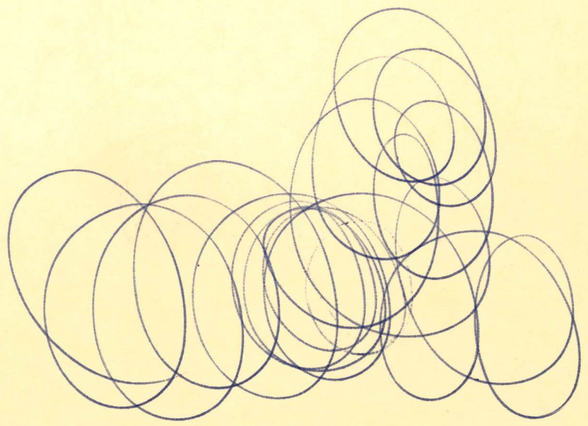
Ec. 162. Tools of Opt.; Sensitivity



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$F = e^{in}$

$\ln F = in$

$\frac{d \ln F}{dn} = i$

$\frac{F'}{F} = i$

$F_n' = Fi$

$F_i' = Fn$

$P = k a^{\alpha}$

$\ln P = \ln K + \alpha \ln a$

$\frac{d \ln P}{da} = \alpha$

$\frac{P'}{P} = \alpha$

$$\epsilon = \frac{P'/P}{1/a} = a \frac{P'}{P} = \frac{P'}{P/a} = \frac{MP}{AP}$$

So, if $\frac{MP}{AP}$ remains const. over some range, use $P = k$

John Twiddy, Med. absences 13 & 15

$i = .063$

$S = .9407 + .8850 + .8325 = 2.658244406$

a) 13.334

b) $13.334 - 11.200 = 2.138$

$$P = k\, a^{\alpha}\, b^{\beta}$$

$$\ln P = \ln k + \alpha \ln a + \beta \ln b$$

$$\frac{P'}{P} = \alpha \frac{a'}{a} + \beta \frac{b'}{b}$$

$$\frac{de^{ix}}{dx} = i \times e^{ix}$$

$$\frac{de^{-ix}}{dx} = -i \times e^{-ix}$$

$$= \frac{-i}{e^{ix}}$$

Eco. 162 Intro

Mon

1. Ideas and skills that do not obsolesce
   Apply all places, all times.
   Transcend tech.
   Comp. Interest, e.g.

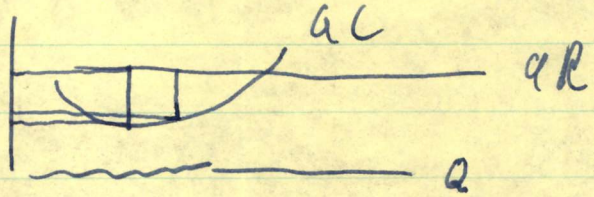
2. Test by experience.
   Apply to solve real problems.

3. Learn how to learn

4. Do problems

5. a. Graph.
   b. Calc. (programmable)
   c. Num.
   d. Tabularly

6. A. Mgmt. as optzn.. 2 or more variables interact.
   1) E.g. Mgrl Acctg.



✓

Max (AR - AC) × Q

✓ 2) Performance std.
   a. Assure solvency, survival
   b. Max value over time
   c. Avoid fallacies.

Ex. of optimizing.

a. Subtracting

TR - TC

~~Expr~~ Rev - Exp = CF

CF - Depr. = Inc.

Inc - Int. = Nett.

b. Mult.

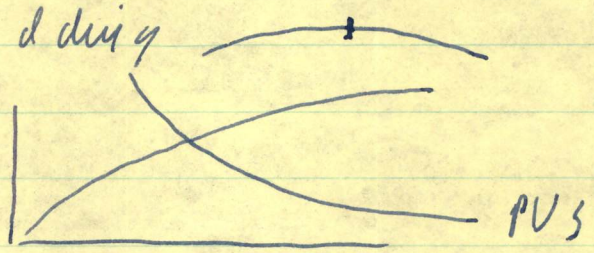
PQ = Rev. (gross)

c. Dividing.



$\frac{S}{I^n} = PV$

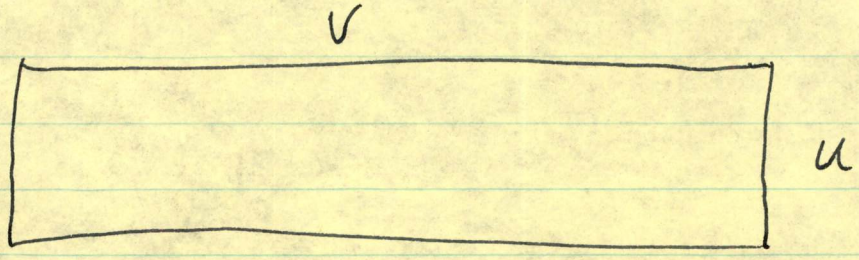
d. Adding



e. Subtracting & multiplying.

