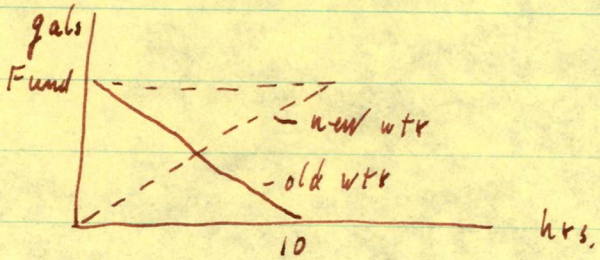
Calc. Mean Pd. of Invt.

Turnover.

Tank holds 10 gals.
Input = 1 gal/hr
Outgo = 1 gal/hr.

In 10 hrs, 10 gals in & 10 gals out.



In 5 hrs half the old wtr. is out. But it took it on ave. 5 hrs to come in, so pd. is 10 hrs.

When dealing w. depreciation or exhaustion alone, is ok to say mean pd. of life is half the total. But when dealing w. continuous turnover, not so.

✓ In either case, ~~[illegible]~~ period = $\frac{\text{Fund}}{\text{Flow}}$, where fund means mean level.

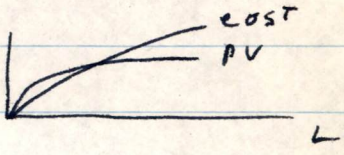
✓ W. const. level in 10 hrs. you change 100% of the wtr., so ea. gal. spends 10 hrs going thru. ~~How do we know?~~

But this assumes perfect market. ~~Too~~ If terminis pariter, assembly problems, get social losses.

Rent as constraint on temporal expansion.

$$\sum R_n((1+i)^{t-n}\ \frac{i}{(1+i)^t-1}$$

$$C_0\ \frac{i}{1-(1+i)^{-t}}$$



Spreading C_0 over time, cb. space.

a. Spreading costs over time
b. Add as 3d dimension to space graph .. a 4th dimension!
c. MC of time. It, too, may differ from AC.
   Replace when MC = MR
d. Rent constrains temporal expansion. O.C. of future.
e. " " scale ecs. -- they involve durability
f. Ave. rent weeds out units too short, despite hi marg. rent
g. Refutes idea ~~too~~ that risk argues vs. intensity
c. Squeezing in interim uses. Use idea of MC of time. Take net value of future use, put mgnl to it.
   When in doubt, fall back on max. P.V.
h. More risk when lengthen into mgnl. periods.

1. "Granular" K.

2. Waiting in lines at Drive-ins.
Queuing – a way of expressing period of K graphically.

3. Water storage.
In soil, water is stored for a week before use.
In river, ~~one year~~ 4 mos.
In secular ground storage, 5 – 20 years. (Takes 3 years for mx. wx. in San Bernardino to affect ground wtr. in Bunker Hill Basin.

4. Popn. Turnover

5. Grad. student Turnover.

K / payroll = K / 1

In the 80-yr. normalized forest, where payroll is 1, K / payroll = K = TVE = 5,887.
If one job earns $30,000, that is $177m / job.
(N.B. This refers only to jobs in planting. Logging is not considered here.)
By contrast, consider the K in a busy restaurant where it turns daily. There, $1 of K does a volume of $365/yr. Round that down to $300 to allow for holidays, & keep the arith. simple. Now, one $30,000 job requires $100 of K. The 80 yr. forest then requires 1,770,000 x as much K / job as the rest.

The contrast is overdrawn, to make a pt. You may whittle it down as much as you please, however, & not escape the message: to make jobs w. lod. K, put the K to work turning ~~too~~ fast, not in timber or slow-turning p.w.

Spr. qtr.

~~Apr.~~ March 30, lustrum begins.

$$\frac{m-k}{ik} = \frac{m}{ik} - \frac{1}{i} = \frac{m}{1-i^{-m}} - \frac{1}{i}$$

$$\left(1+\frac{i}{f}\right)^{fn} = \left(1+\frac{1}{f/i}\right)^{xni} \quad \text{where } x = \frac{f}{i}$$

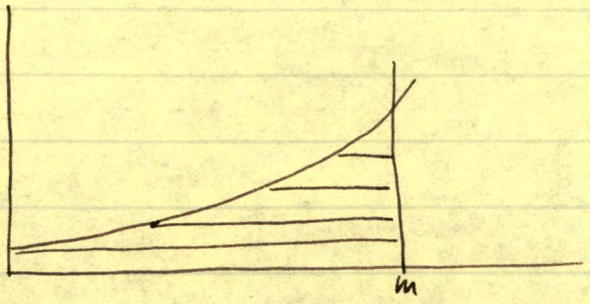
$$\left(1+\frac{1}{x}\right)^{xni} \quad \text{where } x = \frac{f}{i}$$

~~$= e^{ni}$~~

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e$$

$$(1+i)^{n} = \lim_{f\to i}\left(1+\frac{i}{f}\right)^{fp} = e^{p}$$

$$p = \ln(1+i)$$



$$\text{Mean Pd.} = \Big[mk + (m-1)(k)(1+i-1) + (m-2)\,k\,((1+i)^2-1) + (m-3)\,k\,((1+i)^3-1) \Big] \div I^m$$

$$= k\left[1 + I^1 + I^2 + I^3\right] \div I^m$$

$$= K/F$$

Pd. of k is $\frac{K}{F} - Ki = K\left(\frac{1}{F} - i\right)$

$$= \frac{1 - I^{-m}}{i} - (I^m - 1) = \frac{1 - I^{-m} - iI^m + i}{i}$$

$$= \frac{I^m - 1}{i}\left(I^{-m} - i\right)$$

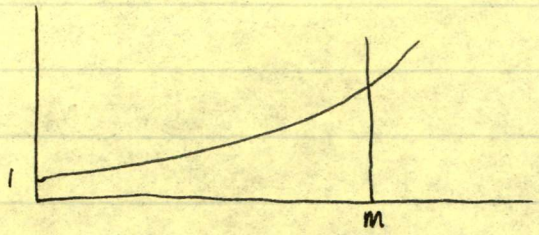
~~Pd of k is~~ $F - Ki = I^m - (I^m - 1) = 1$

Pd. of k is "obviously" m.

Normalized model, $F = m I^m$

$$K = \frac{I^m - 1}{i}$$

$$F - Ki = mI^m - I^m + 1$$



W = Work $\quad \frac{W}{F} = \frac{1}{I^m} = I^{-m}$. F here is not a flow, but a future value.

If normalized, then is one K = area under curve

$= \frac{I^m - 1}{i}$

$P = \frac{K}{F} = \frac{I^m - 1}{i\, I^m} = \frac{1 - I^{-m}}{i}$

Share of K in receipts is $\frac{Ki}{I^m} = \frac{(I^m - 1)}{i\, I^m} \times i$

$= 1 - I^{-m}$

Cow case

a. One cow. ~~If~~ $K_0 = 1$ = ~~the~~ Work

$F = i$ / yr. for 10 yrs, ff. by 1.

