{40} = 1.085^{40} = 26.0$$

$$g) \text{ Deflated Wealth} \quad 26.0/CPI = \frac{26.0}{31.41} = .828$$

$$h) \text{ Change in real wealth: } -17.2\%$$

Land w/o cash flow

$$1.03^{40} = 3.262$$

$$(1.03 \cdot 1.09)^{40} = 102.5$$

$$(1+r)^n(1-t) + t = 1.03^{40} \cdot .69 + .69 \\ = 2.941$$

$$2.941/3.262 = .902$$

$$= 102.5$$

$$102.5 \cdot .69 + .69 = 71.42$$

$$71.42/31.41 = 2.27$$

$$+ 127\%$$

Building

$$a) g=0, t=0 \quad a = \frac{r}{1-(1+r)^{-n}} = .0433$$

$$.0433 \frac{1.03^{40} - 1}{.03} = 3.262$$

$$b) g=.09, t=0 \quad a = \frac{i}{1-I^{-n}} = .12420$$

$$.12420 \frac{1.123^{40} - 1}{.123} \approx 103.56$$

$$c) g=0, t=.31 \quad \left[a(1-t) + \frac{t}{n} \right] \frac{[1+r(1-t)]^n - 1}{r(1-t)} =$$

$$(.0433 \times .69 + \frac{.31}{40}) \times 61.33 =$$

$$(.0299 + .0078) \times 61.33 = 2.3120$$

$$d) (c)/(a) \quad \frac{2.312}{3.262} = .71$$

$$e) \quad \text{same as (b)}$$

$$f) g=.09, t=.31 \quad \left[a(1-t) + \frac{t}{n} \right] \frac{[1+i(1-t)]^n - 1}{i(1-t)} =$$

$$(.12420 \times .69 + \frac{.31}{40}) \times 294.66 =$$

$$(.0857 \times .0078) \times 294.66 = 27.55$$

$$g) \text{ Deflated wealth: } \frac{27.55}{\text{CPI}} = \frac{27.55}{31.41} = .877$$

$$h) \text{ Change in real wealth: } -12.3\%$$

Thanks - will use.

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THE EFFECT OF THE TAX SYSTEM ON THE IMPACT OF GOVERNMENT DEBT

WILLIAM VICKREY

Just as the ultimate impact of changes in taxes with a macro-economic impact, such as the corporate income tax, cannot be determined without considering the concomitant changes in fiscal and monetary policies that could be expected, so the impact of changes in national debt resulting from deficits cannot be determined without looking into the tax structure that will necessarily be involved in ultimately paying off the debt. Indeed fairly drastic differences in the impact of government debt will result from what may appear to be minor changes in the tax structure.

At one extreme, one could consider the case of a community committed to Henry George's single tax on site values, assessed independently of the actual use or condition of the tract involved. If it be assumed that these values vary proportionately over time, any government debt becomes in effect a collective mortgage on the properties. If it be assumed that initially at least the interest cost of the government debt is lower than the rates charged on individual mortgages, or obtainable by added individual investment, it would be to the advantage of the taxpayers in the community for current outlays to be financed by borrowing, rather than by a current land tax levy, up to the point where the debt would be so high in relation to the underlying property values that the marginal cost of the public borrowing would be driven up to the level of the opportunity cost of private capital to the property owners.

~~78~~

This limit would not be expressible in terms of the market value of the property, which would be reduced by the capitalization of future tax payment: if the gross value of a parcel would be \$100,000 in the absence of any government debt, and the share of this parcel in the debt is \$80,000, the market value would fall to \$20,000 and the debt would be 4 times the market value. A buyer could purchase the parcel for \$20,000, whereas in the absence of the public debt he would have had to pay \$100,000, possibly paying \$20,000 in cash and raising the other \$80,000 by an individual mortgage, on which the burden of the financing charges would be somewhat higher than the taxes alternatively required to finance the public debt.