Gain per unit x Q = (Price - AC) x Q

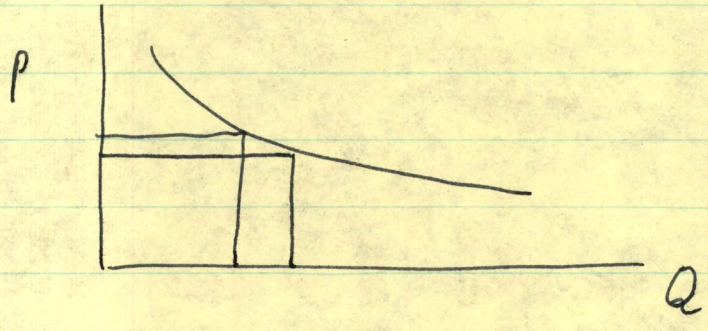
Ec. 162 Non-fuck



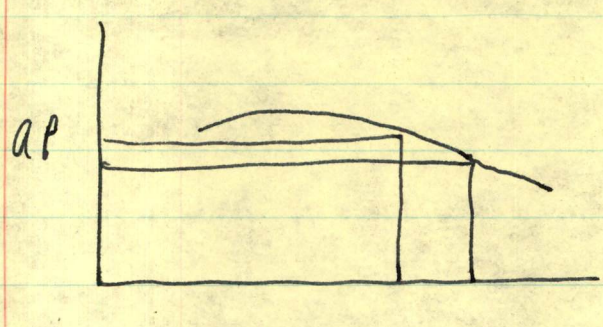
$$(uv)' = u'v + v'u = 0$$

$$u'v = -v'u$$

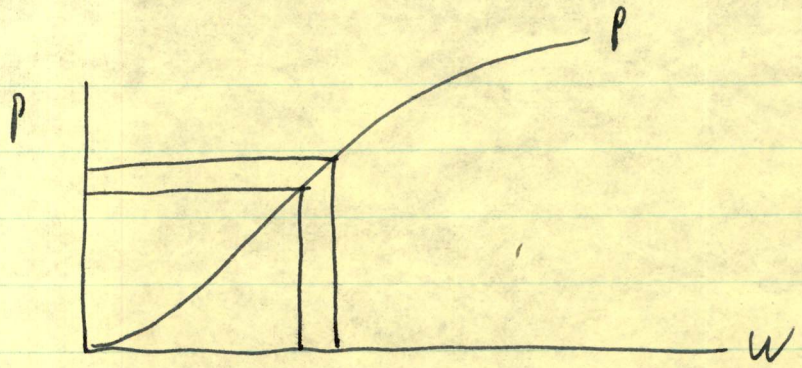
$$\frac{u'}{u} = \frac{-v'}{v}$$



$$-\epsilon = \frac{Q'/Q}{P'/P}$$



$$\epsilon \text{ of produc.} = \frac{P'/P}{W'/W}$$



$$P = aP \cdot w$$

$$P' = w' \cdot aP - (aP)' \cdot w$$

~~$(aP)' =$~~

$$aP = \frac{P}{w}$$

$$(aP)' = \frac{P'w - w'P}{w^2} = \frac{P'}{w} - \frac{w'}{w}\frac{P}{w}$$

$$P' = w' \cdot \frac{P}{w} - \left(P' - w'\frac{P}{w}\right) = 2w'\frac{P}{w} - P'$$

$$2P' = 2w'\frac{P}{w}$$

$$P' = w'\frac{P}{w}$$

$$\frac{P'}{P} = \frac{w'}{w}$$

$$P' = P\frac{w'}{w}$$

Const. returns ?

Ec. 162 Non-fuel.

$a(1+x)\ b(1-x) = ab(1+x-x-x^2) = ab(1-x^2)$

$$\frac{A(1+a)-A}{A} = -\frac{B(1+b)-B}{B}$$

$$\frac{aA}{A} = \frac{-b(B}{B}$$

$$a = -b$$

Measures, Wt. & Vol.

Weight

avoirdupois oz. = 1/16 lb. = 23.3495 grams. (Webster)
troy & apoth. oz. = 1/12 lb. = 31.1035 "

Betrazomatic oz. = $\frac{400}{14.1}$ = 28.369 "
(Funk & Wagnall same as Benz!)

Volume

Fluid oz. = 1/16 pt. = 29.6 c.c. U.S.
1/20 imp. pt. = 28.4 cc

1 cc = .061 c.in.
29.6 cc = 1.806 ci = FL. OZ
16 FL OZ = PT. = 28.890 c.i.

Powers & elasticities.

1. For power functions, the exponent is ϵ.
E.g. $y = x^3$, $\epsilon = 3$.

2. There is one exponential fn., $y = e^x$, whose exponent is also elasticity: $\epsilon = x$.

$$y' = e^x$$
$$y'/y = 1$$
$$\frac{y'/y}{1/x} = x$$

3. For $y = e^{rx}$, $\epsilon = rx$. (If x varies. Cf. 6)

4. For other exponential fns, $y = B^x$, $\epsilon = x(B-1) = xr$
$(B = 1+r)$

$$y = (1+r)^x$$
$$y' = \ln(1+r) \cdot (1+r)^x$$
$$y'/y = \ln(1+r) \doteq r$$
$$\frac{y'/y}{1/x} \doteq xr$$

5. Thus, whether exponent is fixed or variable, it gives you ϵ.

6. $y = e^{rx}$ is symmetrical in r & x.

∴ if r varies, $C = rx$ still holds.

7. But $(1+r)^x \doteq e^{rx}$

Showing that r & x are symmetrical here, too.

$$y = (1+r)^x$$

$$\frac{dy}{dr} = x(1+r)^{x-1}$$

$$y'/y = \frac{x}{1+r}$$

$$\frac{y'/y}{1/r} = x\frac{r}{1+r} \doteq x \ln r \doteq xr$$

2 cylinders. Effect of symmetry.

