54a Correlation analysis & prodn fns.

~~Sect~~ mixed partial completely explained in the general situation. all you need due is put in the special words of the particular situation.

If z is output per acre
x " men " "
y " money " " (other than wages)

Then $\frac{\partial z}{\partial x}$ is a fcn of y (i.e. depends on the value of y)

and is approximately the increase in output due to adding one more man.

Now that increase depends on y, so can be graphed as a fcn. of y, say marginal output of labor vs money per acre. (approximately)

Then $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\frac{\partial z}{\partial x}$ would be the change in this marginal output due to adding one more dollar per acre.

Since $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$, it would also be (appr.) the change in the marginal output of ~~capital~~ money due to adding one more laborer.

Matt

Dear Mase,

$\frac{\partial^2 z}{\partial x \partial y}$ means that z is a function of x and y (and possibly other variables) and that one takes two derivatives as follows: First, one holds x constant, so that z is a fcn. of y alone. Then take the derivative of z with respect to y. (this is called a partial derivative) Your answer depends on y, and also on x, so it is a fcn. of both x and y, denoted $\frac{\partial z}{\partial y}$. Then take this new fcn. of x and y, hold y constant, & take the derivative w.r.t. x. This is called the 2nd mixed partial of z w.r.t. x & y.

There is a theorem which says that if $z(x,y)$ is C^2 (i.e. in every practical case) it doesn't matter in which order one takes the derivatives. i.e $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

example: $z = x^2 y$

$$\frac{\partial z}{\partial y} = x^2 \qquad \frac{\partial z}{\partial x} = 2xy$$

$$\frac{\partial^2 z}{\partial x \partial y} = 2x \qquad \frac{\partial^2 z}{\partial y \partial x} = 2x.$$

example 2. Let x = number of laborers per acre on a farm. (Assume for simplicity that x can vary continuously)

Let y = money per acre devoted to machines, etc.

Let z = output per acre.

Then z is a function of x and y.

$\frac{\partial z}{\partial x}$ is readily interpreted — the increase in output due to the addition of one more laborer.

$\frac{\partial z}{\partial x}$ depends on x and y, and has a derivative w.r.t. y, $\frac{\partial^2 z}{\partial x \partial y}$.

Of course, $\frac{\partial z}{\partial x}$ also has a der. w.r.t. x, written $\frac{\partial^2 z}{\partial x^2}$.

Interfirm prod-n fns are just as valid as ~~in~~ cons. fns derived from interpersonal & intertemporal cfsons.

s468 Heady, Earl O. "Techn. Scale Relationships & Farm Size Policy." SEJ 19(3): 353-64 (Jan '53)

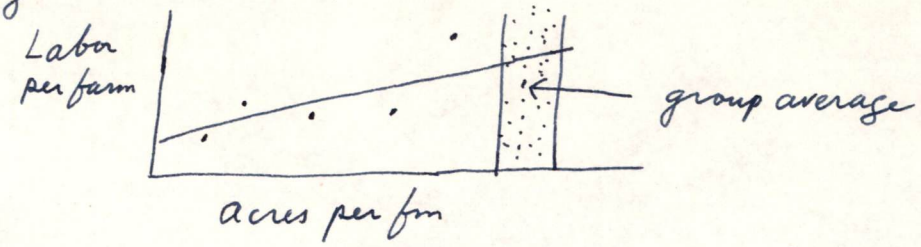
Comes to decr. returns to scale, by elast. of prod-n method. Again, has same elast. over whole range. Recognizes this as defect. But at least returns fall, whole range.

Hesitate to cite, bec comes to such diff. answers.

✓ Put in a note – & in bib.

Ezekiel on correl-n.

I have used group averages to show ~~things~~ how one vble changes w. another. Caution.



But that's not what I did? But I did parallel things.

Probly objections to averages here rise from too small a sample. Census data are whole universe, so ok? Ezek. computes standard deviation, σ. "computed standard error" is then $\frac{\sigma}{\sqrt{n-1}}$, where n is no. of cases. W. census, n is very great, $\frac{\sigma}{\sqrt{n-1}} \to 0$

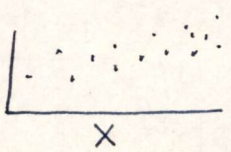
Besides, this is error of estimate. I'm not concerned w. predicting how individ. cases may turn out.

$\sigma \equiv \sqrt{\frac{\Sigma x^2}{n}}$, where x is individ. deviation from mean.

$\frac{2}{3}$ of universe falls w-in σ of mean; 95% w-in 2σ.

* Fit a curve to scatter, then "standard error of estimate" is σ of data from curve.

p. 118

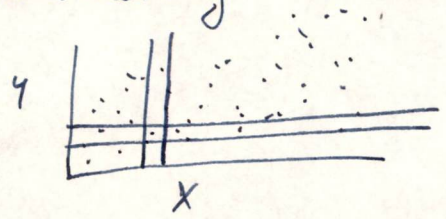


Take the y values alone, w/o ref. to x, get their σ from their mean, σ_y. Now fit regression, get ~~standard error of estimate~~ σ of y, as predicted from curve ~~from it~~, $\sigma_{y\cdot}$. $r \equiv \frac{\sigma_{y\cdot}}{\sigma_y}$ is a coeff. of correl-n. If reg. goes thru all the pts, $\sigma_{y\cdot} = \sigma_y$, $r = 1$. (But I don't like this at all. For horizontal regressions, $\sigma_{y\cdot}$ will be very low, & for steep ones very hi; while σ_y wd not be so much affected by steepness rec. ?)

M

r^2 is "coeff. of determination"; & measures the prop. of variation of y caused by X — for math. reasons not given.

To hold something constant:



Take a slice across scatter, all individ cases where $y = y_1$.

Then get z as $f(x)$ for those cases.

(MNP idea lets you use other cases, too, elim. changes in y by subtraction)

(Could also get z as $f(y)$ by going up vertical slice, thus ~~get idea of tissue relationship~~ find which is stronger)

Ezek. has inc. influenced by cows & acres, & as no. of acres Δs, no. of cows Δs. So plotts inc. as fn (acres), little relation appears. So eliminates influence of cows, thus: plots inc. vs cows, gets regression, $Y = \$694 + \$20(\text{cows})$. The 694 is presumably due to other factors, & influence of mgnl cow is shown by coeff. of regression, $20. (I shd work out regression of income on land, 1900 data, try elim. influence of complements this way, instead of by my rather crude method. Lump complements. But no — can't do it, bcs to eliminate effects this way, must group data by compl.s, to get regression of inc. on compl.s, & they aren't so grouped. Besides, a linear regression shows const. MP thruout range, not much value)

Then tabulates inc. & acres & cows by farm, & subtracts $20 × (cows) from inc., takes regression of adj. ins on acres as shows how acres influence inc.

Ezekiel, 3

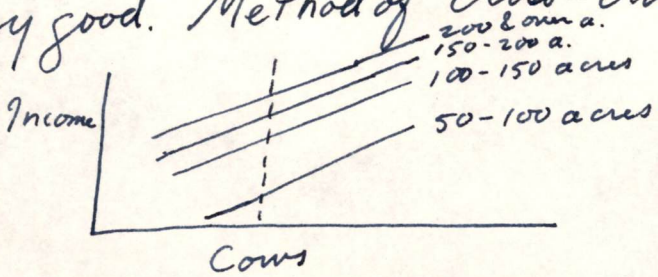
Isn't the statistician pursuing a dream? Influence of acres on income, after eliminating effect of cows. But effect of acre depends on how many cows there are to chew on it, &c.
"Effect of acres on income" means nothing in a vacuum.

151 & above, describes "method of successive elimination" — get effect of cows on inc., subtract from acres on inc.; then take acres-inc. regression & subtract from cow-income, get better cow-inc., w. higher coeff. of regr; then come back & subtract from acre-inc.; & repeat until each stable regression.

But this "method of successive elim." is laborious. Method of multiple correlation gives same result, quicker. (ch. 12)

---

p. 154. Very good. Method of cross-classification.



Above graph comes from table. Distance betw. lines gives idea of MP of land. (Again, census gives no data to operate on this way. But might find some.)

This is just a variation on method of taking slices across a scatter.

Weakness of method: requires very large sample, or no. of cases, to fill up all the possible categories, get signif. ave. in each (but wdn't regression to line of aves take care of this?)

Weeks mentions correlation coeff.s, & measures of degrees of freedom, as formal stat method.

Ezekiel 4

ch. 12, shows method of deriving multiple regression eqn from actual data. It wd take me some study to figure out wheet they did & why. But it is basically same as method of ~~multiple~~ successive elimination, which gives inc. as fn of acres & corns, & then you drop one y-intercept & add the 2 eqns, thus $Y = K + g(\text{acres}) + h(\text{corns})$.

Is based on ass. of linear relations thruout. (& seems to me to take no cognizance of complementation betwn acres & corns)

E.O. Heady, "Production Functions from a random sample of Farms", JFE, Nov 1956, p. 989.

Leg. To understand it, try Tintner, G., "A note on the derivation of production functions from farm records, " Econometrica, Jan 1944 pp. 26-34.

I judge what they have done is take numerous farms of different sizes, compare output on them, see how much the additional resources add to output. But they don't describe their methods. Nor their way of measuring land -- well, it is land plus buildings, he says -- and yet the question the respondent was asked, was for land value -- Heady then assumes he included buildings with it! Pretty sloppy.

Brownlee & Tintner, JFE Aug '44, use land acreage. Also sloppy.

Some of these data appear to help me; others to hurt me. Heady takes pains to emphasize those that hurt me -- that mp of land is larger for larger farms -- but he doesn't define what he means by large and small. And the MP he gets only refers to "the geometric mean" of the data. One would have to analyze the data assiduously to get any meaning from them, and then would probably find the methods were so sloppy they aren't worth much. So skip it for now. Some day, perhaps.

5173

eg

p. 303 Cites Heady, "Production Functions from a Random Sample of Farms", JFE, Nov. '46. This one is a must. From 1939 data.

Heady found that for Iowa farms, they were too large. A 1% increase in all inputs would increase output only .85%.

p. 304. Heady also found marginal productivities per $ of additional input.