If site values do not vary proportionately over time, as when suburban property expected to be developed at some time in the future will increase in value over time at a rate significantly greater than for central city properties on which a substantial rent is currently being realized, then the decision to finance by debt rather than by current taxation will have a significant redistributive effect. In the above example, it would shift the burden from central city to the peripheral property owners. The shift would be greater if the tax is based on current

to each other

rental value than if the base is capital value or market value, which might be an argument for using the capital value rather than the rental value base. This would be in addition to the argument that transactions that are closely indicative of the capital value of the land are more frequent than transactions from which a current rental value can be deduced. Aside from this redistributive effect, the overall efficiency improvement from shifting to debt finance is still significant.

Shift only slightly from a land tax to the widely used tax on real estate inclusive of fixed improvements (what the French characterize as "immeubles") and the situation changes drastically. Under these circumstances, anyone who puts up a building thereby assumes, for the time being at least, an increased share in the overhanging public debt. The effect of this burden may be considerably mitigated by an expectation that future builders will come along and take some of the burden off the original builder's shoulders. But as the debt grows this expectation weakens, and a tipping point may be reached rather suddenly at which this expectation vanishes, the assumption of an added share in the debt burden is expected to be permanent, and new construction comes to a grinding halt.

Something of this sort seems actually to have happened in many locations, particularly in places such as Boston and Newark, in which the central city jurisdiction is small relative to its metropolitan area. Redevelopment could then occur only on the basis of special deals in the form of tax concessions or assessment freezes that permit revitalization of strategic central areas. However, such concessions sometimes run up against objections that they violate constitutional requirements of uniformity. In Milwaukee, indeed, after a project undertaken in response to such a concession was well under way a taxpayer suit overturned the deal on the ground of unconstitutional discrimination, but by then sufficient revitalization had gotten under way that the developers are alleged to have made out fairly well rather than reaping a bonanza.

If we move to a sales tax as the basic tax, the results are less clear. At one extreme, individuals could maintain the same consumption patterns, and adjust their savings and bequests so as to achieve the same results as in the land tax case, at least to the extent that consumption patterns vary proportionately over time. On the other hand, debt finance with lower sales taxes currently and higher sales taxes later on could, in an economy open to trade and capital flows, result in higher current consumption expenditure, either from the change in the relative prices of consumer goods or from liquidity constraints. To the extent that this increase in consumption consists of domestically produced goods, employment will be stimulated and a Keynesian, "functional finance" policy becomes possible as a means of achieving fuller employment.

A less significant reaction in an economy open to migration could involve population shifts.

In a closed economy, already at full employment, avoiding inflation in the face of higher current consumption would require raising interest rates to curtail investment correspondingly, which would mitigate the change in the ratio of future to present prices.

In all but the inelastic case, there will be some decrease in savings reflected in decreases in domestic or foreign investment. Whether this is a gain or loss in overall welfare would depend to some extent on the levels of tax from which one starts. For example, if one starts from a state in which the tax rate is currently high as a result of a pay-as-you-go policy for financing an extraordinary current outlay, whether in repairing the ravages of a hurricane or an earthquake, or building schools and other facilities to serve a burgeoning suburban community, borrowing might equalize current and future tax rates and reduce the loss from distortions.

If instead of a flat retail sales tax or a value added tax we have a progressively graduated expenditure tax, an added element is introduced in that the potential for progressivity will be greater if the overall level of the tax is fairly uniform over time than if it is much lower at one time than another. Otherwise the results are similar to those for the sales tax.

If we go to a tax on earnings, the results may be somewhat similar except for the fact that variations in earnings over a lifetime are more variable than variations in consumption expenditures and the intergenerational transfer aspects will be somewhat larger.