	A	B	
Height, h	10"	6.75"	
Circ., c	$9\frac{1}{2}$"	12.75"	
~~Vol., V ≠~~			
Area, $A = \frac{c^2}{4\pi}$	7.182	12.936	sq. in.
Vol., $V = A \cdot h$	71.820	87.320	cu. in.
Surface, S ≠	109.364	111.935	
~~S/A ★~~			
S/V ★	1.523	1.282	
Listed contents	26.7 FL. OZ. = 1 PT 10.7 FL. OZ.	16.4 OZ = 1.02 LB (Wt.)	

(a 2d cylinder same Wt., is 14.1 OZ (400 grams))

≠ $S = c \cdot h + 2A$

★ $S/V = \frac{ch + 2A}{A \cdot h} = \frac{c}{A} + \frac{2}{h} = \frac{4\pi}{c} + \frac{2}{h}$

$V_0 = A \cdot H$

$V_1 = A(1+\dot{a})\,H(1+\dot{h})$

$= AH(1 + \dot{a} + \dot{h} + \dot{a}\dot{h})$

Calc. is a technique for eliminating the last term, $\dot{a}\dot{h}$.

$\frac{d(AH)}{dx} = AH' + HA' = V'$

$V_1 = V_0 + V' = AH + AH' + HA' = AH\left(1 + \frac{H'}{H} + \frac{A'}{A}\right)$

A 10% Δ in the smaller of A or H has as much effect as a 10% Δ in the larger, but costs less. (Begin w. a square, where all is more obvious.)

Goals:

a. Show functions as elasticities.

b. Show optimizing, relate to.

1. Adding, PV of bldg. + PV of land

2. Subtracting, costs from revs.

3. Multiplying

base × ht.

~~labor~~ price × ~~qty~~. Q

K · Turnover

4. Dividing:

Discounting, max PV

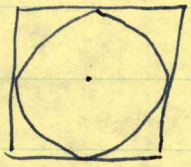
5. Powers

$(1+i)^n = F$. (Note that F is almost equally sensitive to i + n; but volume is much more sensitive to n.

Internal r of r. $r = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1$

c. Sensitivity analysis.

Gain + loss from using round containers.
(or pivot irrign.)



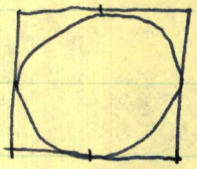
Area of square is B^2 = as
" " circle is $\pi \frac{B^2}{4}$ = ac

$$ac/as = \frac{\pi B^2/4}{B^2} = \frac{\pi}{4} = .785$$

Wasted space = 1 − .785 = .215 of square.

Is it worth going to square container?
Depends on value of shelf space vs. added cont. You are creating .215/.785 = .273 more space, by adding $\left[4B/\pi B - 1\right] = \frac{4}{\pi} - 1$ ~~to circ.~~ = .273 to circ. ~~So?~~ Can that be so?

	Area	Perimeter	P/a
Sq	B^2	$4B$	$4/B$
Circle	$\pi B^2/4$	πB	$4/B$



Take one corner of sq.

$\Delta P = B - \frac{1}{4}\pi B = B\left(1 - \frac{\pi}{4}\right)$

$\Delta A = \frac{1}{4}\left[B^2 - \pi\frac{B^2}{4}\right] = \frac{B^2}{4}\left(1 - \frac{\pi}{4}\right)$

$\Delta P/\Delta A = \frac{B\left(1 - \frac{\pi}{4}\right)}{B^2/4\left(1 - \frac{\pi}{4}\right)} = \frac{4}{B}$

Take whole sq. + circle.

$\Delta P = \cancel{B^2 - \pi B = B(B - \pi)}\ 4B - \pi B = B(4 - \pi)$

$\Delta A = B^2 - \pi\left(\frac{B}{2}\right)^2 = B^2\left(1 - \frac{\pi}{4}\right)$

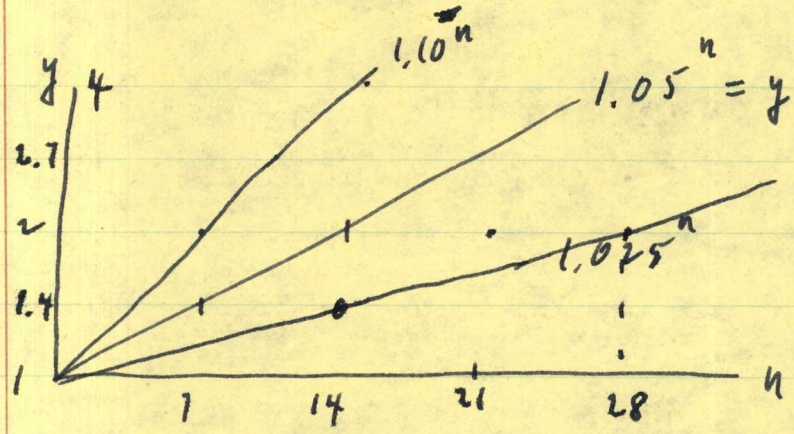
$\Delta P/\Delta A = \frac{B(4-\pi)}{B^2\left(1 - \frac{\pi}{4}\right)} = \frac{4}{B}$

$\Delta P/P = \frac{B(4-\pi)}{\pi B} = \frac{4B}{\pi B} - 1 = \frac{4}{\pi} - 1 = .273$

$\Delta A/A = \frac{B^2\left(1 - \pi/4\right)}{\pi B^2/4} = \frac{4}{\pi}\left(1 - \frac{\pi}{4}\right) = \frac{4}{\pi} - 1 = .273$

amy

Logs + sensitivity



What can we tell about sensitivity from above graph?