If now mortized, $F = mi + \frac{1}{\cancel{\phi}}$

$K = m$

$\frac{K}{F} = \frac{m}{mi + 1} = \frac{1}{i + \frac{1}{m}} = \frac{m}{mi + 1}$

If $m \to \infty$, $\frac{K}{F} \to \frac{1}{i}$, or $Ki = F$, meaning

turnover = i

There are 2 kinds of turnover:

a) job-making

b) Overall, including turnover of ipsegmic K (+ transgenic K).

My App. to TMJ&W deals only w. factor propns., not pd. or turnover.

Wicksell's period is the biol. or corpus life of wine. His construc. is normalizing:

$$K = \frac{I^m - 1}{i}$$

$$Ki = I^m - 1$$

Normalizing conv.: $K_0 = 1$

$K = m$

$Ki = mi$

$I = mi + 1$

Recovery $= I - Ki = mi + 1 - mi = 1$

Pd. $= \frac{\text{Recovery}}{K} = \frac{K}{\text{Recov.}} = m$

Stress $\frac{\text{Flow through}}{K}$

1. Cow case

$$\text{Flow} = mi + 1$$
$$K = m$$
$$F/K = i + \frac{1}{m}$$

2. Tree case

$$F = I^m$$
$$K = \frac{I^m - 1}{i}$$

$$F/K = i\,\frac{I^m}{I^m - 1} = \frac{i}{1 - I^{-m}} = 1\,P\,\overline{m}$$

3. Bldg. Case

$$F = m$$
$$K = \frac{1}{i}\left[m - \frac{1 - I^{-m}}{i}\right]$$

$$\frac{F}{K} = \frac{mi}{m - \frac{1 - I^{-m}}{i}} = \frac{i}{1 - \frac{1 - I^{-m}}{mi}}$$

F/K @ $i = .08$

Cow: $i + \frac{1}{m}$

Tree: $\frac{i}{1 - I^{-m}}$

Bldg: $\frac{i}{1 - \frac{1 - I^{-m}}{mi}}$

Bldg / Tree − 1

	$i = .08$	.04	.08	.04	.08	.04	.04
m							
1	1.08	1.04	1.08	1.04	1.080	1.04	0
2	.58	.58	.561	.530	.7~~4~~38	.702	.325
4	.33	.29	.302	.275	.4~~3~~65	.432	.571
8	.205	.165	.174	.149	.284	.253	.698
16	.1425	.103	.113	.086	.1~~2~~79	.147	.709
32	.11125	.078	.087	.056	.124	.091	.625
64	.0925	.056	.081	.044	.099~~8~~	.062	.409

Findings: ~~a worksheet~~

1. With all 3 ~~patterns~~ types, F/K drops steeply w. m
2. F/K is same for all 3 types when $m = 1$. To correct for this, use const. int.
3. F/K otherwise is always highest for Bldgs, lowest for Trees.
4. Differences among types are not very great. The highest ~~when $i = .04$, $m = 16$, $i = .04$~~ is at $m = 16$, and F/K for Bldg. is ~~7~~ 11% higher than for Tree. Max is between 16 and 8.
5. It follows from (4), major diff. is to be found in operating the bldg. Somewhere I have data saying the op'g empluat. in a bldg. = its constrn. cost. Put in this datum, to show the main difference.

Likewise the cow. ~~It~~ Whatever yields

a reg. flow of F needs to be opd. & maintained.

Table 2: Cow, Tree, & Bldg., F/K, using continuous int, $p_x = \ln(1+i)$

	Cow $p + \frac{i}{m}$	Tree $\frac{p}{1-e^{-pm}}$	Bldg $\frac{p}{1-\frac{1-e^{-pm}}{mp}}$	$\frac{\text{Bldg}}{\text{Tree}} - 1$
	$i = .08$			
m				
1	1.077	1.039	2.052	.975
2	.577	.539	1.052	.952
4	.327	.290	.553	.907
8	.202	.167	.304	.820
16	.139	.109	.181	.661
32	.108	.084	.122	.452
64	.093	.078	.096	.231

Findings, as modified:

1) Same

2) F/K of Bldg. is maximally in excess of Tree when m = 1, being nearly double. This is bcs. K is cut in half.

3) Same – ~~but cow is now a lot more like Tree~~

4) Diffs. among types are greater. Bldg. of short life, (or machine, furnishing, fixture, rolling stock) has double the F/K of Tree or Cow. Cow & Tree are much alike.

5.) Same

Lake turnover in Wisc. - 2 to 9 years.

~~Ludwig.~~ ~~Gerardine & Goya~~

~~vested in the verdant verdure~~ of vicinity.

Leopold. ~~I miss ecologist here.~~

~~Only eco-freaks here are ecists~~

Sewage Turnover. Acorn St. treat. plant: 28 days.

~~HW Waukesha 521-5437~~

Inventory of different ages

Year	1	2	3	4	5	6	7	8
Parcel								
I	$20A_{100}$	$20B_{110}$	$20C_{121}$	$20D_{133}$	$15A_{100}$			
II		$8A_{100}$	$8B_{110}$	$8C_{121}$	$8D_{133}$			
III			$22A_{100}$	$22B_{110}$	$22C_{121}$	$22D_{133}$		
IV				$16A_{100}$	$16B_{110}$	$16C_{121}$	$16D_{133}$	

A = Calves, 0 - 1 yr old
B = Yearlings, 1 - 2 ...
C = 2 - yr. olds, 2 - 3 .. ..
D = 3 - yr. olds, 3 - 4 .. ..

20A means 20 calves
$20A_{100}$ .. .. .. worth $100 @ .

Alt. Vintage methods

Year	Age 0-1	1-2	2-3	3-4
1	A			
2	B	A		
3	C	B	A	
4	D	C	B	A

A Means Vintage yr. 1
B " " " 2
C 3
D 4

Alt. vintage methods

Period	Year 1	2	3	4
I	1	1	1	1
II		2	2	2
III			3	3
IV				4

Nos. in table indicate year born, or vintage.

Period	Yr.-end 1	2	3	4
	1	2	3	4
		1	2	3
			1	2
				1

Nos. in table are age at end of yr.

K:O ratio vs life index, L

L > K/O bcs. K is PV of L

But the excess of L over K/O is income, not cap. recovery.

But recov. is limited to K. But the K is spread over n yrs. (n = L)

Thus, take an 80 yr. bldg. @ 7%

a = 1

PV = 14.22

mean recovery pd. = 65.6 yrs.

n	PV	K/R_e	n - PV ↓	K/R	PV + K/R	PV + K/R_e
80	14.22	65.68	65.78	66.1	80.~~29~~ 32	79.80
70	14.16	55.84	55.84	56.33	70.~~49~~ 49	70.00
60	14.04	46.27	45.96	46.77	60.81	60.31
50				37.41		
40				28.58		
30	12.41	19.76	17.59	20.25	32.66	32.17
20		12.19				
10		5.56				
5		2.64				
1	1	.51				1.51

So you come very close to the mean recovery time by just subtracting: $n - PV \cong$ recov. pd.