Livestock & feed	84¢
Misc. op. expenses	39¢
Equipment	20¢
Labor	08¢
Land	046¢

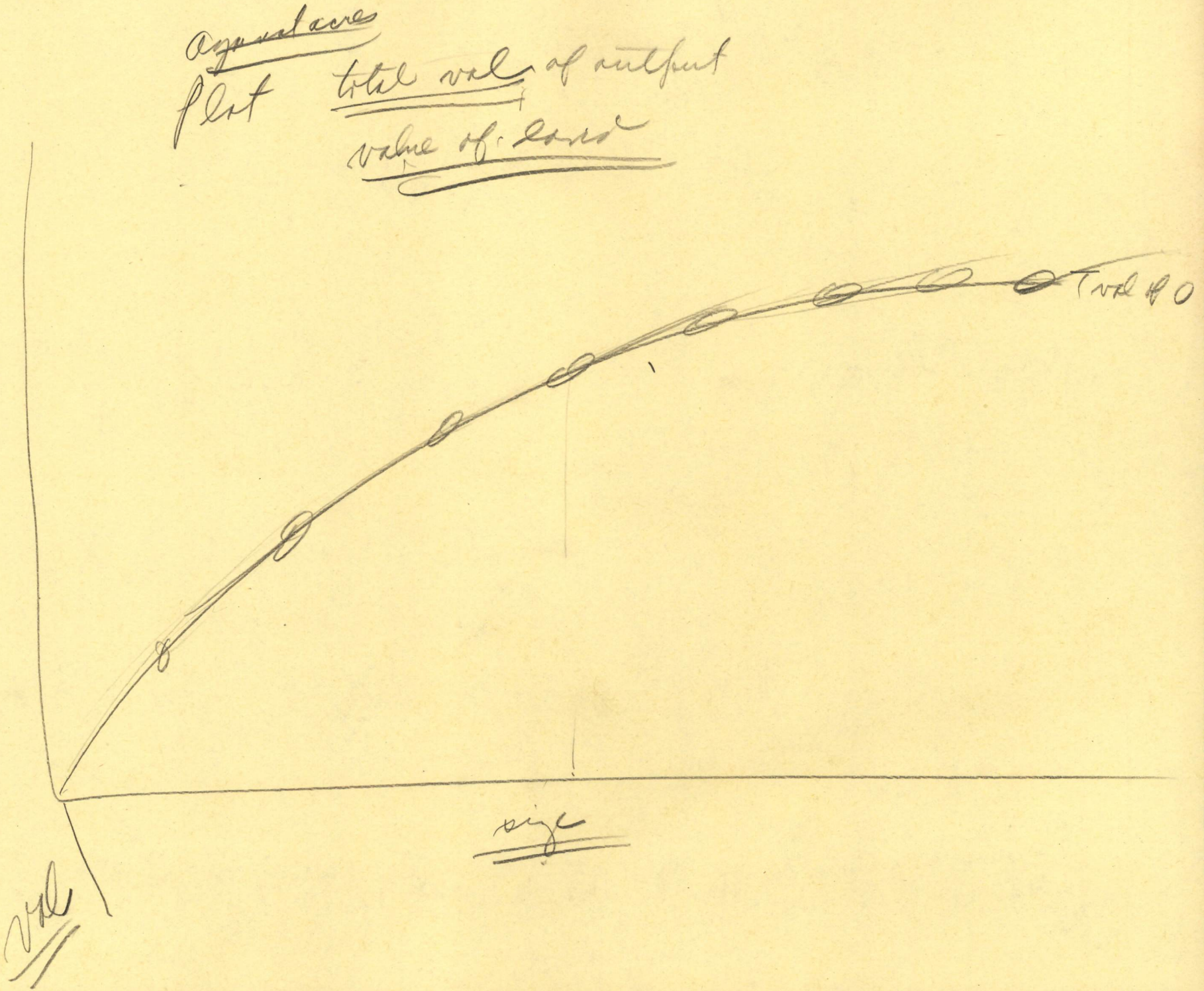
quote this

Schultz: "One may infer from these data that in Iowa in 1939 more livestock and feed, operating items and equipment should have been employed relative to labor & land. The capital market, however, in Ia & also elsewhere, is better developed for renting & borrowing funds to acquire additional land than it is to serve farmers in obtaining the other physical inputs listed above."

S 419

Ed. Bishop claims mp of land is low on small N.C. farms, feed, equipment, etc, is hi.

Check his use of "Cobb-Douglas" pm. fns.


Agricultural
Plot total val of output
value of land
Tot val of O
size
Val

7. Rate of turnover data --- output/assets

Beware fallacy in MP study, fig 7 get. For assets of smaller, called "land", are more cap. turnover faster. But at same time shd be higher costs, to maintain, show up in cost figs.

S460

Leg

Raden, Walter L. Farm Size & Its Relation to Volume of Production, Op's Costs & Net Returns — Southeastern Nebr., 1930-34

Nebr. Bull 346, 1943.

p. 7

Crop acres per farm	Total acres per farm	Receipts (incl. Δ inventories) $	Expenses (Incl. depn & depl of inv.) (excl. farm Labor) incl. family	(Receipts/acre) (Exp./acre) (ANP/acre) ANP/afm
Under 80	112	1,542	1047 + 65 = 1047	
80 -	155	1652	911 + 56 = 967	
120 -	193	2009	1131 + 98 =	
160 -	242	2542	1422 + 164 =	
200 -	300	3055	1691 + 208 =	
240 & over	408	4922	2935 + 254 =	

p 8 Larger fms, bigger up & down of fm inc. In '31 & '32, inc/fm was actually lower on larger fms - per farm! Call were negative in those yrs.

? & how about feed bought off farm?

Size Groups	No. of farms	Acres	Δ acres	Output	Value of bldgs	Annual value of bldgs 10%	R & M	Annual R & M 20%	Live-stock	Annual Livestock	Labor	Fert.	Total Annual Costs	Net output	Δ net output / Δ acres fm	Δ net output / Δ acres fm

Net output

Size groups	No of fms	Acres	$\frac{\text{acres}}{\text{fm}}$	$\Delta\left(\frac{\text{acres}}{\text{fm}}\right)$

Above is figuring MP directly. Simpler might be fig'g AP/acre for each group, figure what MP that implies. Suppose you know 2 APs, how figure MP?

$$A_n = \frac{T_n}{L_n}$$

$$M = \frac{T_{n+1}}{fm_{n+1}} - \frac{T_n}{fm_n} = (A_{n+1})L_{n+1} - A_n L_n.$$ assuming "n" is one unit, but here it isn't.

Given A_n & A_{n+1},

---

g is the acreage (or value) group
f .. .. no. of fms in the group
l .. .. acreage in group
p .. .. total product of group

$$\Delta AP = \frac{P_{g+1}}{f_{g+1}} - \frac{P_g}{f_g} = \frac{AP_{g+1}\, l_{g+1}}{f_{g+1}} - \frac{AP_g\, l_g}{f_g}$$

$$AP_{\text{per acre}} = \frac{\frac{P_g}{f_g}}{\frac{l_g}{f_g}} = \frac{P_g}{l_g}$$

Livestock contribute to output. cost is int. & depletion. Actually fms maintain their stock, by pasturing & feeding. So ideally we wd. count as a cost depletion of stock, & as an output products fed to stock, & Δ'd value of stock pastured. Data on depl. of stock n.a., nor on Δ value from pasture. So 2 alternatives:

1. Count no depletion of stock, but just int. on stock; Don't count value of crops fed to stock on fm as output. This has weakness that some fms bought feed off fm, but is no figure for this. Such fms will thus appear as higher output than should. Probly smaller fms. But is compensated by not count's feed home grown, which wd favor smaller fms (due to pasture on large)

2. Figure depletion of stock as cost, & replacement of it as output — i.e. the replacement that occurred on the fm. That means output fed to stock. (a big element here wd. be horses, incid., this is home grown fuel) Weakness is this allows nothing for pasturing, a method of replenishing stock.

Safest way is choose method least favorable to hypothesis. Then if hypoth. proves out, is sure. This means allow generous depres. on cap., thus Δ costs of smaller more. And disallow products fed to livestock, as small have more of them. This means are assuming as stock maintained, count no depreciation, or turnover, on stock. Assume are all produced on fm, also all their feed. W. no figures on feed or stock purchase, have to ass. that anyway. This will unduly amplify net output from those fms which buy feeders, & depress output ~~Can use L.V., rather than acreage, as more of land~~ from w-n ranches, which are more self-sufficient. Large hld'gs gnrly more self-contained, so these figs, lack of data on off-fm purchases, will make small look better than are.

Anyway, don't count dep. or repl. of livestock (unless want to club in est. of repl., make it as large as possible on small, small as poss. on large, thus dim. any criticism.

Data needed

(Why not get whole U.S.? By value)

Critic says, maybe it is labor or cap. that is less productive on larger firms.

Answer: labor & cap. known to be more productive on large, all agree.

George Mehren says:

To find MP of land by size of fm, get data, simul, fms of diff. sizes. Take output as fn of various inputs, incl. land. Subtract costs of other inputs, thus have MP of land — (of a sort, interfirm)

Method, math.: Output = f(Acres, [labor, cap])

Compute $\frac{dO}{dA}$.

Suggests Sune Carlson, Erich Schneider, as ~~so~~ describing math. of method. Says Weeks can tell me easier than wading thru all that.

So all I need are data on acres, gross output, & other costs, of fms of diff. sizes. Very simple.

E.g. of. 50 acre fm, output $5/acre, costs $4/acre

vs

60 acre fm, output 480/acre, costs 380/acre.

$\Delta A = 10$; $\Delta O = 60 \times 480 - 50 \times 500 = 28800 - 25000 = 3,800$

$\Delta \text{cost} = 60 \times 380 - 50 \times 400 = 22800 - 20000 = 2,800$

$\Delta \text{Net Output} = 3,800 - 2,800 = 1,000$

$\frac{\Delta \text{N.O.}}{\Delta A} = \frac{1,000}{10} = 100.$

Now get such figs for wide range, show MP fully.

Or dub in a few figs from scattered evidence. Observe effects of scale on proportions, probly get something good there. Work out algebra. Pregnant.

Boil down: Size groups. ^ No. of fms. Acres. ~~Acres/fm (comp.)~~. Δ acres/fm (comp.)