At another extreme, consider the case where the dominant tax is a net worth tax on a destination basis. This is not as irrelevant as might at first appear: under inflationary conditions, a tax on nominal accrued income is equivalent to a tax on a base consisting of real income plus a percentage of net worth equal to the rate of inflation. While existing income taxes depart in many intricate ways from a tax on accrued income, under inflationary conditions they do contain a significant net worth tax element. Moreover, as long as the rate of inflation is moderate, including in the income tax base a net worth element resulting from inflation is not an undesirable feature: while the result is not what is ordinarily meant by an income tax, the practical result is a tax that for a given level of revenue and degree of progressivity has a substantially smaller disincentive effect on effort and risk taking than if an attempt were made to restore the tax to the conceptually pure income tax, as by indexing not only those profits conforming to some necessarily arbitrary definition of "capital gains", but all forms of income from capital.

In this case it may be appropriate to distinguish three periods: a period during and perhaps prior to the borrowing in which the reduction in the tax on net worth may induce additions

to net worth out of the tax savings, an intervening period during which the anticipation of future heavier taxes on net worth would induce a reduction in net worth and an increase in consumption, and a final period in which repayment of the debt occurs and the gradual reduction in the present value of the remaining added taxes on net worth leads to a gradual building back of net worth, though possibly not to the same level as would have occurred without the borrowing.

It is difficult to say very much a priori about the net direction of the effect on consumption and the overall level of capital investment, which will depend, among other things, on how the pattern of change in the fiscal policy relates to the life cycle patterns of various individual cohorts. There is first of all the ambiguity of the net effect on savings of a change in the net return on savings, involving the possible opposition of income and substitution effects. There is even the possibility that during the initial borrowing period the reduction in taxes combined with the increased rate of return offered would induce a reduction in consumption and an increase in savings by more than the tax reduction, leading to an increase in total investment in the economy, contrary to the conventional wisdom concerning the effect of government borrowing. But admittedly this outcome seems unlikely under most practical circumstances.

The three essential characteristics of taxes that affect the results are: whether the tax is capitalizable, whether it has distortionary effects, and whether it is levied before or after the market. A graduated expenditure tax is largely non-distortionary non-capitalizable, and levied after the market, for example. The corporate income tax is highly distortionary, but is distinguished, in its macro-economic impact, from a net worth tax, chiefly by being levied before rather than after the market.

In practice, the results are further muddled by the fact the individuals do not visualize with any vividness what taxes are likely to be levied at various times in the future to finance the service of a government debt. Indeed in a real situation it may be difficult to determine, analytically, just which of the array of taxes in effect are to be considered as levied in order to finance debt service. This may be because future policies are genuinely uncertain, but even where tax patterns are relatively simple and fixed there may simply be a refusal to come to grips with a burden deferred to an indefinite future. This "debt illusion," in which government securities figure much more vividly in the calculations of their holders than in their calculations as taxpayers ultimately responsible for the required payments, can be a strong factor in determining the effect of a government debt.

This is likely to be of widely varying importance at various levels of government in the U.S. In many local jurisdictions the reliance on the property tax inclusive of improvements is almost

complete and firmly established, so that the possibilities for debt illusion are limited. At the state level, the tax structure is more complex, often involving a mixture of personal and corporate income taxes, sales taxes, excises, and other levies, so that the identity of the eventual bearers of the burden of financing debt service is less clear, though interstate competition may focus attention and provide avoidance opportunities that are less significant at the national level. At the national level in the U.S., in spite of the often vehement political rhetoric over budget deficits defined in a manner that is almost absurd from any fundamental economic standpoint (one could reduce the Gramm-Rudman defined deficit by selling the Pentagon to a life insurance company and leasing it back), actual expectations as to when and how the debt service will be financed must be vague at best. As major elements of the revenue structure we are faced with an individual income tax full of often quite arbitrary concessions and loopholes constantly under revision, a corporate income tax devoid of economic rationale that is highly distortionary and the burden of which nearly everyone feels is borne by somebody else, and payroll taxes incorporated into a social security system that is part insurance, part redistribution, and part Ponzi or chain-letter scheme, preserved from eventual collapse only by the power of the government to compel the adherence of new cohorts of contributors. The degree of "debt illusion" is likely to be substantial.