Try this at 4%.

n	PV	K/R_c	n - PV	PV + K/R_c
80	23.92	58.13	56.08	82.05
70	23.39	49.31		72.70
60	22.62	40.81	37.38	63.43
50	21.48	32.69	28.52	
40	19.79	25.03	20.21	
30				
20	13.59	11.29	6.41	24.88
10				
5	4.45			
1				

Try $i = .12$

n	PV	K/R_c	n - PV ↓	PV + K/R_c
80	8.33	71.19	71.67	79.52
70	8.33	61.20	61.67	69.53
60	8.32	51.24	51.68	59.56
40	8.24	31.61	31.76	39.85
20	7.47	13.49	12.53	20.96
5	3.60	2.73	1.40	6.33

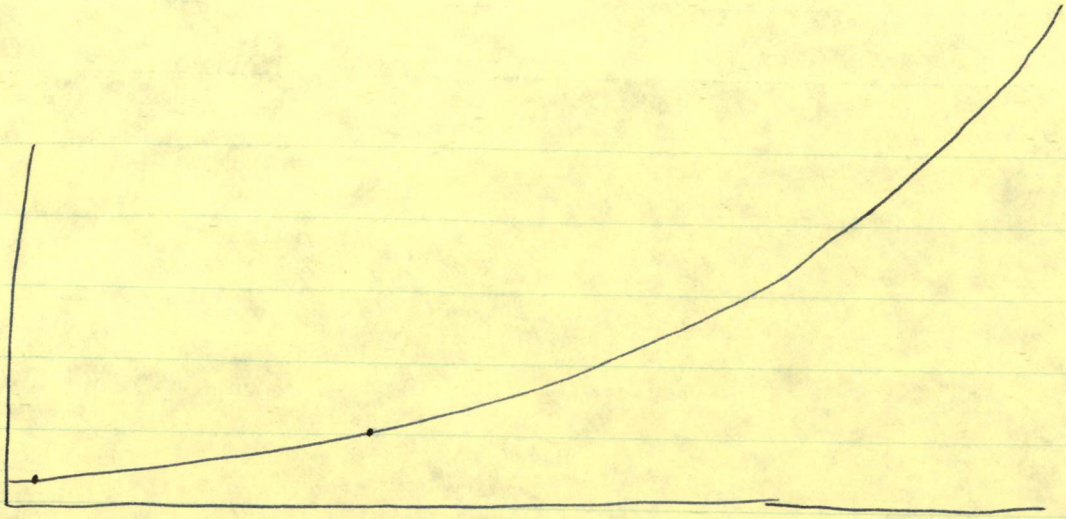
Useful rule.

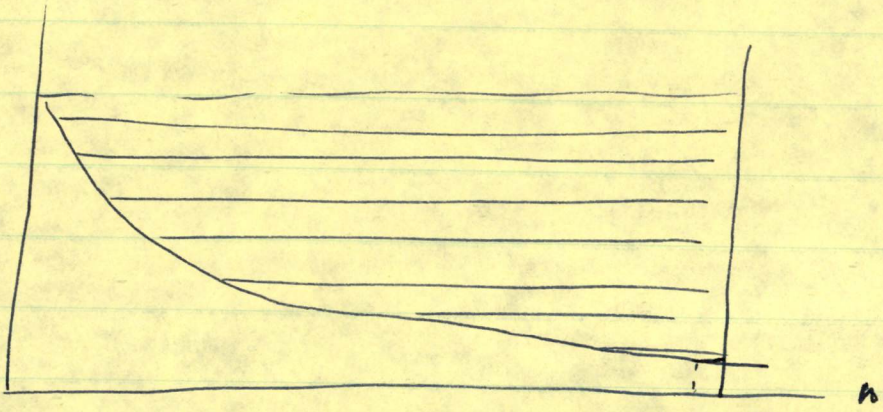
a. For low values of i and n, mean payback is $\frac{n}{2}$ +

b. For higher values, it is n - PV

c. Breaking point is where $n \cdot i \cong 2$

d. Whichever is higher is ~~correct~~ closer.





The mean pd. is a weighted ave. Horiz. values of a, wtd. by ord. of each. That gives you $ · yrs. Then ÷ no. of $, get mean yrs.

But int. is also a mean yrs. -- the vertical summation, same area.

Thus, int. & w. mean pd. That makes sense anyway, independently. If int. is 66 out of every $80 collected; then payback took 66 out of 80 yrs.

Here is why supply of loanable funds responds poorly to Δi!

n	$n/2$	% payback in $n/2$ @ 2%	@ 4%	@ 8%	@ 16%
2	1	49.5	49.02	48.08	46.30
4	2	49.01	48.04	46.16	42.63
8	4	48.01	46.09	42.35	35.58
16	8	46.10	42.22	35.11	23.37
32	16	42.15	34.81	22.73	08.51
64	32	34.50	22.18	8.05	.86

	12%	13%	14%	15%	16%	17%	18%	19%
5							39.80	
10							30.42	
20								
30				15.45		12.29	10.94	
40				9.39		6.78	5.76	

9. PYTN/2

$$\frac{\frac{1-I^{-n}}{i} - \frac{1-I^{-\frac{n}{2}}}{i}}{\frac{1-I^{-n}}{i}} = 1 - \frac{1-I^{-\frac{n}{2}}}{1-I^{-n}}$$

$$= \frac{I^{-\frac{n}{2}} - I^{-n}}{1-I^{-n}}$$

n ~~0~~ 5

i ~~0~~ 6

LRn 57

PD

$i \to \infty$

$n \to 0$

$$\frac{1}{\ln(1+i)}\left[\frac{n\ln(1+i)}{1-(1+i)^{-n}} - 1\right]$$

$$PD = \frac{1}{\ln(1+i)}\left[\frac{n\ln(1+i)}{1-(1+i)^{-n}} - 1\right]$$

K/R e

$i \to \infty$

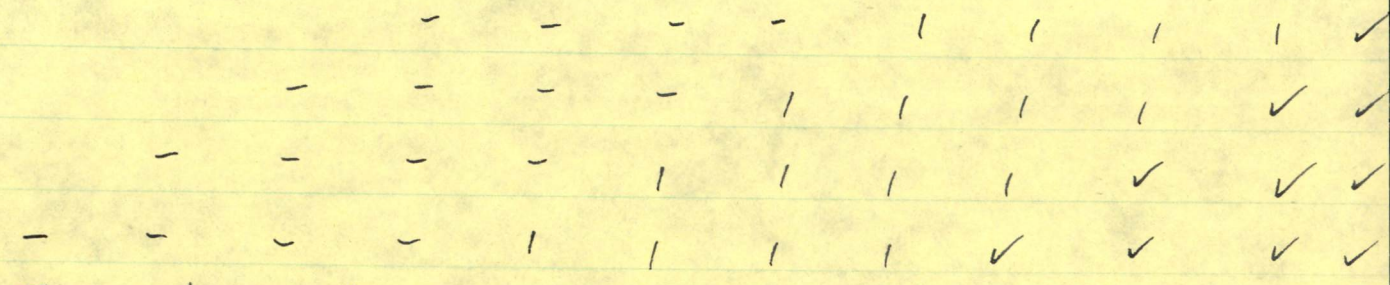
$n \to 0$

$$\frac{n}{1-e^{-n\ln(1+i)}} \quad \frac{-1}{\ln(1+i)} = K/R$$

There 2 programs, evidently arrived at by different routes at different times, are virtually identical.

