



Present Value of Periodic Capital Costs

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PRESENT VALUE OF PERIODIC CAPITAL COSTS
Example of cost of undergrounding water tank¹

Capital costs in perpetuity

Let K_0 be the capital cost of a tank
 i = interest rate
 $I = 1+i$
 n = number of years before replacement
 PVKP = Present Value of K_0 repeated in Perpetuity every n years

$$PVKP = \frac{K_0}{1 - I^{-n}} \quad (1)$$

PVKP is found by multiplying K_0 times the "perpetuity multiplier." This multiplier may be found by summing the geometrical progression $[1 + I^{-n} + I^{-2n} + \dots + I^{-\infty}]$.

$$1/(1-I^{-n}) = \text{"the perpetuity multiplier"}$$

Table I: Some values of the perpetuity multiplier

<u>i</u>	<u>n=</u>	<u>20</u>	<u>40</u>	<u>60</u>	<u>--</u>	<u>100</u>
.03		2.24	1.44	1.20		1.05
.04		1.84	1.26	1.11		1.02
.05		1.60	1.17	1.06		1.01

.10		1.17	1.02	1.00		1.00

Let: A_0 = Capital cost of Aboveground tank
 B_0 = Capital cost of Belowground tank
 ($B_0 > A_0$)
 a = life of aboveground tank

¹These notes were originally written to help a friend engaged in a land-use fight. He wanted to force a developer to underground an eyesore water tank. The developer objected to the higher capital or "up-front" cost that would result. My friend wanted help showing that the cost of undergrounding the water tank was not much more than the cost of the eyesore tank, when you considered the longer life of the underground tank. Hence, you will note bits of advice offered to help him make his case in an adversarial arena. I've shorted some points the other side might make.

b = life of belowground tank
 $(b > a)$

$$PVAP = \frac{A_0}{1 - I^{-a}} \quad (2)$$

$$PVBP = \frac{B_0}{1 - I^{-b}} \quad (3)$$

Example

This example uses a real i.r. = .04. We use a real rather than a nominal i.r. because Equation (1) assumes that K_n etc. are identical with K_0 . That means we are measuring in constant \$, not nominal ones. To be consistent with that we measure the i.r. in real terms, also.

Let $a = 40$
 $b = 100$
 $i = .04$

Then $PVAP = 1.26 \times A_0$
 $PVBP = 1.02 \times B_0$

So, to account for cost of replacement of tanks in perpetuity, add 26% to A_0 , and 2% to B_0 .

$PVAP > PVBP$ whenever

$$1.26A_0 > 1.02B_0$$

or

$$A_0/B_0 > .81$$

Other factors

Operating and Maintenance expenses (o&m). The aboveground tank requires higher o&m, beginning from an early date. Regard them as an annual flow of costs beginning at $n=0$. (The formulas assume the costs of each year are concentrated at the end of each year.) Take the PV of this flow of expenses by "capitalizing in perpetuity," which is done simply by dividing

the flow by i . In the example that means multiply by 25. Add this PV to A. This is a form of "life-cycle costing" (although in the usual dialogue this is done inversely, by annualizing the capital costs -- and for just one cycle).

Since you are multiplying by 25, it is important to get a firm estimate of o&m costs. This is a project in itself, and includes lots of hidden items like overhead, fringe benefits, worker insurance, payroll taxes, etc.

Cost of visual blight. This, too, begins immediately. It is measurable conceptually as the reduced net service flow of affected lands, especially amenity lands like parks and residential.

Unlike o&m expenses, blight costs rise in real \$ in perpetuity. O&m costs rise during the course of each cycle, true, but we have accounted for that indirectly by assuming a short life for aboveground tanks. (In a longer model you could handle that more directly and accurately.) Blight costs rise forever, from cycle to cycle, as the impacted lands are settled, and, rise in real value, reflecting the fact that more and more people are affected by the eyesore.²

Let c = annual cost of blight, year #1
 g = growth of c , annual, as a fraction
 PVc = PV in perpetuity of cost of blight

$$PVc = c/[i-g] \quad (i > g) \quad (4)$$

With g reducing the denominator, you see the possibilities for pushing this figure very high. However there is a real world measure to prevent exaggeration, and that is the selling price of land.

The value of this point is when your adversary claims that speculative land prices overstate the actual value of land, because they are based on assumed future service flows higher than current ones. A common expression for this is that the P/e ratio of land is very high (P/e = price/earnings ratio). Your

²Note to students. This may seem trivial to you now, compared with the worries you have about careers, choosing a mate, where to live, affording a home, etc. However, when you start earning a living you have to consider the crotchets of older people, too, because "they've got all the dough."

answer is that the PVc/c ratio is also very high, and for the same reason: each is equal to $1/[i-g]$.