There is also the prospect that much government debt is likely to be refinanced rather than paid off at maturity out of tax revenues, so that the bearing of the burden of repayment is at least in part deferred to an indefinite future. Analysis of cases in which repayment is indefinitely deferred, in the extreme case allowing for the debt to grow at a rate *pari-passu* with the growth of the economy, tends to be extremely sensitive to minor variations in the assumptions made. The prospect of such refinancing is of course another factor tending to strengthen the debt illusion, as are uncertainties concerning future rates of inflation.

Going from these considerations to a realistic appraisal of the impact of government borrowing under actual conditions is thus likely to involve some fairly intricate analysis as well as some difficult evaluations of reaction factors. In most cases a simplistic viewing with alarm will be largely beside the point.

Growth of Wealth with Inflation + Income Tax (All income reinvested.)

$$r = \text{real growth rate} = .03$$

$$g = \text{inflation rate} = .09$$

$$t = \text{tax rate} = .31$$

$$P_0 = \$1.00$$

$$n = 40$$

$$CPI_{40} = 1.09^{40} = 31.41$$

$$i = r + g + rg = .123$$

$$I = 1 + i = 1.123$$

$a = \text{annual cash flow}$

Savings Account

a) $g=0, t=0$

$$1.03^{40} = 3.262$$

b) $g=.09, t=0$

$$1.03 \cdot 1.09 = 1.123$$

$$1.123^{40} = 102.5$$

c) $g=0, t=.31$

$$[1 + r(1-t)]^n =$$

$$[1 + .03 \cdot .69]^n = 1.021^{40} = 2.269$$

d) (c)/(a)

$$2.269 / 3.262 = .696$$

e) $g=.09, t=0. [(1+r)(1+g)]^n = (1.03 \cdot 1.09)^{40} = 102.5$

$$= 102.5$$

f) $g=.09, t=.31$

$$[1 + (r+g+rg)(1-t)]^n =$$

$$(1 + .123 \cdot .69)^{40} = 1.085^{40} = 26.0$$

$$102.5 \cdot .69 + .69 = 71.42$$

g) Deflated Wealth

$$26.0 / CPI = \frac{26.0}{31.41} = .828$$

$$71.42 / 31.41 = 2.27$$

h) Change in real wealth: -17.2%

$$+127\%$$

Building

a) $g=0, t=0$

$$a = \frac{r}{1-(1+r)^{-n}} = .0433$$

$$.0433 \frac{1.03^{40} - 1}{.03} = 3.262$$

b) $g=.09, t=0$

$$a = \frac{i}{1-I^{-n}} = .12420$$

$$.12420 \frac{1.123^{40} - 1}{.123} \approx 103.56$$

c) $g=0, t=.31$

$$\left[a(1-t) + \frac{t}{n} \right] \frac{[1+r(1-t)]^n - 1}{r(1-t)} =$$

$$(.0433 \times .69 + \frac{.31}{40}) \times 61.33 =$$

$$(.0299 + .0078) \times 61.33 = 2.3120$$

d) $(c)/(a)$

$$\frac{2.312}{3.262} = .71$$

e) Same as (b)

f) $g=.09, t=.31$
$$\left[a(1-t) + \frac{t}{n} \right] \frac{[1+i(1-t)]^n - 1}{i(1-t)} =$$

$$(.12420 \times .69 + \frac{.31}{40}) \times 294.66 =$$

$$(.0857 \times .0078) \times 294.66 = 27.55$$

g) Deflated wealth:
$$\frac{27.55}{CPI} = \frac{27.55}{31.41} = .877$$

h) Change in real wealth: -12.3%

~~Δ~~ Δ of wealth, $n = 40$

Land, no sale.