Output. Specific costs. Costs totalled (comp.). Output minus costs, or net output. Δ net output. Δ net output/fm.

$$\frac{\Delta \text{ net output/fm}}{\Delta \text{ acres/fm.}}$$

What data do we need? What does it prove?

Cf. 80 acre fms vs. 40 a. There is Δ land, Δ cost, Δ output. Thus figure Δ net output from Δ 40 a. Cf to net output of 1st 40. Then take 120 a. Cf to 80. Or, cd. cf to 2 40s. Is certainly no sense to cf. 120 to 80, if 40 is better than 80.

Well, you can say 'the MP of land between 80 a. & 120 a. is x/acre. Meaning those fms that have grown to 120 cd., in gnrl, dwindle back to 80, & accomplish what others do at 80, so last 40 a. in them is adding just that much over 80 - I guess that's ok, to you know what it means.

What data need? ave ~~size~~ acreage in ea. group; ave output; ave costs. I.e., all on a per farm basis.

[Or, on per acre basis? Larger gets less/acre, net. So mgnl net product per acre is what it takes to reduce ave. from what it was on smaller acreage. But you must know how many acres. So set up on per fm basis.]

Need: Size groups. No. of fms. Acres. Acres per fm ~~size~~ (computed) output. Output/fm (comp.). Costs, specified. Costs, total (comp.) Costs/fm (comp.) Output minus costs (comp.), total & per fm.

Δ net output. Δ acres. $\frac{\Delta \text{ net output}}{\Delta \text{ acres}}$.

"Value added" – it is net of raw materials, but not of labor. This makes sense if we are tryg to find nat. output. But as index to MP on site? Usual mgnl analysis of output & costs doesn't distinguish raw materials, i.e. products of other industries, from other costs, or inputs. In production ec., the "value added" concept has little meaning.

Gunn & Douglas.

Main points:

a. They find constant returns to scale.

b. $\frac{\text{MP labor}}{\text{MP "cap"}} = 3$. But labor only gets 60% N.?

Methods crude. Skip valuation problem re quantity of cap. Eliminate land! – think it plays ~~so imp~~ negligible role in mfr.

Cf. contemp. ent.s of diff. sizes; also changes w. time, all ent.s. Dubious

Leg:

Douglas, Paul H., & Gunn, Grace T., "The Production Fn for Am. Mfg in 1919", AER, Mch '44, pp 67-80. Crude. No recognition of valuation problem in figuring quantity of "cap." Omit land.

Same, JPE, Aug '42; & JPE Feb. '43

~~Emphasize~~

Tintner, Note on Derivation of P-n Fns from Fm records, Econometrica, Jan '44. Can't make much of it. Probly shd study up some day. He does distinguish land & improvements

Bronfenbrenner, 'P-n fns: Cobb-Douglas, Interfirm, Intrafirm', Econometrica, Jan '44

Reder, Ec-mtca July-Oct '43

Reder says Douglas says he's shown there is no mnpsnstc expl-n of labor! But I cd. swear Doug. said there was, that was whole point! Who's nuts?

Wrigley, Paul ?. "Intensity of Land Use in Pa." Pa. State College School of Ag, AES, Dept of Ag Ec. Paper No. 1322. Apr. '46. (Mimeo)

p. 2 Is better", + pays to concentrate labor & expense on the better grades of land ..."; use poor less intensively.

p. 13 Gross value of ag. production per acre of fm land, Pa, 1940, was $16.11. Even in top cties only $40-50. Well, for postwar double that, & recall it includes all land in cties, much wood & waste.

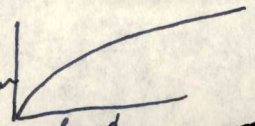
These are from 1940 Census. Try Census, find outputs for fms distr. by size, maybe get something on inputs, work out MP by Mehren method. Choose a state where l.v. holds steady w. size of fm.

1900 gives such data. v. 5, pt 1, pp. 186 ff. You don't need any fancy formula. Figures are fantastic.

mm

For that, use indiff curves. How many have used them? (If too few,

ship) (Can show this much:



Vain search.
~~Gruber.~~

Ia. A E S Res. Bul 298, T W Schultz & O. H. Brownlee, "Effects of Crop Acreage Control Features of AAA on Feed Prodctn in 11 mdw. states.", 1942, p. 683 etc. ascribe part of great Δ of yields to AAA cutbacks.

~~Check Brownlee on AAA~~

~~The Ia. Farm Economist, 1938 & 9, for detailed articles on corn reduction.~~

Schultz, AER, Proc. 30: 136-141
Not there, mis-cited.

pp. 22910. Cal, '35, Unpubl. Census data.

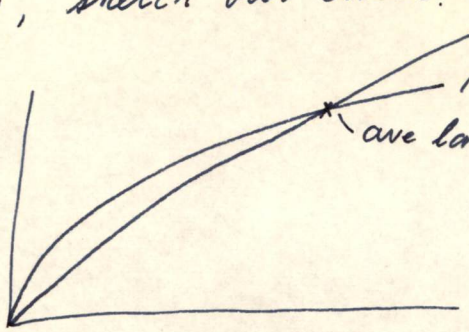
5323

File under regression fallacies. Here, fms classified by no. of men.

No. of fms rpt'g — hired help*	1	5	9	10 & over
	27,424	1,248	246	2,014
Acreage in ~~these~~ fms	5,499,207	1,040,159	257,429	3,651,168
Men hired on them	27,424	6,240	2,214	20,140 or more
~~Hired men per acre.~~ acres per hired man	$\frac{5,500}{27} =$	$\frac{1,040}{6} =$	$\frac{257}{2.2} =$	$\frac{3,650}{20}$ or more

Must figure in family labor to show more acres/man on big

land, which I have, & thus know something about regression of land on labor, which I lack. Then est. beginn'g pt. of regr. of " " ", sketch out curve.



p. 331 – the closer are the scattered pts to a strt. line, the nearer are the 2 regressions to ea. other. (thus, as I have very ~~uniform~~ smooth line for regr. of labor on land, opp. regr. not much diff. – good.

Either ~~eqn~~ regression -- y on x or x on y --- has same "r", if linear. (Ezekiel, p 140-41) (If not linear, is called ρ, they differ)

Now I wonder if, by calc. correlation coeff., can find expression of relationship not subject to bias? "r" = 1 means you can predict y perfectly from x, using the correct regr. eq-n you have found. (Suppose y is constant? Wd you then get perf. correl? -- r = 1 ? Wd it then mean anything? No – for 1st you must find regression eq-n relat'g y to x, as dependent vble.)

p. 471, can use "r" to compute regr. of x on y from regr. of y on x, simple method.

But thru-out, says it is perfectly ok to take regression of one vble on another, w/o any correction, if that is what you want to know. But he might have other ideas, where, as in my case, proportions of factors is what you want to know.

But why can't we say, in 10-20 acres class, ave fm has 3 men. & then say, if we have a fm of 10-20 a., most likely ~~size is~~ am't of labor is 3 men; & why isn't that same info. regression eq'n gives? Least squares curve-fitting is entirely on vertical basis anyway.

Waugh, p. 475, seems like easiest exposition to follow.

When you get a regression line, then, what does it mean? It can't work both ways. If you have ~~0 to acres~~, ~~say~~ 5 acres, say -- not thinking of grouped data now. — ~~curve~~ curve tells you to expect 1.2 men; if have 1.2 men, expect 5 acres? - no, expect 1. So what does curve mean?

Use Waugh 475 to ask qns of experts.

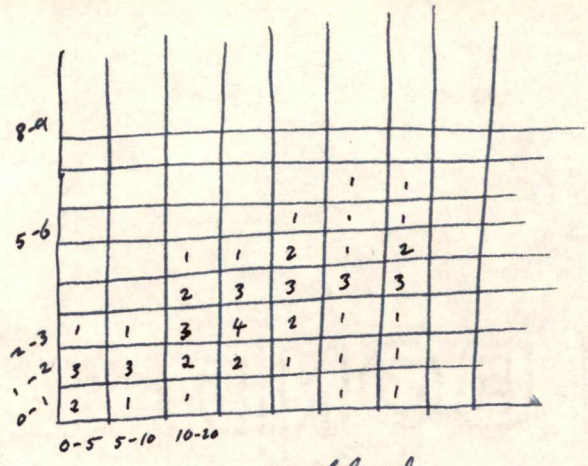
Waugh, p 298, says you choose one variable as indep; other as dep., depending on what you want to show. (I suspect maybe regression line depends on which you choose as indep?)

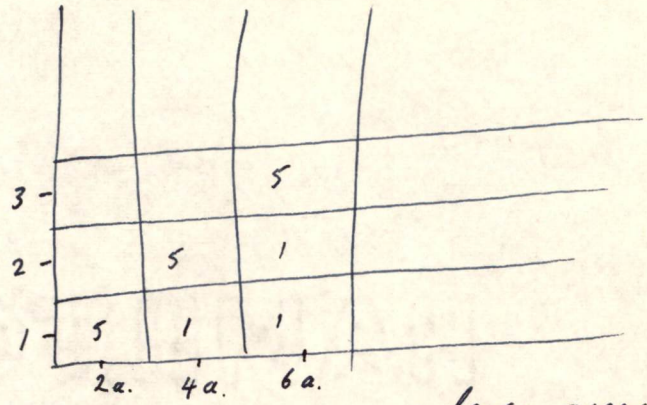
p. 300, proceeds ~~by taking~~ to correlate age & blood pressure by plotting age groups on abscissa, & ave. blood pressures on ordinate Refers to line fitted to those points as a regression line, p 303

Then fits one, p. 313

p. 329-333 -- Explicitly states, regression dependence is only one way — can't turn eq-n around, like ordinary math. eq-n. Regression of X on Y differs sharply from regression of Y on X. They cross at. pt. rep'g averages of both. (In other words, I cd. take ave. fm size, & ave. am't of labor/fm, & spot that on regression line of labor on

Mary-write Gordon





Here is e.g. where you get larger acreages having less men per acre; larger men-groups having more men/acre.

If we had data available broken down as above, there is a correlation formula for grouped data (Croxton & Cowden, p. 676-7) that is neutral as to which group is chosen as crit. of size — i.e. gives approx. same results as if used individual points, fitted curve to them.