1. Sens. of y to n Δs w. r, rate of growth.

2. " " y to r Δs w n

Compound powers + sensitivity analysis

$$y = a x^{\alpha} z^{\beta}$$

$$\frac{dy}{dx} = a z^{\beta} \alpha x^{\alpha - 1}$$

$$\frac{dy/dx}{y} = \alpha / x$$

$$\frac{y'/y}{x'/x} = \alpha$$ as before. "ϵ of prodn." $= \frac{MP}{aP}$

Ditto for β

$\alpha + \beta =$ % returns to scale

(Problem for me: how write above eqn. to show substn. between $x + z$?)

Let W = Work
L = Land
K = Kapital
y = output

$$y = a W^{\alpha} L^{\beta} K^{\gamma}$$

1. If have L of "hi eff. + low cap.", then $MP_{w} << aP_{w}$, so α is small.

In conclusione

$$A = \ln B$$

$$A(1+a) = \ln[B(1+b)] = \ln B + \ln(1+b)$$

$$1+a = 1 + \frac{\ln(1+b)}{\ln B}$$

$$a \ln B = \ln(1+b)$$

$$B^a = 1+b$$

$$b = B^a - 1$$

$$a = \frac{\ln(1+b)}{\ln B}$$

$$e^A = B$$

$$e^{A(1+a)} = B(1+b)$$

$$e^{A+aA-A} = 1+b$$

$$e^{aA} = 1+b$$

$$aA = \ln(1+b) \doteq b$$

$$R = k\, W^{\alpha}$$

$$\frac{\frac{dR}{dw}}{R} = \frac{k\,\alpha\, W^{\alpha-1}}{k\, W^{\alpha}} = \alpha\, W^{-1} = \frac{\alpha}{w}$$

$$\ln R = \ln k + \alpha \ln w$$

$$\frac{d \ln R}{dw} = \alpha \frac{w'}{w} = \frac{\alpha}{w}$$

2. MP might be hi, but MC also hi, so beware. MP etc. do not tell us everything.

3. This is horizontally integrated analysis, i.e. instant. in time.

4. If have bldg. expanded into ∴ of "low quality", this implies kind of hi capacity, $MP_K \leq AP_K$, so γ is hi.

5. If ϕ mix is heavy on K, p. 6 is not enough to tell you that s_i will force bigger changes on you. # 1-4 above are gropings for this, but ? need more or other tools.

Circ. as fn. of area (cost as fn. of capacity)
(Shows "mgnl. ~~fin~~ l.r. fixed ~~area~~ cap. cost"

$C = 2r\pi$

$A = \pi r^2$

$C = 2\pi r = 2\frac{\pi r^2}{r} = 2A/r$

$$\frac{dC}{dA} = 2\left[\frac{A}{r}\left(\frac{A'}{A} - \frac{r'}{r}\right)\right] = 2\frac{A}{r}\left(\frac{1}{A} - \frac{r'}{r}\right)$$

$$r = \left(\frac{A}{\pi}\right)^{\frac{1}{2}} = \pi^{-\frac{1}{2}} A^{\frac{1}{2}}$$

$$\frac{dr}{dA} = \pi^{-\frac{1}{2}} \tfrac{1}{2} A^{-\frac{1}{2}}$$

$$\frac{r'}{r} = \frac{\frac{1}{2} A^{-\frac{1}{2}}}{A^{\frac{1}{2}}} = \frac{1}{2A}$$

$$\frac{dC}{dA} = \frac{2A}{r}\left(\frac{1}{A} - \frac{1}{2A}\right) = \frac{2}{r}\left(1 - \frac{1}{2}\right) = \frac{1}{r}$$

$$\frac{dC/dA}{C} = \frac{1}{2\pi r^2} = \frac{1}{2A}$$

$$\frac{dC/C}{dA/A} = \frac{1}{2}$$

To Find E w.r.t. Cost, where Cost ∝ Surface.
(Then, as last step, do backwards)

1. Use square as simple e.g., to show procedure & rationale.

Product = Area
Cost = Surface (~~in that~~ same as Perimeter, in 2-d)

$A/S = \frac{B}{4}$.

$\left(\frac{A'}{S'} = \frac{2B}{4} = \frac{B}{2}\right)$

$A = S \cdot \frac{B}{4}$

$\frac{dA}{dS} = \frac{B}{4}\left(\frac{dB/4}{dS}\right) = \frac{B}{4}\frac{1}{4}\frac{1}{2B} = \frac{1}{32} = .0313$

$\frac{dB}{dS} = \frac{1}{\frac{dS}{dB}} = \frac{1}{2B}$

E.g. $B_0 = 10$
$S_0 = 40$
$S' = 1$
$S_1 = 41$
$B_1 = 10.25$
$A_1 = 105.0625$
$A' = 5.0625$

$A = B^2$ $\qquad S = 4B$

$\frac{dA}{dB} = 2B$ $\qquad \frac{dS}{dB} = 4$

$$\frac{\frac{dA}{dB}}{\frac{dS}{dB}} \stackrel{?}{=} \frac{dA}{dS} \stackrel{?}{=} \frac{2B}{4} = \frac{B}{2}$$