Normalized models.

A. Inventory, w/o capd. int.



Each dose = 1

Pd = 4 mos. = $\frac{1}{3}$ yr.

Fund = 4 mos. "inflow" = Pd. × flow

In mos., 4 × 1 = 4

In yrs., $\frac{1}{3}$ × 12 = 4

Flow = T × Fund = $\frac{1}{4}$ Fund

In mos., 1/4 × 4 = 1 = Flow/mo

In yrs., 3 × 4 = 12 = Flow/yr.

Pd = Fund/Flow = 4/1 = 4 mos.

T = Flow/Fund = 1/4 = 1/4/mo.

K/O = Fund/Flow = Pd. = 4

This is assuming the beginning "inputs" are v.a. in the opn., hence not output.

In fact, much of these are other things: raw mtls; machine hire; etc. In bldg. constrn, less than 20% is v.a. on the job.

Let w, a %, be the % of flow that is v.a.

on the job. Then output of v.a. = $F \cdot w$. Then:

$$K/O = F_{und}/w \cdot F_{low} = Pd/w$$

B. Normalized Forest, w. Capd. interest

		1	...	I^{n-4}	I^{n-3}	I^{n-2}	I^{n-1}
	1	I	...	I^{n-3}	I^{n-2}	I^{n-1}	1
1	I	I^2	....	I^{n-2}	I^{n-1}	1	I
1	I	I^2	I^3	....	I^{n-1}	I	I^2

Pd. between plantings = n yrs.

$$F_{und} = 1 + I + I^2 + \dots + I^{n-1} = \frac{I^n - 1}{i}$$

Outflow = I^n

Inflow: Planting = 1

Interest = $I^n - 1$

$P + I = I^n$ = outflow

$$F_{und}/Outflow = \frac{1 - I^{-n}}{i} = \text{"Gross period"}$$

$$F_{und}/P = \frac{I^n - 1}{i} = \text{Labor period}$$

n = naive labor period (overlooking capd. int.)

→ P is recovery of K in each yr: $P = I^n - i\Sigma K$

→ $\frac{P}{\text{Fund}}$ = Turnover of labor content = $\frac{i}{I^n - 1}$

If $n = 40$; $i = .072$

$T = .0048$

Pd = 210 yrs.

→ ~~Delete~~ Insert p. 3A

[Share of v.a. by $P = \frac{1}{I^n} = I^{-n}$

Share of v.a. by $K = \frac{I^n - 1}{I^n} = 1 - I^{-n}$]

[Add Terra, T, at $T = P$, and share added by P gets much lower yet.]

C. Normalized depreciating K. $n = 40$

$$\text{Fund} = \Sigma = \frac{1 - I^{-40}}{i} + \frac{1 - I^{-39}}{i} + \dots + \frac{1 - I^{-1}}{i}$$

$$= \frac{1}{i}\left[n - (I^{-1} + I^{-2} + \dots + I^{-40})\right]$$

$$= \frac{1}{i}\left(n - \frac{1 - I^{-n}}{i}\right)$$

Gross Inflow = $\frac{1 - I^{-n}}{i} + i \cdot \text{Fund} = \frac{1 - I^{-n}}{i} + n - \frac{1 - I^{-n}}{i} = n$

Gross Outflow = n

Cap. recovery = $n - i \cdot \text{Fund} = n - n + \frac{1 - I^{-n}}{i} = \frac{1 - I^{-n}}{i}$

After you have built up this normalized complex, the whole costs you so much int. on ΣK each yr. that you only recover $\frac{1}{\frac{I^n - 1}{i}}$ or $\frac{i}{I^n - 1}$ of the Fund each year, as free K, to reinvest. The remainder goes for current income purs

Before this, for n yrs. you are putting K in, getting nothing out.

Following this you can liq. — but only slowly over n yrs.

$$\frac{\text{Gross Flow}}{\text{Fund}} = \frac{n}{\frac{1}{i}\left(n - \frac{1 - I^{-n}}{i}\right)} = \frac{ni}{n - \frac{1 - I^{-n}}{i}} = \frac{i}{1 - \frac{1 - I^{-n}}{ni}}$$

$$\frac{\text{Int Flow at } n=0}{\text{Fund}} = \frac{\frac{1 - I^{-n}}{i}}{\frac{1}{i}\left(n - \frac{1 - I^{-n}}{i}\right)} = \frac{1}{\frac{n}{1 - I^{-n}} - \frac{1}{i}}$$

Same as cap. recovery / Fund

Example, $n = 40$; $i = .072$

$$\text{Fund} = \frac{1}{.072}\left[40 - \frac{1 - \frac{1}{16}}{.072}\right] = 13.89 \times 26.98 = 374.74$$

$$\text{Cap. recovery} = \frac{1 - \frac{1}{16}}{.072} = 13.02$$

$$\text{Turnover} = \frac{13.02}{374.74} = .035$$

Pd. = 28.78 yrs. (a lot shorter than 210, and shorter than 40)

i. Fund = 26.98

n = gross rev. = 40

Share of int. $= \frac{26.98}{40} = 67\%$

Share of $K_0 = \frac{13.02}{40} = 33\%$

Share of v.a. on site = W x .33

Periods (use p for i)

n	i = .01	.02	.04	.08	.16	.32
2	~~1.0~~	1.0	1.0	1.0	1.05	1.1
4	~~2.0~~	2.0	2.1	2.1	2.2	2.4
8	4.1	4.1	4.2	4.4	4.8	5.4
16	8.2	8.4	8.8	9.6	10.9	12.6
	53%	55%	60%	69%	80%	89%
32	16.8	17.7	19.3	22.0	25.5	28.4
64	35.4	38.6	44.2	51.5	57.3	60.4
128	77.2	88.5	103.4	115.0	121.3	124.4

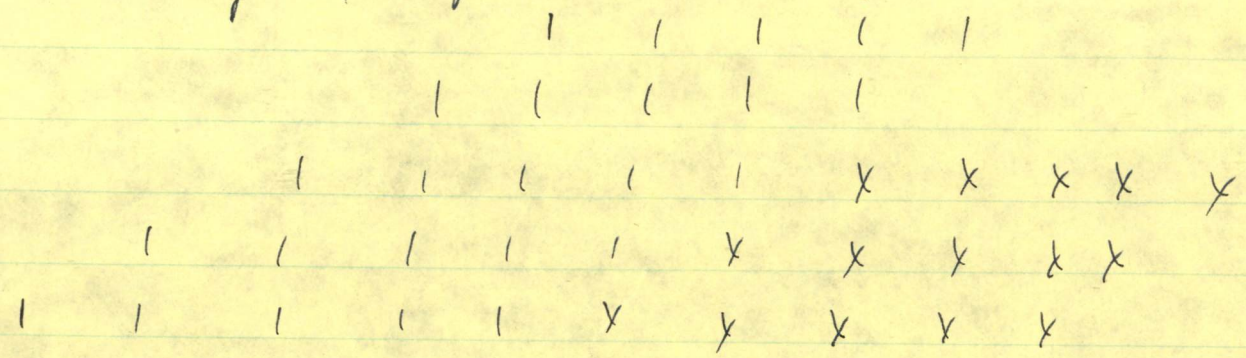
Turnovers

n	i = .01	.02	.04	.08	.16	.32
2	~~1.0~~ .997	~~1.0~~ .993	~~1.0~~ .987	~~1.0~~ .975	~~1.0~~ .953	.91
4	.50		.487	.476	.455	
8	.247		.238	.227	.209	
16	.122		.113	.104	.092	
32	.059		.052	.045	~~.003~~ .039	
64			.023	.019	.017	
128			.010	.009	.008	

Basic Normalized Models.