But all I have are frequencies in classes of acreage ... & those lumped in one aggregate, not separated into diff. labor classes, as this formula requires.

If I wanted to spend time, cd break down ea. freq. by acreage classes into diff. labor classes, by some assumed normal distrib. From that, get ~~correl~~ ~~regress~~ correl. coeff.; & regression eq-n.

But don't take time now. Just regret we lack data to work out regression of labor on land. Fall back on gross output.

II, 2, OK !!

Present data by land classes, for labor & cap.

To these data there may be some objection. Some those wise in ways of stat. may observe that there will inevitably be ~~various~~ random variations in props. of land, labor & cap. — conceivably for valid ec. reasons of op'g ec. in crops of diff. types & resource requirements — & whatever factor we take as more of size, will be a tendency for figs to show a higher prop. of it to others in fms of large 'size', so msred. etc.

Granted their is such a tendency; & it has made our correlation of smallness w intensity more striking than it wd be otherwise. Still it by no means follows that is the only reason the data proved out as they did. The correlation is much too pronounced for such an expl.

But lest anyone doubt, & suspect... Let us ~~look at~~ ~~error he~~ relate intensity to size, when size is defined msred not by land, but by a msre that is neutral as among diff. factors: gross output.

Now if intensity varied haphazardly, w. gross output might say was just due to diff crops call'g for diff. intensity. But when intensity is so markedly related to size, suggests our other hypoth. Even then, does not prove it, but certainly is consistent w it.

II, 2.

Here are efforts to squeeze as much as possible from the data at hand. Maybe I try too hard, for this stage of diss. Better, just present basic evidence, say it is consistent w. diff. MP.s.

✓

Ftn.

It may be urged there is a certain random variation of intensity to be expected, from diff. requirements of diff. kinds of production.

...

& furthermore, those op'ns requir'g less cap/acre will nat. spread over more acres, from perf. good op. reasons, bes a, there are fewer mgmt problems per acre; & b, it requires more land to complement nec. items of equipment in this line.

So we natch find ~~less dense~~ firms which have higher prop of land also tend to have more land, for perf. good op'g ecs.

Now is some truth in that — tho I am skeptical that those factors alone wd cause some tendency for large farms to be less intense. can carry it very far — but let us then see if that explains the contrasts of density.

Let us msre firm size by total value of assets. [Doing so, I wd think it reasonable to ~~we shd~~ expect, if op. ecs were only factor at work, larger bundles of total assets wd. be more intense ... bes intense firm, things closer together, can administer more. (dubious — land easier to manage)]

Do so — large still less intense.

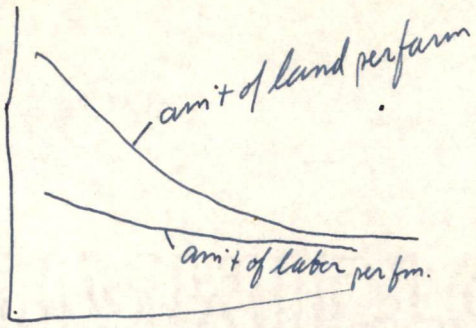
Then take gross output. Note how that is vs. us, bes includes as large firms w hi turnover

so let us abstract from that factor by msrj size in other ways:

by gross r.e. value; & by gross output.

Note that by doing so, we not only ~~abstract from the~~ remove the favorable bias from our data, but introduce unfavorable one. ... (so? If so, express it.)

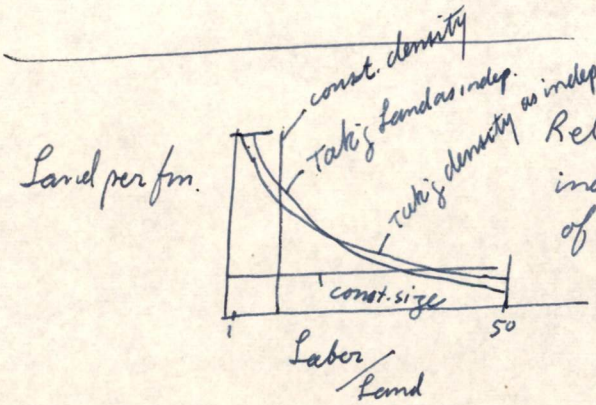
Even so, data are in our favor.



Intensity (Labor/land v.



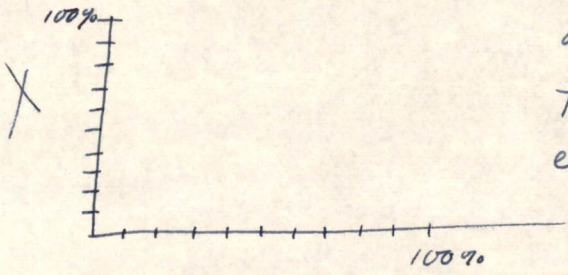
Amt of Land in Op-n.



Relationship is same, regardless of which is indep. vble. Until we start using classes, instead of ea. ~~corresponding~~ fm., & corresponding intensity.

Then, if land/fm is indep. vble, will get greater contrasts than if intensity is indep.

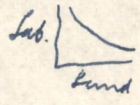
For take largest group of fms. - all over $10,000 of land, say. Say their ave density is 1 man/acre. Then rank by density, take lowest group, extend them until ave. density is 1 man/acre. Ave fm size will be less than ave. of first, largest group of fms. Bes some of largest won't be in this least dense group; & some smaller ones will be.

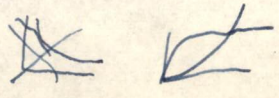


Take smallest group by size. Say it has 50 men/acre. Then take most dense fms, in order of density, until get down to ave. density of 50. Fms will be larger.

Take land as indep. gives greater contrast of sizes, as density changes.

" density " " " " " " density, " size " "





So by taking land as indep. vble, we are showing less change of density than we wd. by taking density as indep. vble.

I.e., we are understating the contrasts of intensity from fm to fm; & overstating their correlation w. farm size.

I.e. if we had data ranked in order of intensity, we wd find ~~much~~ greater diffs. of intensity between the most intense & the least intense groups. & smaller diffs in size.

(Ideally, we wd have a 1 to 1 correspondence for ea. fm. Then wd have exactly the same set of pts, either way)

These great contrasts of size of o-op fm land hold'gs in themselves lend credence to idea that MP of land varies from hold'gs to hold'gs ... for it is scarcely plausible that opt. op. scales shd vary so much. / that one thg cd use the 300th acre to better effect than another the 10th. assume, for arg., hyp., opt op. scale tends to medium; then MP low for very large; hi for very small.

There is a 2d fact that supports this interpretation: these large hold'gs are gnrly used much less intensively than the small. I.e., prop. vary.

(That is consistent w hyp that very large get large bes of inv. purchases of land, rather than thru ec.s in op.g machines or organizing men. If it were latter, they wd not shy away from using more men & machines But put this off to pt. 3 -- perhaps a brief allusion here is enough)

Give data, by land class limits. Labor, & cap.

Now a certain variation in intensity is to be expected, due to diff. op. needs, etc. Furthermore, we should expect that variation to be such that larger hold'gs, when size is msred by land alone, were slightly less intensely used. Due simply to fact that 2 op'ns of same gross output, if one is land intensive & another labor intensive, land intensive falls in larger size group. (cont.)

But we shd hardly expect such extreme variations.

However, lest anyone think .... trickery ... stacking cards in our favor .. take another more.

a certain random variation in intensity is to be expected, (from diff. op. needs, etc.)

But is there any reason, (from op. needs) to expect it to be related to size?

That depends on how we mse size. If take land as crit, yes — tho not so much as find.

But if take neutral msre, gross output, is no such bias in data. (or if take intensity as crit., relate to size)

These facts do not prove MP land lower on larger ~~from~~ holdings for not every kind of agric. calls for same proportions of factors, & maybe MP land as hi in one ~~set-a~~ w. few complements as in another w many. & our misc of "size", by am't of land, has been such as to call these naturally land intensive op-ns 'large' -- & thus lead to desired conclusion.

Very well. We have not proved diff. MP. We did not hope to w data at hand, but only to show existing conditions are ~~exist~~ compatible w such a conclusion, & highly suggestive of it.

But lest it be thought we reached even that limited conclusion merely by statistical prestidigitation, let us observe that it need not be that way, merely by choosing am't of land as criterion of size. It is quite conceivable that larger landholdings should be more intensely used than small ones. - & many have said they were.

Besides, acreage is not same as l.v. -- (tho it inclines that way.)

~~Recheck notes on RM Haig.~~

"owner-op. landholdings" – always use that term.

Introduce II, 2.

~~Explain~~ Explain briefly what you will show, & what it proves.

Recall II, 1. Great contrast of size of landholdings, owner-op. We figure maybe reason is land mkt tie-in. – so some buy more land than can use best.

If that is so, then ~~certainly expect large land~~ one manifestation wd. be that large holdgs are less intensely used – combined w smaller proportions of labor & cap. – Brodey, for a th Now this does not prove the hyp. true – there are other possible explanations of less intense use of large owner-op. landholdgs. But it certainly lends credence to it, esp. if disproportion is extreme. (& with it, lends credence to assoc. hyp., which is our main current hyp., that M° farms differ & again, if large o-op l-hldgs are more intense, our hyps. must be wrong. – & there is stat chance they might be --- not foregone conclusion otherwise.)

Intensity varies

&

as proportion of land Δs, everything tends to get bigger — land most of all, of course, but also others.

Thus ~~passing~~ indicating that one cause of different intensities is land holdings too small to spread opt team over; & on other hand, holdings too large to mnge

For otherwise why wd intensity be so closely related to size?

II, 2

A. Evidence
B. Implications

A. A second basic fact suggesting that ~~90%~~ of land varies from ~~one holding to another~~ farm to farm is the great contrast of use intensity from farm to farm. Putting it another way, the proportions in which farmers combine land with labor & capital vary markedly from farm to farm. And these variations are related to size: ~~by~~ ~~any criterion of size~~ ~~the more land per unit of labor &~~ ~~capital~~ the larger the farm, the less labor & the less capital it employs per unit of land value.

Some readers, well-versed in farm production economics, may think that almost too obvious to bother proving. But there is a widespread belief to the contrary, even among those who should know better. E.g. ... Levy, Comm. Cl., find others. (but they say large are cap intensive, not labor. So deal separately w labor & cap.?