B	$\frac{dA}{dB}$	$\frac{dS}{dB}$	$\frac{dA}{dS}$	S
10	20	4	5	40
50	100	4	~~400~~ 25	200
100	200	4	~~800~~ 50	400

dA	dS	$\frac{dA}{dS}$
20	4	

$B = 10$

$A = B^2 = 100$

$dA/dB = 2B = 20$

$121 - 100 = 21$

$S = 4B = 40$

$dS/dB = 4$

$dA/dS = \frac{20}{4} = 5$

~~100~~ 50 $\qquad$ 100

2500 $\qquad$ 10000

100 $\qquad$ 200

$2601 - 2500 = 101$ $\qquad$ ~~201~~

200 $\qquad$ 400

4 $\qquad$ 4

$\frac{100}{4} = 25$ $\qquad$ 50

$\frac{B}{2}$ $\qquad$ 5 $\qquad$ 25 $\qquad$ 50

$27

	$\frac{1+g}{1+b}$	$1+g-b$	$\frac{1}{1-g+b}$		$\frac{g-b}{1+b}$	$g-b$	$\frac{g-b}{1+b-g}$	g	b
	1	2	3	4	5	6	7	8	9
1	.95455	.95000	.95238		-.04545	-.05000	-.04762	.05	.10
2									
3	1.04762	1.05000	1.05263		.04762	.05000	.05263	.10	.05
4									
5	.85714	.80000	.83333		-.14286	-.20000	-.16667	.20	.40
6									
7	1.16667	1.20000	1.2500		.16667	.20000	.25000	.40	.20
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AMPAD EFFICIENCY LINE No. 2636

10/19/82
Ec. 162A

4. Quotient Rule

a. We are given:

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{u' \cdot v - v' \cdot u}{v^2}$$

(You can derive this from product rule. They are consistent. Just let $y = uw$ $\left(w = \frac{1}{v}\right)$.)

b. To conform this to common sense & a workable rule:

First term. $\frac{u'v}{v^2} = \frac{u'}{v} = \frac{u'}{u}\frac{u}{v}$

2d term $\frac{v'u}{v^2} = \frac{v'}{v} \cdot \frac{u}{v}$

$$\therefore y' = \frac{u}{v}\left(\frac{u'}{u} - \frac{v'}{v}\right) = y\left(\frac{u'}{u} - \frac{v'}{v}\right)$$

$$\frac{y'}{y} = \frac{u'}{u} - \frac{v'}{v}$$

c. To find new quotient, y_n:

$$y_n = y_0 + y' = y_0\left(1 + \frac{y'}{y_0}\right) = y_0(1 + g - d)$$

$g = \frac{u'}{u}$ or % growth of sum.

$d = \frac{v'}{v}$ or % discount

In general: $\frac{1+g}{1+d} \cong 1 + g - d$

d. E.g., effect of $\in$ of prodn. on productivity.

$g = 10\%$
$d = 5\%$
$P_n = P_0(1 + .10 - .05) = P_0(1 + .05)$

a 5% gain in productivity.

e. E.g., int. & infln., effect on PV

$$P = F_{10}\left(\frac{1}{1+i}\right)^{10} \cong F_{10}(1-i)^{10}$$

Effect of $g + d$: $P = F_{10}(1 - i + g - d)^{10}$

If $g = d$, no change.
If $g > d$, PV rises, + effect is same as lowering i

f. Real int. rate, r

$$\frac{1}{1+r} = \frac{1+g}{1+i}$$

$$1 - r = 1 - i + g$$

$$r = i - g$$

g. Caution!

As w. all calc., rule is exact only for very small changes; + usable

Here, very small means less than 10%, and sometimes less than this — if you are raising to more than the 15th power.

E.g.

~~E~~ are saying: ~~[illegible]~~

$$\frac{1+g}{1+d} = 1 + g - d$$

If $g = 0$, $\quad \frac{1}{1+d} = 1 - d$

But this is not exact. The rt. side is smaller. ~~greater.~~ ~~The ratio is:~~

To see how great an error that makes, take ratio of larger to smaller.

~~$\frac{1/(1+d)}{1-d} = (1-d)(1+d) = 1-d^2$~~

$$\frac{1/(1+d)}{1-d} = \frac{1}{(1+d)(1-d)} = \frac{1}{1-d^2}$$

d	d^2	$\frac{1}{1-d^2}$	$\frac{1}{(1-d^2)}^{20}$
.01	.0001	1.0001	1.0020
.05	.0025	~~.9975~~ 1.0025	1.0513
.10	.0100	1.0101	1.2226
.20	.0400	1.0417	2.2624

Write this column on board

Give last column. Students do double problem, to learn use of calculators.

a. Duplicate path to answers
b. Confirm by calc'g the long way

d	$(1-d)^{20}$	$(1+d)^{-20}$	$\frac{(1+d)^{-20}}{(1-d)^{-20}}$
.01			
.05	.3585	.3769	1.0513

Berkeley's ghosts return to haunt!

h. So, when can we use the rule?

1. To know direction of change in quotient
If $g \doteq d$, change $= 0$, etc.

2. When g & d are small.

3. When g & d are close to each other (& same sign)

4. When not going to raise to power over 10 or 15.

5. When precision not needed.

6. When no calc. available — i.e. in conference or debate.

4 kinds of optimizing, and many more.