$$P_0 = 1$$

$$F_{40} = 1.09^{40}$$

$$a = ? \quad 1 = \frac{a}{i-f}, \quad \text{so } a = i-f = r = .03$$

$$\Sigma F = \cancel{.03 I^{40}} + .03(1+f)$$

$$= .03(1+i)^{40} + .03(1+f)(1+i)^{39} + .03(1+f)^2(1+i)^{38} + \dots + .03(1+f)^{39}(1+i)$$

$$= .03 \left[(1+f)^{40}(1+r)^{40} + (1+f)(1+f)^{39}(1+r)^{39} + \dots + \right.$$

$$\left. .03(1+f)^{39}(1+f)(1+r) \right]$$

$$= \cancel{.03 \left[(1+i)^{40} + (1+i)^{39} \right]}$$

$$= .03 \left[(1+f)^{40}(1+r)^{40} + (1+f)^{40}(1+r)^{39} + (1+f)^{40}(1+r)^{38} \dots \right]$$

$$\text{Series is } .03 \left[1 + (1+f) + (1+f)^2 + (1+f)^3 + \dots + (1+f)^{39} \right]$$

$$FV = .03 \left[(1+i)^{40} + (1+f)(1+i)^{39} + (1+f)^2(1+i)^{38} + \dots \right]$$

$$= .03 \left[(1+r)^{40}(1+f)^{40} + (1+f)(1+f)^{39}(1+r)^{39} + \dots \right]$$

$$= .03(1+f)^{40} \left[1 + (1+r) + (1+r)^2 + \dots \right]$$

$$= .03(1+f)^{40} \frac{(1+r)^{40} - 1}{r}$$

~~Wrong~~

Repeat:

$$\text{Weight} = .03(1.09)^{40} \frac{1.03^{40} - 1}{.03} + 1.09^{40}$$

$$= 1.09^{40} [1.03^{40} - 1 + 1] = 1.09^{40} \times 1.03^{40} = I^{40}$$

Insert 2A

$$t = .31$$

Series is $.03 \cdot .69 [1 + (1+g) + \dots + \text{etc.}]$

$$FV = .03 \cdot .69 [(1+i(1-t))^{40} + (1+g)(1+i(1-t))^{39} + \dots + 1]$$

Discussion

$$1 + i(1-t) \stackrel{?}{=} (1+r(1-t))(1+g(1-t))$$

$$= 1 + g(1-t) + r(1-t) + rg(1-t)^2$$

$$i(1-t) \stackrel{?}{=} g(1-t) + r(1-t) + rg(1-t)^2$$

$$i \neq g + r + rg(1-t)$$

$$i > g + r + rg(1-t)$$

Very small error - can accept, if convenient

$$1 + i(1-t) \equiv 1 + j$$

$$= .03(1-t) [(1+j)^{40} + (1+g)(1+j)^{39} + (1+g)^2(1+j)^{38} + \dots]$$

Insert 2B

$$= .03(1-t) \left[\frac{(1+j)^{40} - (1+g)^{40}}{1 - \frac{1+g}{1+j}} \right] = .03 \times .69 \frac{(1+.12 \times .69)^{40} - 1.09^{40}}{1 - \frac{1.09}{1+.12 \times .69}}$$

Δ of wealth.

Land, no sale, no tax, no infln., no rise of l.v.

$$P_0 = 1$$

$$F_{40} = 1$$

$$q = Pr = r = .03$$

$$F = .03 \left[(1+r)^0 + (1+r)^1 + \dots + 1 \right] \text{ all } + 1 = .03 \frac{(1+r)^{40} - 1}{.03} + 1$$

$$= (1+r)^{40} = 3.262$$

"main part"

$$ab^n + a^2b^{n-1} + a^3b^{n-2} + \dots + a^n b =$$

$$r = ab^{-1} = \frac{a}{b}$$

$$f = ab^n$$

$$S = ab^n \frac{1 - \left(\frac{a}{b}\right)^{n+1}}{1 - \frac{a}{b}} = \frac{ab^n - a^{n+1}}{1 - \frac{a}{b}}$$