Again, some readers, wise in the ways of statistics, may observe "It all depends on what you take as the criterion of size. If you msre size by the amount of land area or value in a farm, naturally those fms whose mgrs have subbed land for other resources will fall in the upper size brackets, & also be land-intensive. But if you chose the am't of labor as your criterion of size, you might find the largest fms, so msred,

(It need not be so)

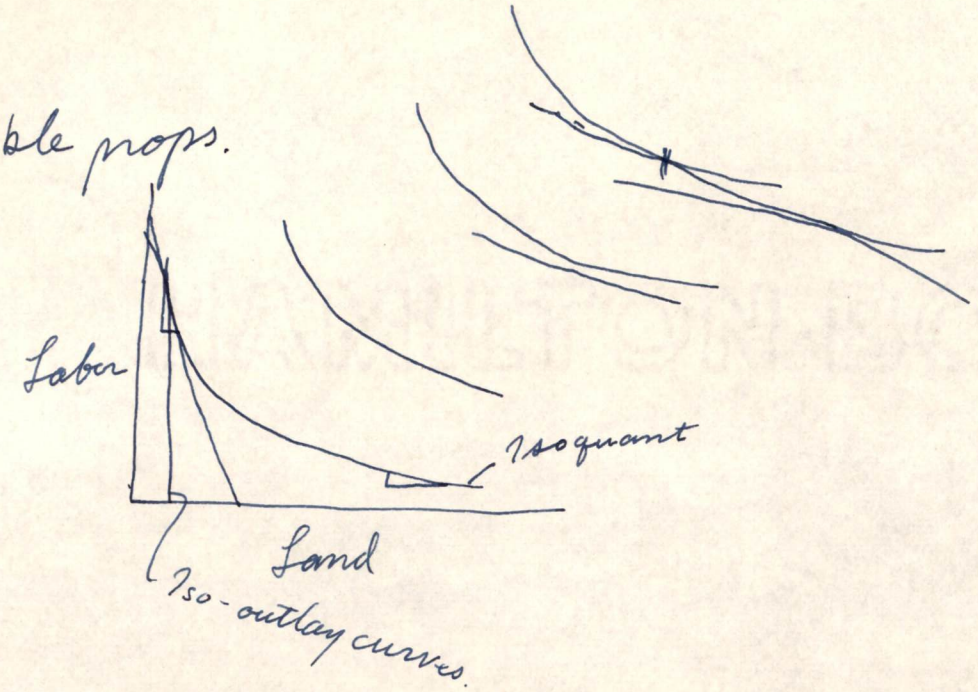
are labor intensive"

On that, 2 comments

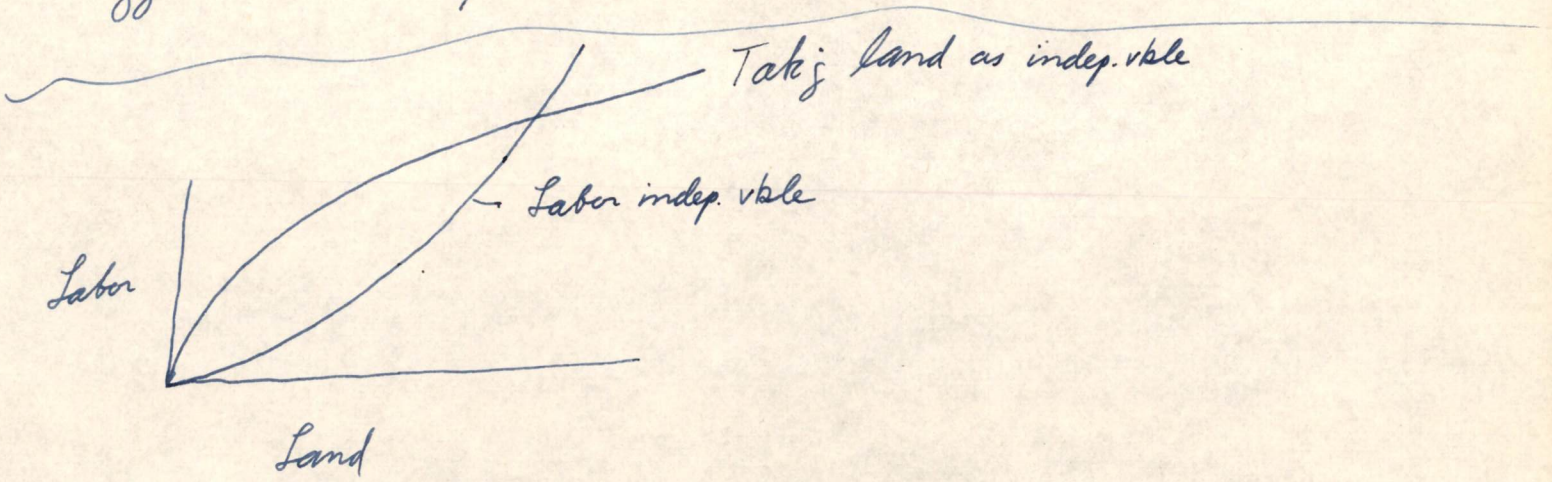
1. That is exactly a pt I am try'g to make: those who have large hold'gs are often subbing land for labor & cap, rather than Δ's all 3 together, & Δ's output & that does not detract in the least from the suspicion that large hold'gs are less intensely used, w lower MP of land. Rather, tends to corroborate it.

Our hypoth, at least as briefly as we have revealed it thus far, does not require that larger firms be less intensely used when size is mixed by something other than the amt of land. We have only said that those who buy primarily as an investment will tend to have more land than they can put to best use. Sometimes that will be because, tho they have little land, they are preoccupied elsewhere. But more often it will be bcs they have more land than they can mnge most effectively, even giving it their full attention. So, gnrly, we expect these undersused holdings to be larger, in terms of land alone

Vble props.



Cd. present this near beginng, showg land cheaper to bigger. Then say data are consistent w this.



II, 2. A. Evidence
B. Implications: MP higher where more complements handy. Vble props.

A. If MP of land really ~~differs from~~ increases from small to large holdings; & if our hyp explains why; then expect proportion of land-value/labor to Δ with size of hold'g; & also, to a lesser degree, land-value/cap.; & also l.v./gross output

Must be careful how we msre size. If by l.v., natch expect larger fms, so msred, to be more land intensive

That wd not prove much.

So msre size by gross output — a neutral measure, which can be Δd by more labor, cap., or land.

They ~~H~~ do~~H~~, when we msre size by acreage. Present evidence. — also, some who believe otherwise -- e.g. Levy, Comm. Club.

~~Then~~

But some may think that not signif. ... no matter what we took as criterion of size, wd find that most conc. But not so. Fact is, no matter what we take, as criterion, still find land most conc., tho less so.

But take a criterion that is neutral as to factors.

Gross output. ... etc.

(But how prove fewer bldgs than land? Try old census. Also, note particular land intensive crops.

As to land & bldgs, take a criterion that is neutral as between the 2: value of land plus bldgs.

MNP ~~~~ has not been used re scale. But seems admirably suited to problem, & esp to integrate it w trad-nl evaluation in terms of elements, & study implications re interfirm (overall social) effic. of ~~land use~~ as affected by scale.

Approach: I've examined works of - --- & find no analytical framework exactly suited to analysis present qn. Hence contrive one — very simple.

Crux of matter is selecting unit on abscissa. ....

Surprised none has developed MNP curve as tool of analysis ecs of scale. Or leave this out, express only admiration for these men.

List works consulted:

S. Carlson (S425)

Chamberlin, E.H. (S426) (comes close but doesn't get there)

Stigler, G., (S427, 428)

Samuelson (S429) ~~HB171 S28~~

Black, J.D. (S430, 431) ~~HD171 B45~~ ~~S561 B55~~

Heady ~~HD HH H4~~ & ~~S561 H4~~ ~~HH5686 AS H62 1949~~

~~Joan Robinson~~ ~~HB 201 Soc. Sci. R6~~

Lerner - ~~HB34 L46, also in soc sci & Gi.~~ HD5406 L415, HD82 L37

~~Kaldor~~

Boulding ~~HB171~~ .5 B64 L955 ~~Soc. Stud. & Gi.~~

Black, Clemson, Sanford Wilcox ~~AFCL, main cat.~~

Weintraub.

Black, J.D. & Albert G. Prod-n Org-n N4: H. Holt & Co '39 ~~Gi~~ HB 171 .5 B53

Hicks

Machlup

Robertson

Edgeworth

~~Agnbks Chipota~~

Bain.

Machlup, Fritz. "Marginal Analysis & Empirical Research."

5465 AER 36(4): 519-54. (Sept '46)

p. 530 Uses mgnl net product.

"The use of the word "net" is preferred in order to stress the fact that additional output will rarely be produced efficiently by increasing the employment of one particular factor while leaving all other outlays unchanged; as a rule, some other adjustments will be appropriate. That "marginal productiv ty" refers regularly to a net revenue product has been clear to economic theorists for over fifty years." Cites Marshall, Principles, 8th ed, p. 521: "the net product. . . is the net increase in the money value of . . . total output after allowing for incidental expenses."

HH

Allows in MP any adjustment; a, pure subst-n, no Δ output; b, Δ output, other factors nearly const. c, Δ other inputs, even to where other inputs absorb most of gross mp

p. 531-2, Mnp does not deduct any cost of hiring factor itself.

✓ re o.c. of land

As everywhere, no hint of carrying thru to whole scale picture.

?

Put matter of ch. 3 pp 51-6 in % terms.

Before max ANP, Δ land by 1% (w. opt. compls) & Δ output > 1%.

After " " ~~" " , i.e. < 1%~~ " " " " " " < 1%

That is rough. How handle other inputs? By subtracting their cost from output.

Now it is no norm to achieve const. partial returns to one factor, unless others are all free.

But is a norm to achieve const. net returns to one factor.

Let a 1% Δ land bring a 1% Δ net output.)

Intro – bring in conflict in o.c. ideas. – o.c. of land, or funds in title. Both seem right.

Conflict due to indivis. O.c. of land is ~~ult~~ basic. Other is individual. L in society where indivisible, maybe social cost too. Will take trad-nl o.c. of land, show how indivis. obstructs it.