Add, subtract, mult, divide.

Ln diff. curves only apply to adding.
Thus they lock us into a clumsy apparatus of limited value.

~~x~~ To max $A^N = B$

$$[A(1+a)]^{N(1+a)} = B(1+b)$$

$$\frac{A(1+a)^{N(1+a)}}{A^N} \stackrel{?}{=} (1+a)^{(1+a)}$$

$$A(1+a)^x \stackrel{?}{=} A^x \cdot A_a^x$$

$$[10 \cdot 1.05]^2 \stackrel{?}{=} 10^2 \cdot .5^2$$

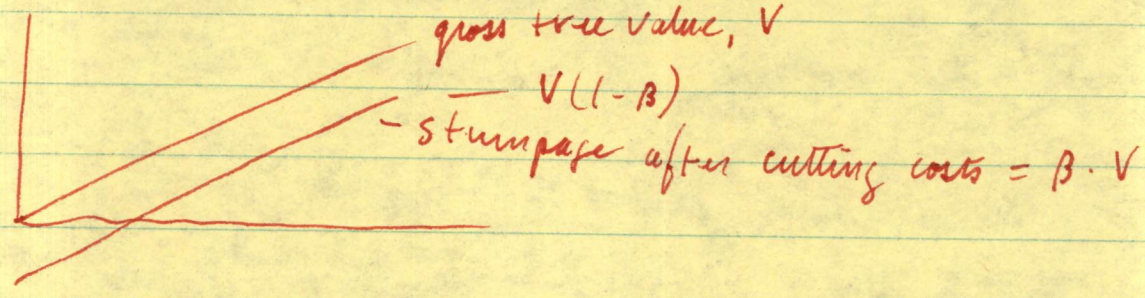
$$10.5^2 \stackrel{?}{=} 100 \cdot .25 = 25$$

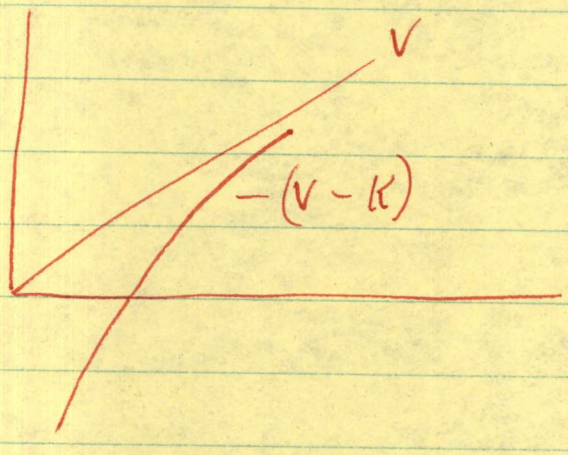
$$110.25 \neq 25$$

$$\pi(r+\Delta)^2 - \pi r^2 = \pi\left[r^2 + 2r\Delta + \Delta^2 - r^2\right] = \pi\left(2r\Delta + \Delta^2\right)$$

$$\frac{\Delta A}{\Delta \Delta} = \frac{\pi(2r\Delta + \Delta^2)}{\Delta} = 2\pi r + \Delta \longrightarrow 2\pi r$$

Use semi-log scale to show sensitivity to i.r. + front + rear-end costs.





Back to diminishing rets. Some implics.

1. Deals w. progns; not scale. Is partial anal.

2. Deals w. all prodn. not just farming.

E.g.s.

Urban bldg.: ht. of one bldg; no. of d.u. per tract.
Dams, height of
Energy used by smelters
Water used by Fontana.
Damage - effic. of pollutants: conc. or dispose

3. Wages indep. of AP, hence of productivity as usually expressed.

4. Profits are higher when MP/AP labor is lower, & elast. of prodn. is lower.

E.g. Amerada, payroll = say 10%.
So a 1% Δ labor in eq. yields a 1/10 % ΔP.

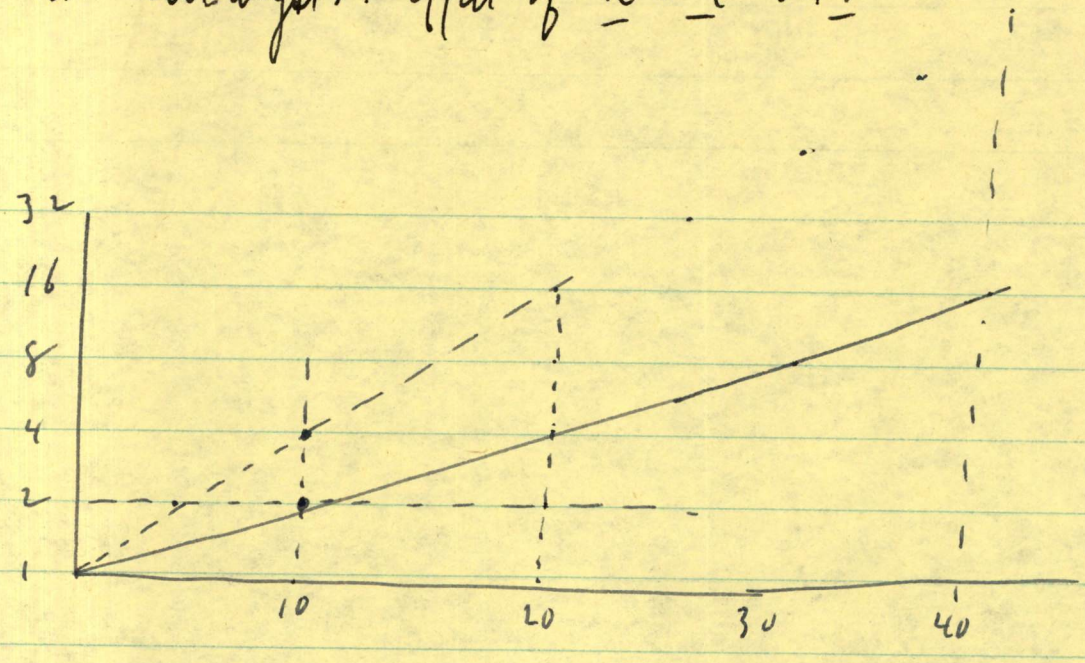
Thus they are insensitive to labor cost.