So you may measure the PVc by the decline in the current value of land. With residential and parkland the annual "service flow" is often hard to measure, or even explain to doubters. But current land prices are familiar; and they are the valid relevant measure.

With parkland you can push this a bit and claim a higher likely value for g , because of growing scarcity as population rises and other parks are neglected.

Relative weight of infinite future replacements

The PV of the first replacement, K_n , is more than half the PV of the infinite chain, for most civil works. As a result, most analysts neglect the rest of the infinite chain. But the rest of the chain is not negligible; and it strengthens your case to measure and include it. Let us find the ratio of PVK_n (replacement #1) to $PVKP \times I^{-n}$ (all replacements beginning from and including #1, but excluding K_0).

$$PVK_n = K_n I^{-n} \quad (5)$$

$$PVKP \times I^{-n} = K_n \frac{1}{I^n - 1} \quad (6)$$

$$PVK_n / PVKP \times I^{-n} = (I^n - 1) / I^n = 1 - I^{-n} \quad (7)$$

This ratio is the reciprocal of that shown in Table 1. Invert it, and you can use Table 1 (inverted, it represents the same thing, basically, n years later). At 3%, replacements after the first one add 44% to the value of all replacements, assuming 40-year life; and 5% to the value of all replacements assuming a 100-year life.

Comparisons using annualization

Another way of comparing the cost of works with different lives is by annualization. Capital costs with interest are

leveled out. There are two basic methods, which are really the same to a mathematician, but require different handling.

Sinking Fund Factor (SFF)

a = annual deposit into sinking fund

K_n = target sum to accumulate after n years

$$a = K_n \frac{i}{I^n - 1} \quad (8)$$

For $n = 100$, a is obviously much smaller than for $n = 40$. The comparison is not fair, and greatly exaggerates the benefit of deferred replacement, but could be used a), to exaggerate deliberately, for effect; b), to communicate with a group whose mindset forces you to express everything in annual terms (like people who think only in terms of their monthly payments, not the true total cost); to counter an adversary who bases comparisons on the IPF (below).

Installment Plan Factor (IPF)

This gives the level annual payment, a , required to recover K_0 over n years, with interest on the unrecovered balance at all times. In explaining it, premise that the tank is 100% debt-financed (as it probably will be). You are figuring the annual debt service cost: the levelized cost to retire the principal, and pay interest on the unrepaid balance each year. IPF is much less sensitive to n and i than is SFF, because it is based on paying off a present sum, rather than accumulating to pay off a future one.

$$K_0 \times \text{IPF} = \frac{K_0 \times i}{1 - I^{-n}} \quad (9)$$

Table 2: Values of IPF (%)

<u>i</u>	<u>n=</u>	<u>20</u>	<u>40</u>	<u>60</u>	<u>100</u>
.03		6.72	4.33	3.61	3.16
.04		7.36	5.05	4.42	4.08
.05		8.02	5.83	5.28	5.04
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.10		11.75	10.23	10.03	10.00

The fractional change in IPF going from $n = 100$ to $n = 40$ is the same here as it is for the perpetuity multiplier. However a person using IPF is more likely to use a nominal instead of a real i.r., with results you see in the last row where $i = .10$. As you see, this appears nearly to eliminate the advantage of lower periodic debt service payments otherwise attached to the longer-lived capital asset. Your adversary is then ready to stress that you pay nearly as much per year, and pay it 60 years longer! He can total up the sum of all payments and compare A with B, to the great disadvantage of your case. Then you have the burden of explaining why they should use a real i.r., which at the moment seems to me harder here than with future replacements. Even then, the sum of payments will be much greater if you finance over 100 years.

If your adversary forces you into this position, you'd be better off to finance both tanks over 40 years, and just admit that yours will cost more per year. Then you can take another approach: stress that with the underground tank, the community will then have an asset to last another 60 years. Point out that they pay 20% more, and get 150% more years of service. For an even bigger number, tell them your tank lasts 250% as long as the other one. Keep them from thinking about interest rates at all, if you can: they work against your case. If they must think of interest, get them to use real, rather than nominal rates.

Who Really Pays?

At some point, if you're prevailing, ask your own conscience if you want to impose this extra capital cost on poor home buyers. They do have interest payments to make, and some can only borrow at high rates. Paying off lenders does not really eliminate interest costs, it just shifts them from explicit to implicit form. That means that the ratepayers or taxpayers - whoever pays off the bonds - now have an equity interest in the capital. They have put their own capital into

it, even though at the same time they may be - some of them surely will be - borrowing at 19% on credit cards, and other forms of high-cost credit. The poorer they are, the higher the rates they probably pay. It may ease your conscience a bit to realize that much of the cost is not shifted to buyers (although everyone will tell you it is, by Pavlovian reflex). Actually, if you get in early, the cost is shifted to and borne by the original land seller: the developer will pay the seller less.