✓ A virtue of mrp of land analogy to mp, L is not misleadg. For prop-ns do change, just slower.

✓ Here is a reason for preferring mrp of land to other inputs

~~Re Chamberlin's criticisms~~

Bain. Joe S. Pricing, Distrib'n, & Employment N.Y.: Henry Holt & Co. '48

No help on l.r. MP

There is label for me - l.r. mp

I am much surprised to find that none of these authors has ever developed techniques just fitted to present

S430 p. 564. Black uses composite units of land, labor & cap.,
but doesn't say how composed. (Wicksteed used "cap-&-labor" composite
in 1894. Stig., prod.-& distn Thies | S427)

Lerner uses MVP, pp 109, 112, 115. Seems to feel no expl. nec. Defines it as ? wd. Is given as solution ~~to~~ to alloc-n problem where opt. bundles of factors vary between alt. uses. No opt. size application. That is my contrib.n.

"On the Meaning ... 5463

Machlup, p. 172 Wieser & Pareto objected to holding all const. in MP. Moved toward np. Hicks gave it that name. Marshall first used term, but Marshall didn't separate it from MP. Hicks "Theory of Wages" in Economica 12 (1932)

Hicks "Principle of Variation" let proportions vary in ea. combo., use for long run.

Marshall: "The net product ... is the net increase ~~of~~ in the money value of ... total output after allowing for incidental expenses." Princ. of Ec, 8th ed, ~~p. 527~~ p. 521

So his "net product" is mgnl net product.

~~re land~~

p. 154. "If some definite size ~~is taken~~ (of land) is taken as the smallest unit, it is done so, not bes of any limited divisibility of land, but bes of the limited div. of its compl. factors."

5463

Machlup

Pigou, 5461

p. 121, uses mnp to define = mgnl, & max. social dividend. – but nowhere defines mnp. Only Lerner does so.

– altho has a chap. hdg on def. of mnp.

Stigler, s 428 (Thy of P.)

p. 102

✓ "A difference between alternative cost and the value of the marginal product in any firm or industry is proof of inefficiency, and the magnitude of the difference is a clue to the extent of the inefficiency."

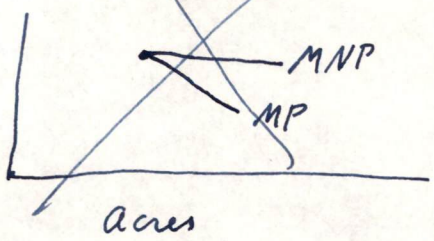
✓ "Efficiency he defines as actual output / maximum output, from given resources.

He's very good, pp. 101 ff., on efficiency, and alternative costs. Leaves suspicion that his concept of alternative costs is a little crude. He mentions using resources in other industries. More feasible alternatives may often be, I would say, using them by other plants in this industry; or using them better by same plant. This last is especially necessary re forestry problem, e.g.)

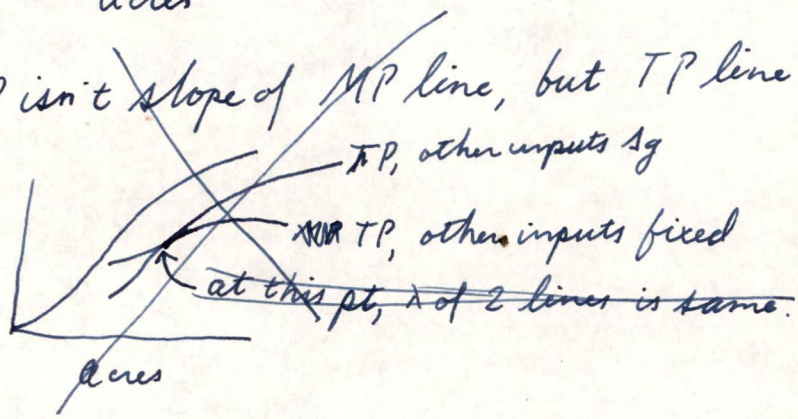
p 221 ff. J B Clark, marked pioneer, in allowing full variability of form.

5464, Robertson

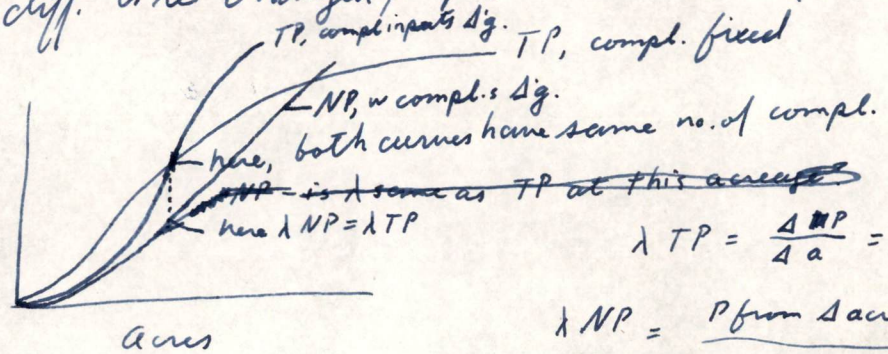
MP is slope of a line, at a pt. MNP & MP wd have diff. slopes, no?



But MP isn't slope of MP line, but TP line.



MP is Δ output on new acre – reduced output on orig. acres. This -Δ output is Σ MP of labor moved to new acre. W. MNP, supply new labor from outside, subtract its cost. This cost is same as Σ MP of labor from original lands which now needn't be moved. Only diff. is re changed props, which is no effect.



$$\lambda TP = \frac{\Delta MP}{\Delta a} = \frac{P \text{ from } \Delta \text{ acre} - \Delta P \text{ on orig. acres}}{\Delta a}$$

$$\lambda NP = \frac{P \text{ from } \Delta \text{ acre} - \text{cost of hiring complements}}{\Delta a}$$

As $\Delta a \to 0$, the 2 λs approach equality.

When $\Delta a > 0$, to rt. of this pt. of = λ, λ NP is greater, bes get greater P from Δ acre, & cost is less (bes w. fixed compl's, their MP rises)

To left of this pt, v.v., λ TP is greater

p. 117 In defining mp, Stigler lets other factors, which are fixed, be fixed in quantity, but adaptable in form. Thus, adding 11th ditch digger with constant capital, can have 11 shovels, costing same as 10 original shovels. (But this makes it a long run concept, no? And if that, why not let quantity of others vary as well as form? Actually, what I have done in chap. 3 is unify solution of proportions and scale, and take artificiality from it. Let all vary. One might define at least 3 different MP concepts: 1, other factors fixed in form and quantity; 2, others factors fixed in quantity, not form; 3, others variable in both form and quantity. (4, others variable in quantity, not form).

Scale line approach, I cd. use, is common- Bouldg. e.g., uses.

It really takes output as indep var, asks what inputs you'll use. Must know their costs to answer. My method lets you not know land cost.

Validity of my parameter type concept corrob. by Bldg p. 762, labor-land analysis.



(over)

Qn. for Matt. re S. Carlson.

Suppose $x = f(a, b, c, d \ldots g)$

Want to know dx [when a…g all Δ by const. %, "m".]

So get $\frac{dx}{da} \ldots \frac{dx}{dg}$

Take Δ of ea. input ma, mult. x its mp, $\frac{dx}{da}$, add up, get dx

(A): $dx = \frac{dx}{da} ma \ldots + \frac{dx}{dg} mg$

S. Carlson does this, p. 17.

But: 1) m is finite. (Ed. it be infinitesimal?) (Yes, that is answer I think) (Then don't need homog. to make work)
And, 2) as each input changes it changes mp of others. These ~~offset~~ oppose ea. other, but do they offset?

1. Since m is finite, $\frac{dx}{da}$ etc. all change during movement & ~~$\frac{dx}{da}$~~ $\frac{\Delta x}{\Delta a}$ doesn't $= \frac{dx}{da} ma$, but is probly smaller, due to decr. returns.

2. ~~When~~ When Δa, raises ~~$\frac{dx}{da}$~~ $\frac{dx}{db}$ etc.

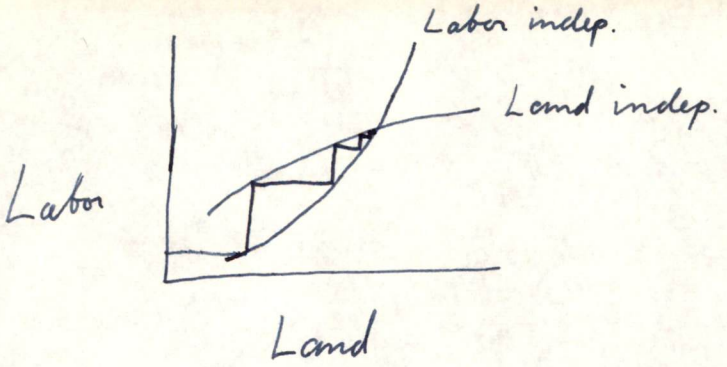
So is eqn (A) valid?

If not, what cd be done? Make $m \to 0$? Isn't ass. of fixed proportions too limiting even then?

Is it valid to conclude from (A) that $x = \frac{m}{\frac{dx}{x}} \left[\frac{dx}{da} a \ldots \frac{dx}{dg} g\right]$

--- i.e. that when ~~$\frac{dx}{x}$~~ $= m$, $x = \Sigma mp$?

also: what is $\frac{d^2x}{dv_1\, dv_2}$?



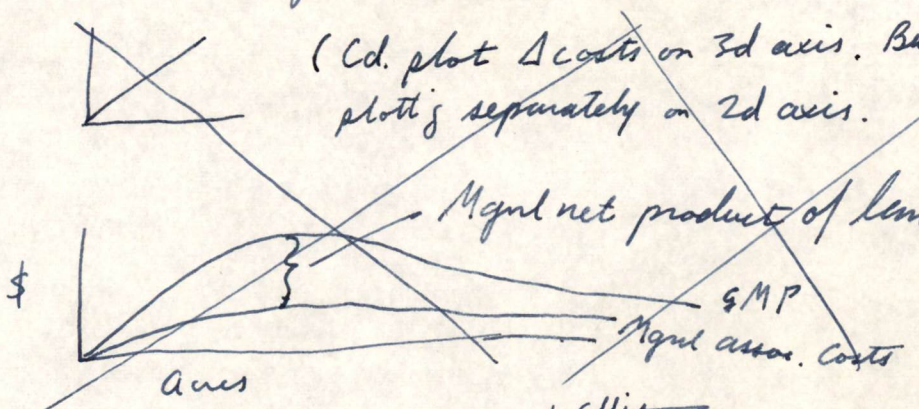
Pick fixed land, then if below opt. scale combine much labor, for, as land is fixed, must approach ec. of scale by using it intensively.