— They can drop a few men and not reduce output by a large %, even tho AP is high (but not many men)

— Payroll is small % of total costs. They can smile at min. wage laws. They benefit by hurting small competitors.

—

Sens. Analysis: effect of R + n on r



$r = R^{\frac{1}{n}} - 1$

$1 + r = R^{\frac{1}{n}}$

$\ln(1+r) = \frac{1}{n} \ln R$

$r \approx \frac{\ln R}{n}$

n	R	r
10	4	14.81
20	4	7.18
10	8	23.11
20	8	10.96

n	$R \rightarrow$ 2	4	8
5	14.87	31.9	51.6
10	7.2	14.9	23.1
20	3.53	7.2	10.96
40	1.75	3.53	5.34

R	$\ln R$	$R \cdot \ln R$
1	0	0
2	.693	1.386
4	1.386	5.5
8	2.079	16.6
16	2.773	44.4

11/3/82. Loose Ends

1. Log scales

$F = (1+i)^n \qquad (P=1)$

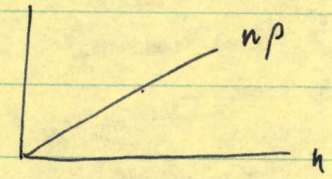
$\ln F = n \ln(1+i) = np$

$\frac{d \ln F}{dn} = \ln(1+i) = p$

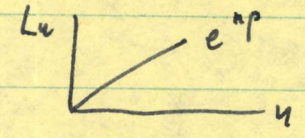
$F = e^{pn}$

$\ln F = pn$

$\frac{d \ln F}{dn} = p$



$k = \frac{np}{n} = p$



2. Max'g quotient $\frac{S}{(1+i)^n}$

Max when ~~growth~~ growth = 0

Growth = $g - d$

~~Qn = Q0~~ $Q_1 = Q_0 \frac{1+g}{1+d}$

Max. when $Q_1 = Q_0$ when $g = d$

But $d = i$

Max when $g = i$

This is exactly accurate.

$g = .05$

$i = .10$

$R_0 = 1$

$R_1 = \frac{1+g}{1+i} = .95455$

$R_1 \approx 1 + g - i \quad .95000$

$R_1 \approx 1 + \frac{g-i}{1+i} = .95455$

$R_1 \approx \frac{1}{1+i-g} = .95238$ — closer!

$R_1 \approx \frac{1}{(1+i)(1+g)} \approx .95694$ — also closer!

$(1+i)(1-g) = 1+i-g-gi$

$$\frac{\left(\frac{1+g}{1+i}\right)^{50}}{\left(\frac{1}{1+i-g}\right)^{50}} = \left(\frac{.95455}{.95238}\right)^{50} = 1.00227^{50} = 1.1202$$

$1 + x = \frac{1}{1+i-g}$

$x = \frac{1 - 1 - i + g}{1+i-g} = \frac{g-i}{1+i-g}$

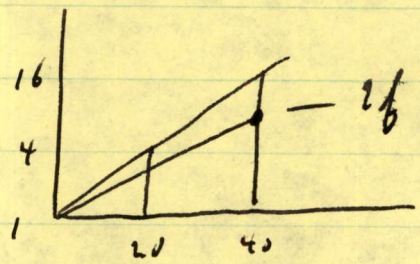
1. Leverage

n	$\frac{i}{(1+i)^n - 1}$	R_n/K
1	20.0	
2	9.7	
4	4.6	
8	2.1	
16	.85	
32	.27	

---

2. That it is better to reduce n than to Δ R/K

If Money doubles in 10 years. ~~Double n + it~~ an incl. 20 Δn Double n + n doubles Double n again. + n quadruples. Etc.



— If n doubled when n doubled, i will fall

Leverage.