$$S = b^n + ab^{n-1} + a^2b^{n-2} + \dots + b a^{n-1}$$

$$f = b^n$$

$$r = a/b$$

$$S = b^n \frac{1 - \left(\frac{a}{b}\right)^n}{1 - \frac{a}{b}} = \frac{b^n - a^n}{1 - \frac{a}{b}}$$

$$a = 1+g$$

$$b = 1+j$$

$$= .0207 \frac{1.0828^{40} - 1.09^{40}}{1 - \frac{1.09}{1.0828}} = .0207 \frac{24.0952 - 31.4094}{.0066}$$

$$= .0207 \times 1099.91 = 22.77$$

This result is too low, even lower than bldg. = 27.55.

Land w/o sale.

$$a) g=0, t=0 \quad a = .03, \quad a \frac{I^t - 1}{i} + 1 = .03 \frac{1.123^4 - 1}{.123} + 1$$

$$b) g=.04, t=0 \quad P = \frac{a}{i-g}$$

$$a = P(i-g) = P(r) = .03$$

$$a \frac{I^t - 1}{i} + 1.04^t$$

Real wealth is maintained, plus only inc. is rec'd w/o any tax on phantom inc.

Land w/o cash flow
Basis indexed.

f) $102.5 \times .69 + .69 \times 31.41 = 70.73 + 21.68 = 92.41$

g) $92.41 / 31.41 = 2.942$

h) 194 %

S = salvage value

Land

a) $g=0, t=0$

$a=.03, S=1$

$$.03 \times \frac{1.03^{40} - 1}{.03} + 1 = 1.03^{40} = 3.262$$

b) $g=.09, t=0$

$$.03 \times \frac{1.03^{40} - 1}{.03} \times 1.09^{40} + 1.09^{40} =$$

$$1.03^{40} \times 1.09^{40} = 103.56$$

c) $g=0, t=.31$

$$.03(1-t) \frac{[1+r(1-t)]^n - 1}{r(1-t)} + 1 = [1+r(1-t)]^n = 2.264$$

~~.03 \times .69 \times 61.33~~ Same as Savings Acct!

Bes. of assing. that reinvst. is taxed like Sav. Acct.

d) $\frac{(c)}{(a)}$

$$2.264 / 3.262 = .696$$

e) Same as (b)

f) $g=.09, t=.31$ $.03(1-t)$

* $\textcircled{+} S = .03 [1 \times 1.09^{40} + 1.09 \times 1.09^{39} + 1.09^2 \times 1.09^{38} + \dots + 1.09^{39} \times 1.09]$ Thus, mult. ea. term by 1.09^{40}

A of wealth w. infln., inc. tax & index.

Ex $r = \text{real growth rate before tax} = .03$

$g = \text{infln.} = .09$

$t = .31$

$P_0 = 1$

$n = 40$

$$CPI_{40} = 1.09^{40} = 31.41$$

1. Saving Acct.

a) $1.03^{40} = 3.262$

b) $1.03 \cdot 1.09 = 1.123$
 $1.123^{40} = 102.5$

$$CPI_{40} = 1.09^{40} = 31.4$$

c) ~~Wealth~~ after tax,
~~max~~ $g = 0$

$$[1 + r(1-t)]^n = (1 + .03 \cdot .69)^{40} = 1.021^{40} = 2.269$$

$$2.269 / 3.262 = .696 \quad (\approx 1-t, \text{a coincidence})$$

~~2.269~~
 In general, $\left(\frac{1 + r(1-t)}{1+r}\right)^n = \left(\frac{1+r-rt}{1+r}\right)^n = \left(1 - t \frac{r}{1+r}\right)^n$

a decreasing fn. of n .

d) Growth of wealth after tax

Before tax: $3.262 - 1 = 2.262$

after tax: $2.269 - 1 = 1.269$

$$1.269 / 2.262 = .561$$

Growth of wealth

1, 2

In general, growth of wealth is:

$$(1+r)^n - 1$$

Growth after tax is $(1+r(1-t))^n - 1$

$$\frac{\cancel{(1+r)^n - 1}}{\cancel{(1+r(1-t))^n - 1}} = \frac{(1+r(1-t))^n - 1}{(1+r)^n - 1}$$