Purpose: show how often K employs labor

1. Inventory, w/o capd. interest



Pd. = $\frac{4}{4}$ mos. = $\frac{1}{3}$ yr. (make it 4 mos, bcs. 4 = $\frac{12}{3}$, even)

1 = $\frac{1}{4}$K

Fund = $\frac{1}{4}$ = Pd × Flow. (4·1 or $\frac{1}{3}$·12)

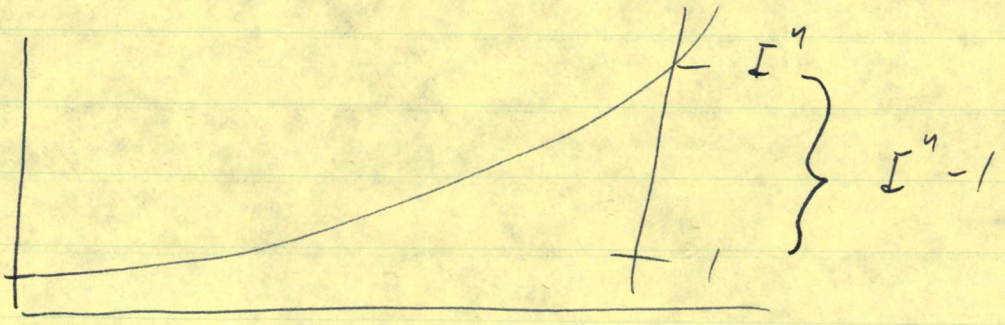
Flow = 1 = Fund × Turnover

Turnover = $\frac{1}{\frac{1}{4}}$ / mo = 3/yr.

K/o = $\frac{1}{4}$ (but v.a. = 0 in this model? No, the 1 is pd. to add value. There is v.a., but no interest $.)

2. Normalized forest. $1+i \equiv I$

$$V = 1 + I + I^2 + I^3 + \dots + I^{n-1} = \frac{I^n - 1}{i}$$



In normalized forest of 60 yr. pd. 7.2%, $I^n = 64$. Sell for 64: 1 is turnover of the labor content.

Yrly. int. is $i \cdot \frac{I^q - 1}{i} = I^q - 1$

3. Normalized indl. plants, $n = 40$

$$\Sigma = \frac{1 - I^{-40}}{i} + \frac{1 - I^{-39}}{i} + \cdots + \frac{1 - I^{-1}}{i}$$

$$= \frac{1}{i}\left[40 - \left(I^{-1} + I^{-2} + \cdots + I^{-40}\right)\right]$$

$$= \frac{1}{i}\left[40 - \frac{1 - I^{-40}}{i}\right]$$

$i = .072$

$$= \frac{1}{.072}\left[40 - \frac{1 - \frac{1}{16}}{.072}\right] = 13.89\left(40 - \frac{.94}{.072}\right)$$

$$= 13.89\left(40 - 13.02\right) = 13.89 \times 26.98 = 374.74 = \Sigma K$$

$$K_0 = \frac{1 - 1.072^{-40}}{.072} = \frac{1 - \frac{1}{16}}{.072} = \frac{\frac{15}{16}}{.072} = 13.02$$

~~$i \cdot \Sigma K = 15/16 = .94$~~

$i \cdot \Sigma K = 26.98$

$\frac{13.02}{40} = .326$ $\quad$ $\frac{26.98}{40} = .675$

C.f. = 40

Bldg. cost = 33%. i.e. $\frac{1}{3}$ goes to pay for K_0.

Int. = 67%

Turnover is cap. recov. ÷ ΣK

$\Sigma K = 374.74$

Recov. = 13.02

$$\frac{13.02}{374.74} = .035$$

$$Pd = \frac{1}{T} = 28.78 \text{ yrs.}$$

More generally:

$$\text{Recovery} = \frac{1 - I^{-n}}{i}$$

$$\Sigma K = \frac{1}{i}\left(n - \frac{1 - I^{-n}}{i}\right)$$

$$T = \frac{R}{\Sigma K} = \frac{\frac{1 - I^{-n}}{i}}{\frac{1}{i}\left(n - \frac{1 - I^{-n}}{i}\right)} = \frac{1 - I^{-n}}{n - \frac{1 - I^{-n}}{i}}$$

$$= \frac{i}{\frac{ni}{1-I^{-n}} - 1} = \frac{1}{\frac{n}{1-I^{-n}} - \frac{1}{i}}$$

$$= \frac{1}{\frac{ni - 1 + I^{-n}}{i(1-I^{-n})}}$$

$$Pd. = \frac{\Sigma K}{Rec.} = \frac{\frac{1}{i}\left(n - \frac{1-I^{-n}}{i}\right)}{\frac{1-I^{-n}}{i}} = \frac{n}{1-I^{-n}} - \frac{1}{i}$$

$= \frac{1}{i}(n \times PF - 1)$. But use $\frac{1}{\rho}\left(\frac{n\rho}{1-e^{-\rho n}} - 1\right)$