1. % Growth of $(1+i)^n$ wrt n is $\frac{i(1+i)^n}{(1+i)^n} = i$

% growth of $(1+i)^n - 1$ is $\frac{i(1+i)^n}{(1+i)^n - 1} = \frac{i}{1-(1+i)^{-n}}$

2. Value of a periodic annuity

$$P = \frac{R_n - K_0(1+i)^n}{(1+i)^n - 1} = K\,\frac{R/K - (1+i)^n}{(1+i)^n - 1}$$

(Use this ~~it~~ to show PV is more sens. to n than to i!) (Or to R/K

n \ R/K	2	4	8	16
1	.95			
2		2.89		
4				
8				
16				

There is reason to believe the dividend of Gump's ~~Gumfinkels~~ Risinos will rise indefinitely ~~at~~ by 5% annually. Stocks are capitalized at 9%. What is its value? Dividend

$$PV = a_0 \sum_1^\infty \left(\frac{1+r}{1+i}\right) = a_0 \sum_1^\infty \left(\frac{1.05}{1.09}\right) = a_0 \sum_1^\infty (1 + .05 - .09)$$

$$\frac{1+r}{1+i} \cong 1 + r - i$$

$$R(1+r) = \frac{T(1+t)}{B(1+b)}$$

$$1 + r + b + br = 1 + t$$

$$r(1+b) = t - b$$

$$r = \frac{t-b}{1+b}$$

$$1 + \frac{c - b}{1 + b} = \frac{1 + b + c - b}{1 + b} = \frac{1 + c}{1 + b}$$

$$\frac{1}{1 + b - c} \stackrel{?}{=} 1 - b + c$$

$$1 \stackrel{?}{=} (1 - b + c)(1 + b - c)$$

$$1 \stackrel{?}{=} 1 - b + c + b - b^2 + bc - c + cb - c^2$$

$$1 \stackrel{?}{=} 1 - b^2 - c^2 + 2bc$$

$$= 1 - b(b - c) - c(c - b)$$

$$= 1 - b(b - c) + c(b - c)$$

$$= 1 - (b - c)(b - c)$$

$$\neq 1 - (b - c)^2$$

$$(b - c)^2 \stackrel{?}{=} 0$$

$$(b - c)^2 > 0$$

$$\therefore \frac{1}{1 + b - c} > 1 - b + c$$

$(b-c)^2$	b	c	$\frac{1}{1+b-c}$	$1-b+c$	$\frac{1+c}{1+b}$
.0009	.05	.02	.9709	.97	.9714
.0036	.10	.04	.9434	.94	.9455
.0144	.20	.08	.8929	.88	.9000
.0900	.50	.20	.7692	.70	.8000

~~Dog + Food (cause) + Clown~~ eat eat

Models

Shotgun

T - formation

I - "

End - around

Punt Formation

Single Wingback

End - around

Fox Trot

Jig

Waltz

Rhumba

Tango

$$r = \frac{t - b}{1 - b}$$

$$R(1+r) = \frac{T(1+t)}{B(1+b)}$$

$R(1 + t - b)$ = new ratio



$$PV = a\,\frac{1-\left(\frac{1-g}{1+i}\right)^n}{(1+i)\left(1-\frac{1-g}{1+i}\right)} = a\,\frac{1-(\,)^n}{\frac{i+g}{\cancel{1+i}}}$$

if the first term is $\underline{a}$, rather than $\frac{a}{1+i}$, then the denom. becomes $\frac{i+g}{1+i}$ which is what corresponds to using

$\frac{1}{1+i+g}$ for $\frac{1-g}{1+i}$ in num.

$$\frac{R'}{R} = \frac{t-b}{1+b} \quad ---$$

$$\frac{1}{1+i+g} \stackrel{?}{=} \frac{-i+g}{1+i} \qquad 1 - \frac{-i-g}{1+i} \neq 1 - \frac{i+g}{1+i} = \frac{1+i-i-g}{1+i} = \frac{1-g}{1+i}$$

If $r = \frac{1-g}{1+i}$ then the change from 1 to r is

$$-\frac{i+g}{1+i}$$

$$V = \frac{a + qV}{i}$$

$$Vi - Vq = a$$

$$V = \frac{a}{i - q}$$

---

1. $Vi = a + Vq$. You could begin by saying this is self-evident postulate.

2. PV in perp. $= a \frac{1}{1 - r}$ This is basic. Safest proof

$$a = \frac{1}{1+i}$$

$$r = \frac{1+q}{1+i}$$

$$P = \frac{1}{(1+i)\left(1 - \frac{1+q}{1+i}\right)} = \frac{1}{i - q}$$

~~Ken~~

Managerial Econ.

$1 \cdot 10 = 10$

$1 \cdot 1.20 \cdot 10 \cdot 1.01 = 10 \cdot 1.20 \cdot 1.01$

$MV = y$

$M(1+m)\,V(1+v) = y(1+y)$

$(1+m)(1+v) = 1+y$

$1+m+v+vm = 1+y$

$m+v+vm = y$

m	v	vm	vm/(m+v)
.01	.01	.0001	.005
.25	.25	.0625	.125
.50	.50	.25	.25
.75	.75	.4312	.2875
1.0	1.0	1.0	.50
2.0	2.0	4.0	1.00

As Bishop Berkeley remarked, Newton murdered vm, but "these murdered quantities will return to haunt".

---

1. $\dfrac{d(MV)}{dx} = M'V + V'M$

2. $\dfrac{d\ln(MV)}{dx} = \dfrac{d(\ln M)}{dx} + \dfrac{d(\ln V)}{dx} = \dfrac{M'}{M} + \dfrac{V'}{V}$

Perhaps this is what you meant? If so, OK — but beware the "murdered infinitesimals".

Sum Rule

$$A + B = C$$

$$A(1+a) + B(1+b) = C(1+c)$$

$$A + Aa - A + B + Bb - B = C + Cc - C$$

$$Aa + Bb = Cc$$

$$c = \frac{Aa + Bb}{A+B} = \frac{A}{C}a + \frac{B}{C}b$$