For large $n \rightarrow \left(\frac{1+r-rt}{1+r} \right)^n =$

$$\text{For large } n, \rightarrow \left(\frac{1+r-rt}{1+r} \right)^n = \left(1 - t \frac{r}{1+r} \right)^n \rightarrow 0$$

$$\text{For } n=1, = \frac{r(1-t)}{r} = 1-t$$

Growth after gains tax:

$$(1+r)^n (1-t) + t - 1 = (1-t)((1+r)^n - 1)$$

$$\frac{(1-t)((1+r)^n - 1)}{(1+r)^n - 1} = 1-t$$

c) $g = .09$. $1.123^{40} = \cancel{103.6} 102.5$
 Wealth after tax is $\cancel{[(1+r)(1+g)]^n}$

$$[1 + (r + g + rg)(1 - t)]^n$$

$$= (1 + .123 \cdot .69)^{40} = (1 + .085)^{40} = 26.0$$

$$26.0 / 102.5 = .254$$

$$26.0 / 31.41 = .828, \text{ a loss of wealth.}$$

$$\text{Change in real wealth: } -17.2\%$$

2. Land price, ~~g~~

a) $1.03^{40} = 3.262$

b) $(1.03 \cdot 1.09)^{40} = 102.5$

c) Wealth after tax, $g = 0$

$$(1.03)^{40} \cdot .69 = \cancel{2.251} + .69 = 2.941$$

$$\cancel{2.941} / 3.262 = \cancel{.902} .902$$

d) Growth of wealth

Before tax: $3.262 - 1 = 2.262$

After tax: $2.941 - 1 = 1.941$

$$1.941 / 2.262 = .858$$

e) $g = .09$ $1.123^{40} = 102.5$

Wealth after tax is:

$$102.5 \cdot .69 + .69 = 71.42$$

$$71.42 / 102.5 = .70$$

$$71.42 / \text{CPI} = 71.42 / 31.41 = 2.27$$

$$\text{Change in real wealth} = +127\%$$

$$2.27 / 3.262 = .70$$

$$2.27^{\frac{1}{40}} - 1 = .021$$

f. Land gain with indexing

Wealth after tax is:

$$102.5 \cdot .69 + .69 \cdot 31.41 = 70.73 + 21.68 = 92.41$$

$$92.41 / 102.5 = .90$$

$$92.41 / \text{CPI} = 92.41 / 31.41 = 2.942$$

$$\text{Change in real wealth} = +194\%$$

$$2.942 / 3.262 = .90$$

$$2.942^{\frac{1}{40}} - 1 = .0273$$

[~~x/~~ Land, cash flow = $a = 0$

$$a) 1.072^{40} = 16.14$$

$$b) 14\% \\ 1.1712^{40} = 730.3$$

c) Gains tax

$$730.3 \cdot .69 + .69 = 503.91 + .69 = 504.6$$

$$504.6 / 730.3 = .691$$

$$504.6 / 45.26 = 11.15$$

$$11.15^{\frac{1}{40}} - 1 = .062$$

d) Gains tax indexed

$$730.3 \cdot .69 + .69 \cdot 45.26 = 503.91 + 31.23 \\ = 535.14$$

$$535.14 / 730.3 = .733$$

$$535.14 / 45.26 = 11.824$$

$$11.824^{\frac{1}{40}} - 1 = .064$$

Much ado about nothing!