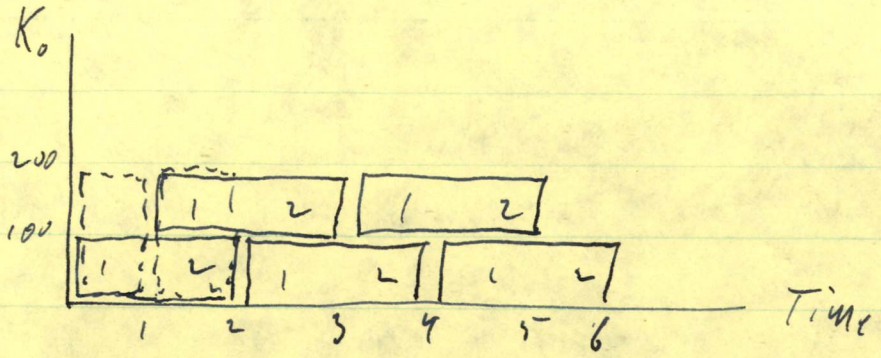
$i \to 0$

$n \to \infty$

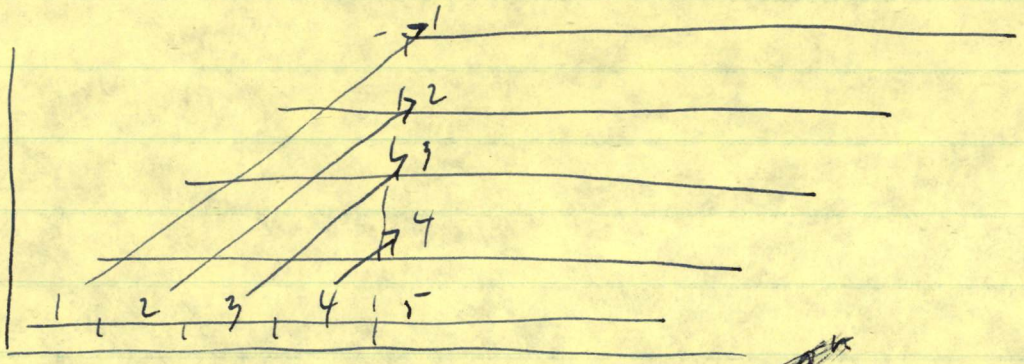
But $e^{-\rho n} = I^{-n}$

$e^{\rho} = I = 1+i$

$\rho = \ln(1+i)$



The whole process exists at one time.



Mult.

$$
\begin{array}{ccccc}
 & & 1 & & \\
 & 1 & .8 & .8^2 & \\
 & 1.0 & .8 & .8^2 & .8^3 \\
1 & .8 & .8^2 & .8^3 & .8^4 \\
1 & .8 & .8^2 & .8^3 & .8^4 \quad .8^5 \quad \cdots
\end{array}
= \frac{1}{1-.20}
$$

$$
\frac{1}{1-.20}
$$

$$
\begin{array}{cccccc}
 & & & & 1 & \\
 & & & 1 & 1.1 & \\
 & & 1 & 1.1 & 1.1^2 & \\
 & 1 & 1.1 & 1.1^2 & 1.1^3 & \\
1 & 1.1 & 1.1^2 & 1.1^3 & \underline{1.1^4} & = \dfrac{1.1^4-1}{.1} \\
 & & & & \| & \\
 & & & & \dfrac{1.1^4-1}{.1} &
\end{array}
$$

$$
\frac{1-1.1^{-4}}{.1} \qquad \frac{1-1.1^{-3}}{.1} \qquad \frac{1-1.1^{-2}}{.1} \qquad \frac{1-1.1^{-1}}{.1} =
$$

$$
\frac{1}{.1}\left[4 - \left(1.1^{-1} + 1.1^{-2} + 1.1^{-3} + 1.1^{-4}\right)\right]
$$

$$
= 40 - 10\left(\frac{1-1.1^{-4}}{.1}\right) = 10\left(4 - \text{ca. } 3.5\right)
$$

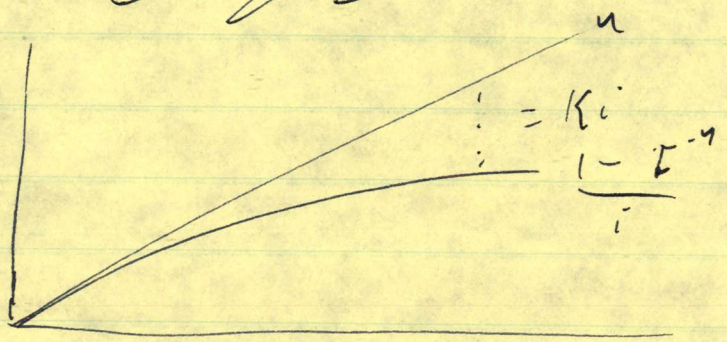
$$K = \frac{1 - I^{-4}}{i} + \frac{1 - I^{-3}}{i} + \frac{1 - I^{-2}}{i} + \frac{1 - I^{-1}}{i}$$

$$= \frac{1}{i}\left[4 - \left(I^{-1} + I^{-2} + I^{-3} + I^{-4}\right)\right]$$

$$= \frac{1}{i}\left(4 - \frac{1 - I^{-4}}{i}\right)$$

$$Ki = 4 - \frac{1 - I^{-4}}{i} = 4 - \frac{1 - I^{-4}}{i}$$

$$1 - I^{-n} = 1 - \frac{1}{I^n} = \frac{I^n - 1}{I^n}$$



$$1 + \frac{1}{1.2} + \left(\frac{1}{1.2}\right)^2$$

$$r = \frac{1}{1.2}$$

$$S = \frac{1}{1-r} = \frac{1}{1-\frac{1}{1.2}} = \frac{1}{\frac{.2}{1.2}} = \frac{1.2}{.2} = 6$$

$$\frac{1}{1.2} = .83$$

$$\frac{1}{1-.83} = 6$$

$$\frac{1}{1-.8} = 5$$

Ec. 100B, Prof. Saffuery

Class notes. Wed. Jan. 28, 1981. 1st installment = 5

Paycheck time, (concluded).

1. Errors, omissions, and clarifications, in notes of Jan. 21.

a. An omission. Put "1st allotment = 4" on heading.

b. An error, p. 3, Eq. (2), should read:

$$K = \sum_{n=0}^{12} (1+i)^n = \text{etc.} \qquad (2)$$

This is because the first term of the progression is 1, rather than $(1+i)$. Remember the formula from which we derived the result:

$$S = 1 + r + r^2 + \dots + r^{(n-1)}$$

We have to watch the end points of the progression, to avoid perky minor errors.

The object of beginning with 1 is to arrive at the simplest possible final form. If we have to or want to begin with $(1+i)$, however, we adjust by multiplying (2) by $(1+i)$. Or, we can add $[(1+i)^{12} - 1]$, which gives the same result.

This is mainly of interest to purists & technicians, and changes nothing of importance.

c. Likewise, Eq. (3) should read:

$$K = \sum_{0}^{n-1} (1+i)^n = \text{etc} \qquad (3)$$

We are now valuing capital at the beginning (not the end) of each week. For cohort #1, the value is 1, not $(1+i)$; etc.

d. Clarification of Equ. (4)

When we divide K by V, the $V = (1+i)^n$ (and not $V = (1+i)^{13}$). I mentioned this in class, but some may have missed it.

e. On p. 7, Eq. (9), I use "S" (Sales) as the divisor. This is the same as "V" (Volume) used in (4) and (7). For consistency, change S to V in (9).

2. A second approach to showing that Payback Period = Capital ÷ Volume ($P = K/V$)

On Jan. 21 we used a "going concern" model to show this. You might ask, what if we don't have a going concern? So let's look at just one cohort of broilers going through its cycle from investment to liquidation.

To avoid imposing a lot of detail on you, we'll just do case 1, as on p. 3 of Installment #4. This is called "PIPO" (Point-Input, Point-Output). The name refers only to inputs other than capital, which are concentrated at one point of time, when $n = 0$. We have seen that there are also regular doses of capital inputs each year, as unpaid interest is plowed back. We'll keep the name "PIPO" to remind us of how easy it would be to make the mistake of thinking the payback period is $\underline{n}$, when really it is on average less than $\underline{n}$.