V. v. for labor.

Along stepladder line, findg opt size becomes a matter of starting w some acres, adding opt men, then takg men as fixed, add opt land, & so on until no more change.

This technique involves concept that for every amt of land, is an ideal complement of other factors. Given that concept (as expounded in Bldg, p. 762) I can combine the doses, subtract cost of others. Let land be indep. vble, others reach best adjustmt to various amts of it.

(Cd. plot Δ costs on 3d axis. But this adds little to plottg separately on 2d axis.



~~For MNP, check Pigou, other Engl. welfare ec. sts. Lerner's use of MNP suggests Engl. use it.~~ ask Ellis

~~this parallel treatment of it. costs & its products~~

Leg. McPherson, W.W., "Economic Concepts and Analysis of Costs" JFE 33: 195

5427

Stigler, Production & Distribution Theories. Obviously unsympathetic to Marshall, re "net product". But a, doesn't really say what Marshall was talking about; b, concerns distribution & Euler's theorem, not problems such as I'm especially concerned with right now; c, is after various quirks of Marshall, not essential to present matter. Marshall was struggling vs. mp theory somehow, by insisting that should let other factors vary, & Stigler doesn't like this

✓ p. 356 Stigler concludes that "Marshall's net and marginal products are identical". This means Marshall's use of the terms, that is all. From quotes, it does indeed seem that when Marshall wrote "net product" he meant increased product from one more unit; and at first he subtracted some associated costs, and later did not. Stigler's conclusions refers only to Marshall's terminology, which has some odd quirks and lack of clear definition. In no way implies that general concepts of net and marginal are identical.

(At bottom of lrcost curve you have linearity, instantaneously, but not nec. homogeneity. At top of my lranp of land curve, have linearity in respect to net product of land. Homogeneity doesn8t matter there. Other resources are getting their mps, with this fixed land complement; the land is getting its ~~mpmp~~ mnp, and perhaps also mp too. Perhaps other factors are getting their mnps, too? Perhaps at this point mnp of other factors equals anp? Figure this out, could be article.)

Weeks agrees w Stig & Marshall on "2d order of smalls".
I don't? What approached this? Not assoc. costs, but
reduced values of MPs of other factors, I think. Weeks
thinks Machlup's def. of MRP ok.

p. 436, Interesting: $Y = \alpha L^x M^y E^z$, where L M & E are inputs,

(S 461) Y is output. Then x, y, z, ~~se~~ give $\frac{\%\,\Delta Y}{\Delta L, M, E}$, & if ~~$\Delta x + \Delta y + \Delta$~~

$x+y+z = 1$, const. returns. But they don't let results vary w size of firm, nor let prop-ns vary, & don't explain well what they mean.

Carlson's $\mathcal{E}$ is not so simple. He assumes <u>all</u> inputs to Δ by <u>same fixed</u> %, "m", & $\mathcal{E} \equiv \frac{\frac{dx}{x}}{m}$. Here is a rather artificial def. I'd say, requiring all inputs to Δ by same %. Better to let <u>one</u> input Δ by a %, add opt. complements of others, subtract cost, maximize net. (Isn't he misusing calculus? $\frac{dx}{dv_i}$ is a rate at a point; but "m" is a finite %, can't apply rate over that spread w/o change.)

(No one can draw a p-n. fn. w several inputs, <u>unless</u> specifies <u>one</u> as indep. vble & <u>limit</u>)

Basic eq-n re which I have doubts: $dx = \frac{dx}{dv_1} m v_1 \dots + \frac{dx}{dv_n} m v_n = m \Sigma \frac{dx}{dv_n}$

~~Another Carlson error.~~

Upshot: $x\left(\frac{\frac{dx}{x}}{m}\right) = \Sigma \frac{dx}{dv_k}$ So when ~~$m \to$~~ $\frac{dx}{x} = m$, $x = \Sigma$ mps.

When $\frac{\frac{dx}{x}}{m} > m$, $x > \Sigma$ mps, & v.v.

Thoughts on prod-n ec., from S. Carlson. 5425

"The" prod-n fn is, let $x \equiv$ output & $v \equiv$ inputs

$$x = \phi(v_1, \ldots v_n)$$

But can't graph that, gives no unit on abscissa. Must take one input, v_k say, let that be the unit. Must have a unit to graph.

S. Carlson p. 15 makes explicit, in prod-n fn. all factors are variable, & method of organizing factors changes w. scale. (Stigler lets it change (even w/o scale)

Next he defines MP as partial derivative. Ok, but now how let all vary?

If graph ea. input separately can get just 2, on a 3-d graph. (Incidentally, a 3-d graph is much easier for me to read if 3d axis goes back, not out toward front) (Carlson's 3-d, p. 18, reads very easily)

p. 17, defines ε (epsilon) as "function coeff.", $\frac{dx/x}{dv/v}$, i.e. $\frac{\%\,\Delta x}{\%\,\Delta v}$, or elast. of prod-n. The dx is what you get when all inputs Δ by same %, "m": (Obviously if $\varepsilon = 1$, const. returns, etc.

For eg., $\varepsilon \leq 1$ for all factors, i.e. MP < AP, ea. individ. factor in stage of dim. ave. ret. Simul., factors as whole in stage of const. ave. returns. (Which means, Δ land by one unit, & assoc. factors to MP = MC, & the % Δ costs = % Δ output. But this involves ~~putting a value~~ attaching a 'cost' to the land, to make commensurable w. other inputs, to compute % Δ inputs. How define % Δ inputs when in commens.?)

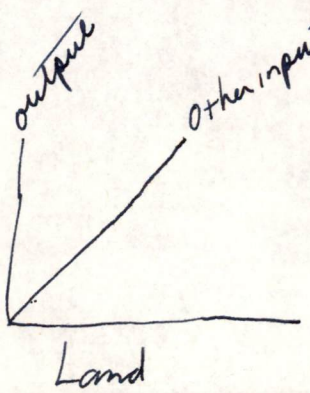
Chamberlin, E. H. "Proportionality, Divisibility & Economies of Scale."
S426 QJE 62(2): 229-62

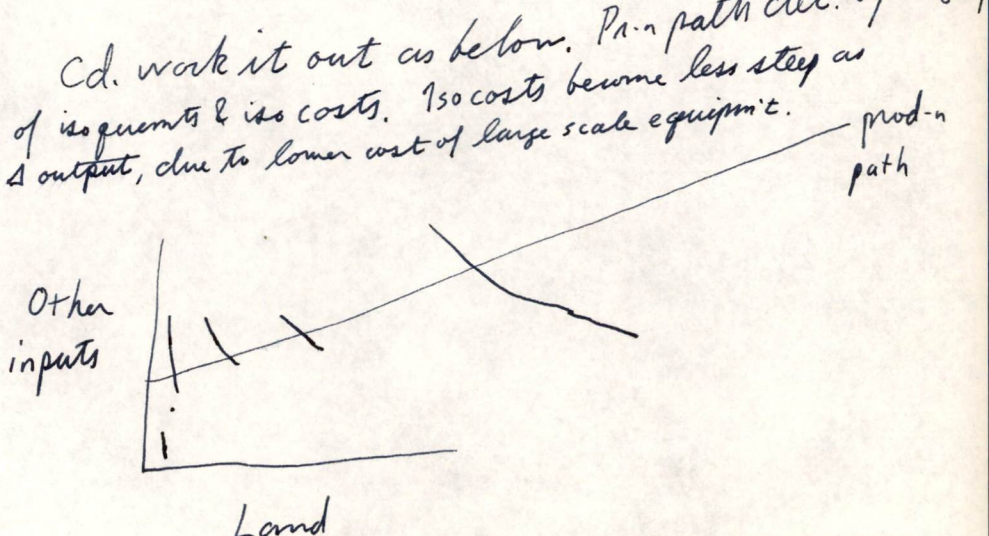
p. 252. Bottom of long run cost curve. Euler applies at bottom of a constant proportions curve. Only at bottom of LRAC do these coincide.

bcs Euler requires const. prop-ns.

Ch. says it adds nothing to say ec. of scale due to indivisib., bcs perf. div. wd. simply mean absence of ec. of scale. Probly a good pt. He runs it into ground.

Cd. work it out as below. Pr-n path det. by tangency of isoquents & iso costs. Iso costs become less steep as ∆ output, due to lower cost of large scale equipm't.





Old story of mixing land with cap.

s519, p. 8. Cobb-Douglas functions, classified inputs into 5 groups: Cropland, labor, workstock and equipment, productive livestock and feed, and current operating expenses.

Note there is no separate buildings category. They must have included it with "cropland". No wonder they conclude these farms don't much need more "cropland" !

Find basic description of "Cobb-Douglas functions".

"Cobb-Douglas" functions, says Cleon, follow a particular equation, with parameters to be filled in from observation. Deriving from Douglas discovery in the '30s that many actual functions more or less followed this form. Using it subsequently I judge involves a considerable a priori assumption that functions follow this form. Also, what is independent variable?

In working out MP of land, note are not adding homog. resources, bec location differs.

Take this into acct by defin'g MP net of xptn costs.

But this changes MP concept a bit. To keep units homog. to individ. fmrs, they must keep gett'g larger as more distant from farm. Only then may MP be other than individualized to specific land units.