Indexing will make diff. when $g \gg i$.
Now, use r for real return; reserve i for nominal
int. rate: $i = r + g$

3. Depreciable K. $P_0 = 1.00$

$$a) \quad a = \frac{.03}{1 - 1.03^{-40}} = .0433$$

~~b)~~

~~c)~~ Wealth ^{w/o} ~~after~~ tax, $g = 0$: 3.262
FVF = 75.40

$$.0433 \times \frac{1.03^2 - 1}{.03} \approx 3.262$$

(Assumes all inc. reinvested.)

b)

c) Wealth after tax, $g = 0$
Tax life ~~= 40~~ $\equiv d = 40$ (use 40 to simplify)

$$\left[a - t \left(a - \frac{1}{d} \right) \right]^{FVF} = \left[a(1-t) + \frac{t}{d} \right] FVF$$

$$= \left(.0433 \cdot .69 + \frac{.31}{40} \right) \cdot 75.40 = 2.8371$$

$$2.8371 / 3.262 = .86$$

d)

$$e) \quad q = .09$$

$$Cf. = a[1 + 1.09 + \dots]$$

$$i = r + q + rq = .123$$

$$FV = a[1 \cdot 1.123^{39} + 1.09 \cdot 1.123^{38}] \quad \dots \text{hard!}$$

Defer.

4. Xmas tree? N.B., it gets cap. gains, wd. be indexed, also full salvage con, etc.

S = Salvage Value
 c = current year
Land

(I could/should have real rent grow too, at 2% / yr.) (or break into 2 parts.)

a) $g=0, t=0$ $a = .03, S = 1$

$$.03 \times \frac{1.03^n - 1}{.03} + S = 1.03^n = 3.262$$

b) $g=.09, t=0$ $a = \sum .03(1.09)^t$? Or, $a = r + g + rf = i = .123$

$$S = 1.09^{40} = (1+g)^n = 31.41$$

✓
$$.03(1 \times 1.09^{39} + 1.09 \times 1.09^{38} + \dots + 1.09^{39} \cdot 1.09)$$
 ✓

PV:
$$\text{sum} = \frac{.03(1+g)}{(1+r)(1+f)} + \frac{.03(1+f)^n}{(1+r)^n(1+f)^n}$$

FV:
$$\text{sum} = \frac{.03(1+f)(1+r)^n(1+f)^{n-1}}{(1+r)^n(1+f)^n} = .03[(1+g)(1+r)]^n = .03 I^n$$

$$\Sigma = .03 \frac{I^n - 1}{i}$$

$$.03 \times \frac{1.123^n - 1}{.123} + S = .03 \cdot 833.82 + 31.41 = 25.0145 + 31.41 = 56.42$$

Won't work unless I begin w. $a = .123$

or

$$.03 \times \frac{1.03^n - 1}{.03} \times 1.09^n + 1.09^n = 1.09^n(1.03^n - 1 + 1) = 1.09^n 1.03^n$$

$$\frac{1.03^{40} - 1}{.03} \times 1.09^{40} = 75.4 \times 31.4 = 2,368.$$

$$\frac{1.123^{40} - 1}{.123} = \cancel{834} 833.8$$

For easy calc., deduct all deprec. at $l/2$, (l = tax life.)

$$\begin{aligned} \text{Wealth} &= (1-t)a \frac{R^n - 1}{r} + t(1+r)^{(n-\frac{l}{2})} \\ &= a(1-t) \frac{R^n - 1}{r} + t(1+r)^{(n-\frac{l}{2})} \end{aligned}$$

or

$$\begin{aligned} &= a(1-t) + \frac{t}{l} \frac{R^l - 1}{r} R^{(n-l)} \\ &= a(1-t) \frac{R^n - 1}{r} + \frac{t}{l} \frac{R^l - 1}{r} R^{(n-l)} \end{aligned}$$

or

$$= a(1-t) \frac{(1+r+g+rg)^n - 1}{1+r+g+rg} \quad \dots$$

$$\text{let } (1+r)(1+g) = i$$

then

$$= a(1-t) \frac{I^n - 1}{i} + \frac{t}{n} \frac{I^n - 1}{i}$$

$$= \left[a(1-t) + \frac{t}{n} \right] \frac{I^n - 1}{i}$$