A picture will help:

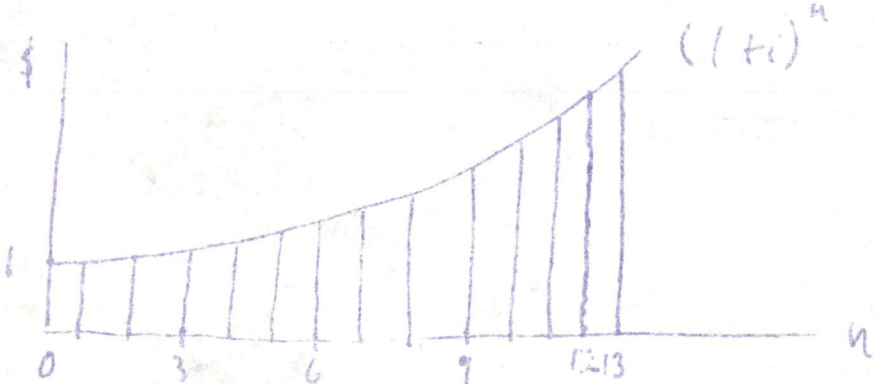


Fig. 1

Eq. (2) is the sum of the columns under the curve. In Eq. (4) we divided this sum by the last column (the right side of the last column,

to be exact, or $(1+i)^{13}$.) We said this ratio of K/V was the average payback period.

The sum of the columns under the curve is the area under the curve. This area can also be calculated as the sum of horizontal bars under the curve, where each bar is one dose of input times its respective payback period, or k·n; (the subscript "i" here is not the interest rate; it means the individual unit.)

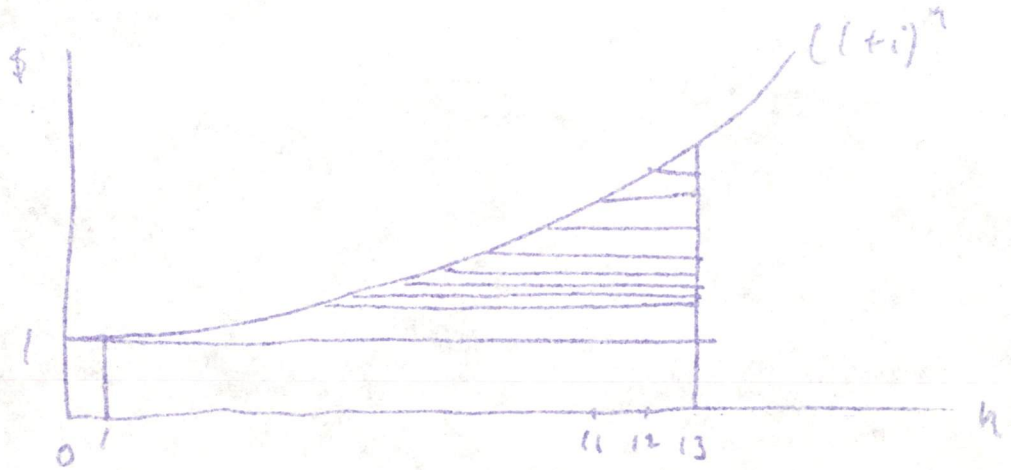


Fig. 2

I have left out the first few bars — they are too close together to draw, except for the very bottom one. [The little triangle in the top right is excluded, because our summation formula values capital only at the beginning of each year. In integral calculus they correct for this minor inaccuracy. Here, we

accept the ~~crude~~ approximation based on columns with squared-off tops -- it is plenty accurate for most purposes.]

Now observe that the area under the curve is the sum of the horizontal bars, or $\sum_{i=0}^{12} k_i n_i$, where i is the date of each dose of capital, and n_i is its individual payback time. We already know the area under the curve, so we have:

$$\sum_{i=0}^{12} (k_i n_i) = \frac{(1+i)^{13} - 1}{i} \qquad (10)$$

We can also see that $\sum_{0}^{12} k_i$ is the height of the last column, which we know is $(1+i)^{13}$

$$\sum_{0}^{12} k_i = (1+i)^{13} \qquad (11)$$

(10) is the sum of payback periods weighted by the amount of capital with that period. (11) is the sum of all capital. Dividing

(which is the sum of the weights)

(10) by (11) therefore gives the average payback period of all the capital:

$$\frac{(10)}{(11)} = \frac{\sum_{0}^{12}(k_i \cdot n_i)}{\sum_{0}^{12} k_i} = \frac{(1+i)^{13}-1}{i(1+i)^{13}} = \frac{1-(1+i)^{-13}}{i} \qquad (12)$$

But (12) is the same as (4). Q.E.D.
The average payback period equals the capital/output ratio, or K/V.

3. Shorter periods are more labor-intensive.

In Eq. (1), p. 3, Installment #4; and Fig. 1, p. 3, Installment #5, we see that output is $(1+i)^{13}$, or generally $(1+i)^n$.
In the PIPO case, labor is applied once, when $n=0$, and is equal to \$1 in value. Labor's share of output is therefore:

$$\frac{\text{Wage Bill}}{\text{Output}} = \frac{1}{(1+i)^n} = (1+i)^{-n} \qquad (13)$$

Capital's share is:

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Note that capital's share increases with n; labor's share falls ~~with~~ as n rises. This is not because the rates of wages or interest change: they don't. It is because capital adds a higher share of the value of output as n rises.

We can now perceive the power of

short payback periods to employ the labor in two ways:

a) Shorter payback means more payoffs are advanced by the same capital;

b) Shorter payback means production is more labor-intensive.

4. Effect of higher interest rates on choice of payback periods.

The interest rate is the economic constraint or discipline that makes us economize on capital when it is scarce. The following diagrams illustrate how this works. We will discuss them in class.

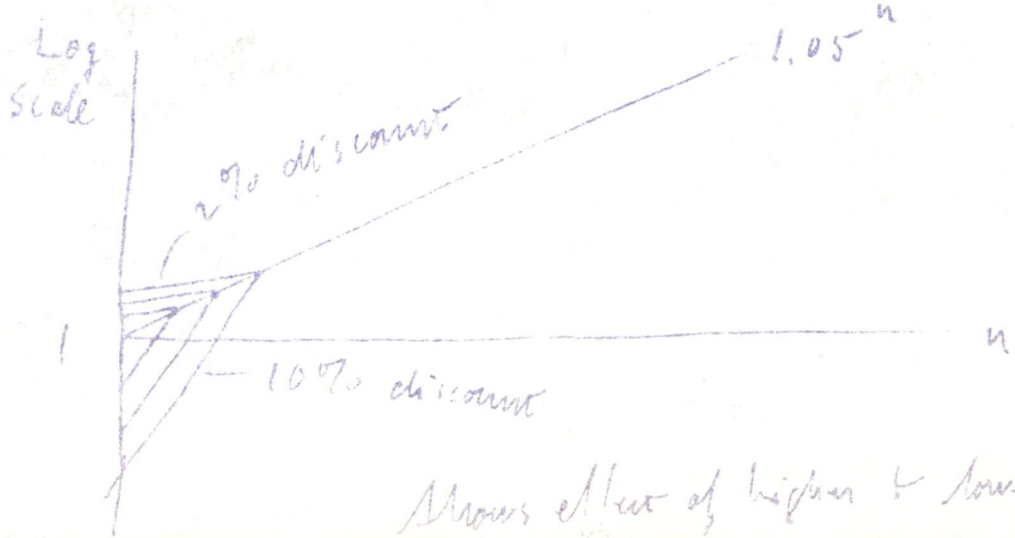


Fig. 3

Shows effect of higher & lower i.



Fig. 4

Shows effect of higher wage rates

Ec. 100B, Prof. Jeffrey

Class notes, Wed. Jan. 28, 1981. Installment #5

Payback time, (concluded).

1. Errors, omissions, and clarifications, in notes of Jan. 2nd.

a. An omission. Put "Installment #4" on heading.

b. An error, p. 3, Eq. (2), should read:

$$K = \sum_{n=0}^{12} (1+i)^n = \text{etc.} \qquad (2)$$

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Payback Time (concluded), p. 2

This is mainly of interest to purists & technicians, and changes nothing of importance.

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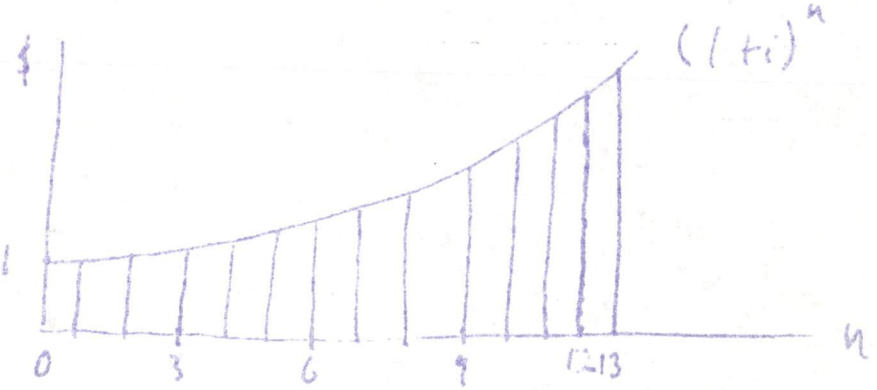


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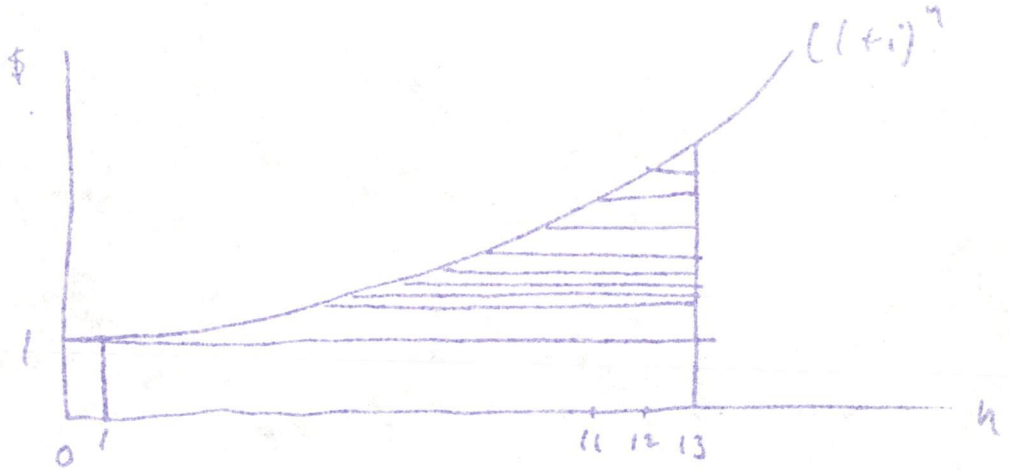


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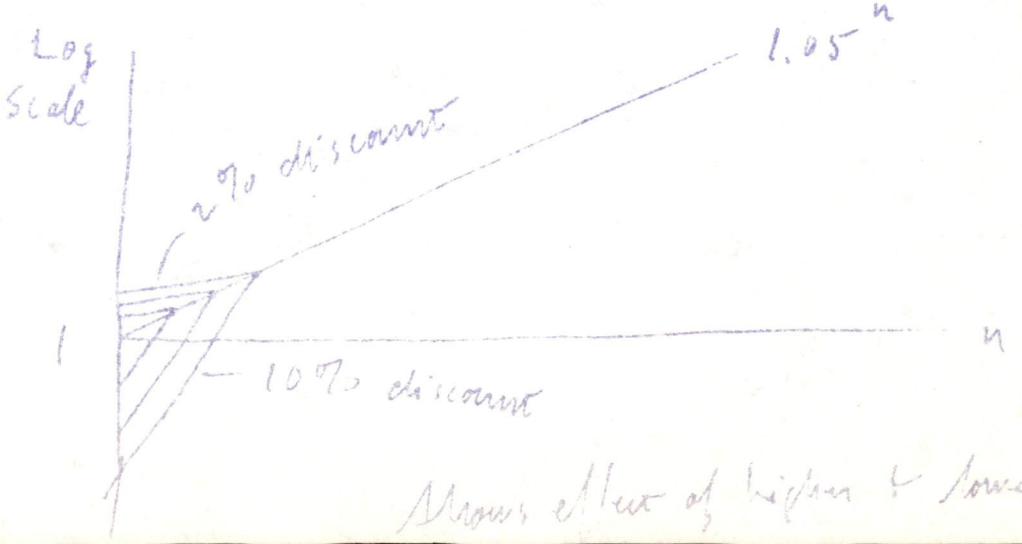


Fig. 3

Shows effect of higher & lower i.

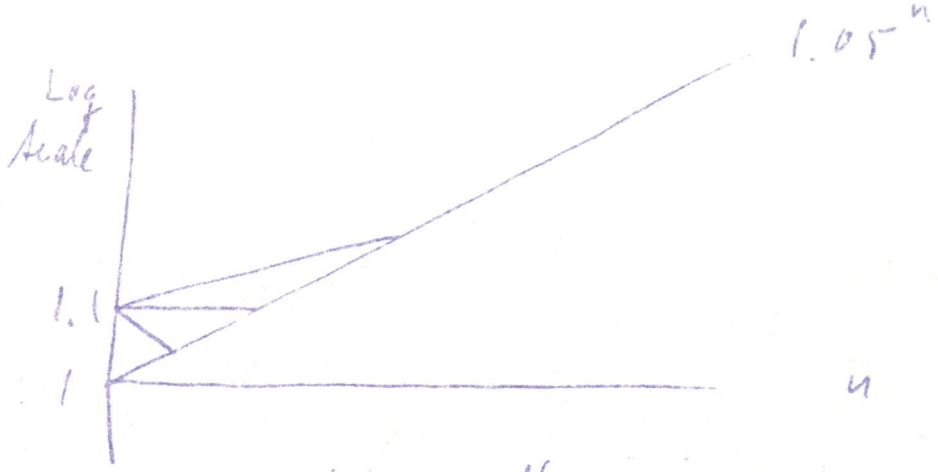


Fig. 4

Shows effect of higher wage rates