(Men & machines are probly much that way, too)

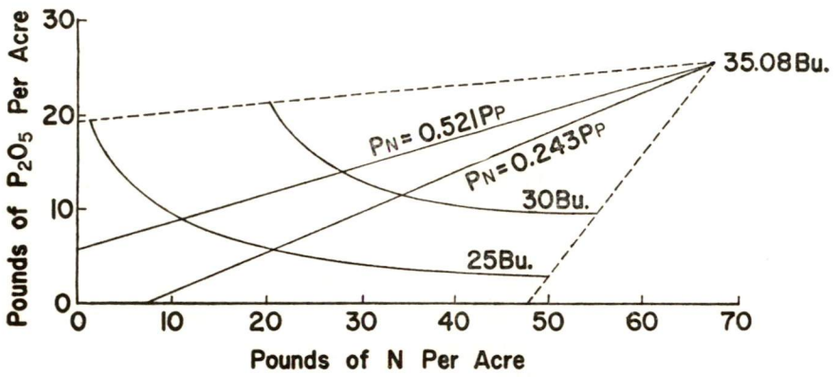


Figure 1. Isocline-isoquant map indicating least cost combinations of nutrients. Dashed lines are ridge lines.

cussed by Doll[3], Heady and Pesek[4], Mason[5], and Stemberger[6]. Also, Hildreth[7] has used a maximum likelihood technique to estimate crop response to fertilizer.

When presenting isoquant and isocline maps, it is often informative to depict ridge lines. Ridge lines define the limits of substitution between the nutrients, e.g. the limits beyond which yield cannot be maintained at a given level by replacing one nutrient with the other. Beyond the ridge lines, both nutrients must be added to maintain or increase the yield level. Ridge lines can be derived by setting $\frac{\delta Y}{\delta N} = 0$ and by solving for N at various levels of P, and by setting $\frac{\delta Y}{\delta P} = 0$ and solving for P at various levels of N. Ridge lines predicted by the quadratic equation presented by Tolton *et al.* are shown in Figure 1.

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[3]J. P. Doll "Evaluation of Alternative Algebraic Forms for Production Functions." Unpublished Ph.D. Thesis, Iowa State College, Ames.

[4]E. O. Heady and J. T. Pesek "Some Methodological Considerations in the Iowa-TVA Research Project on Economics of Fertilizer Use" in **Economic and Technical Analysis of Fertilizer Innovations and Resource Use** (ed. E. L. Baum, E. O. Heady, J. T. Pesek & C. G. Hildreth) Iowa State College Press, Ames 1957, pp. 158-166.

[5]D. D. Mason "Functional Models and Experimental Designs for Characterizing Response Curves and Surfaces" in **Methodological Procedures in the Economic Analysis of Fertilizer Data** (ed. E. L. Baum, E. O. Heady & John Blackmore) Iowa State College Press, Ames 1956, pp. 78-98.

[6]A. P. Stemberger **Economic Implications of Using Alternative Production Functions for Expressing Corn-Nitrogen Production Relationships** North Carolina Agr. Expt. Sta. Tech. Bul. 126, 1957.

[7]C. G. Hildreth "Point Estimates of Ordinates of Concave Functions" **Journal of the American Statistical Association** Vol. 49, 1954, pp. 598-619.

By setting $\frac{\delta Y}{\delta N} = 0$ and $\frac{\delta Y}{\delta P} = 0$ and solving them simultaneously, the input rates at which the predicted maximum yield occurs are obtained. This nutrient combination also defines the point on the input plane where all isoclines, including ridge lines, converge. This point for the data under consideration is shown in Figure 1; the maximum predicted yield of 35.08 bushels of barley per acre occurs when P_2O_5 and N are applied at 25.85 and 67.68 pounds per acre, respectively.

The experiment from which the barley yields were derived was replicated 10 times. Fertilizer experiments in which regression analysis is to be used can be conducted utilizing more rates or combinations of rates of the fertilizer nutrients with a reduction in the number of replications. Box[8] and triple cube[9] designs are now being used in fertilizer response studies. These designs reduce the number of treatment combinations and experimental plots necessary for conducting factorial experiments with 2 or 3 variables. Also, regression equations permitting economic analyses of the fertilizer rates can readily be fitted to the data from these designs. The Box design reduces the number of treatment combinations of a 5 x 5 x 5 factorial experiment from 125 to 15 treatments per replication. Using the triple cube, a production surface of a 5 x 5 x 5 factorial can be estimated with 31 treatment combinations. Such designs can conserve research resources used in conducting fertilizer experiments in a given field, greenhouse, or laboratory experiment.

The capital position of the farmer and the cost of applying fertilizer should be considered. Under an unlimited capital position it would be desirable to apply the rates of fertilizer specified by the maximum profit position. However, under a limited capital position lower rates should be applied. A method concerning this latter problem has recently been developed by Pesek and Heady[10]. It provides a means of estimating the lowest rate of fertilization that should be recommended to a farmer. The *lowest* rate is stated to

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[8]R. J. Hader, M. E. Harward, D. D. Mason, and D. P. Moore "An Investigation of Some of the Relationships Between Copper, Iron, and Molybdenum in the Growth and Nutrition of Lettuce: I. Experimental Design and Statistical Methods for Characterizing the Response Surface" **Soil Science Society of America Proceedings** Vol. 21, 1957, pp. 59-64.

[9]T. E. Tramel "A Suggested Procedure for Agronomic-Economic Fertilizer Experiments" in **Economic and Technical Analysis of Fertilizer Innovations and Resource Use** op. cit., pp. 168-175.

[10] John Pesek and E. O. Heady "Using Data on Crop Response to Fertilizers to Calculate Rates Resulting in Highest Returns on Investments in Fertilizer and its Application" **Soil Science Society of America Proceedings** Vol. 22, 1958, in press.

be the rate which returns the highest net return per dollar invested considering prices and the cost of fertilizer application.

Finally, the inherent errors of experimentation should be fully recognized. The predictions of economic optima are point estimates of unknown reliability. Confidence limits for such estimates define the degree of reliability of the estimates and should be included in an analysis of experimental data.

Mason Gaffney

(*Reprinted from Canadian Journal of Agricultural Economics, Vol. VI, No.* 1, 1958)

NOTE ON *PHYSICAL AND ECONOMIC RELATIONSHIPS INVOLVED IN FERTILIZER USE*[1]

ROBERT D. MUNSON and JOHN P. DOLL[2]

TOLTON, Gilson, and Hedlin's article deals with the economics of fertilizer use, a subject in which there is growing interest. Because it may be used as a model for future agronomic-economic research, we would like to clarify a portion of their analysis and provide further information which may be useful in planning and conducting such research.

The isocline and isoquant map presented in their paper could lead the reader to an erroneous conclusion concerning the shape of isoclines derived from a quadratic equation. As explained in their article, isocline equations are obtained by equating the marginal rate of substitution to the inverse of the price ratio of the fertilizer nutrients. When this is done for their equation

$Y = 16.4643 + 0.8114P + 0.2389N - 0.0208P^2 - 0.0025N^2 + 0.0039NP$, the resulting isocline equation is:

$$\frac{\frac{\delta Y}{\delta N}}{\frac{\delta Y}{\delta P}} = \frac{dP}{dN} = \frac{0.2389 - 0.0050N + 0.0039P}{0.8114 - 0.0416P + 0.0039N} = \frac{Pn}{Pp} \qquad (1)$$

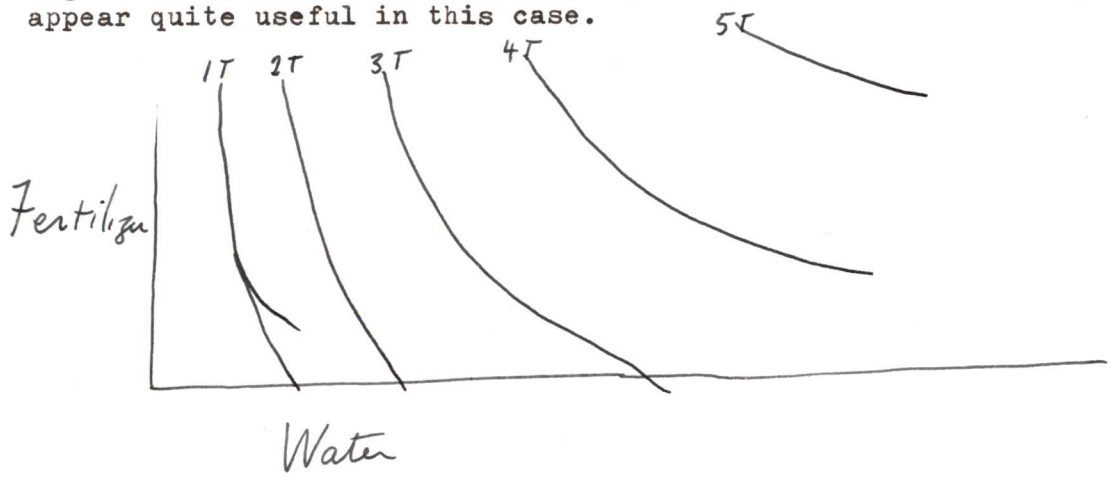
This equation can be simplified further by solving for N as a function of P and the price ratio or by solving for P as a function of N and the price ratio. However, it can be seen from examination of isocline equation (1) that the relationship between the variables is linear. Therefore, the quadratic equation presented in the article must predict linear isoclines which pass through the nutrient axes and converge at the point of maximum yield. The isoclines in Figure 1, derived from their equation, show this relationship.

However, there are functions which predict curved isoclines. These functions might include cubic terms, square root terms, or other transformations of the original variables. Problems involved in selecting functions for use in fertilizer research have been dis-

[1]By Tolton, Gilson & Hedlin. This Journal Vol. V, No. 2, 1957.
[2]Tennessee Valley Authority

Knetsch, Jack L., and Parks, W.L. "Interpreting Results of Irrigation Experiments -- a Progress Report." TVA Report N° T 59-1 AE. Knoxville, 1958.

Response of yields to fertilizer (nitrogen) plus water, latter measured negatively as drought. Production isoquants result, also give results as regression equation. Show yields respond to water a good deal, mores at higher levels of fertilizer. The isoquants appear quite useful in this case.



After a while, requires ~~a good~~ more fertilizer to get results from more wtr. And as you get more wtr, $1 fertilizer is more effectin, natch.

This experiment suggests map schedules of wtr may tail out a long way, — and this is at heart of theoretical determination of opt. distrib. net.



$$\text{Yield (Tons)} = 3.01 + 0.1506N + 0.0010D - 0.0023N^2 - 0.0007D^2 - 0.0005ND$$

Where N is Nitrogen, D is Drought. Note that a little drought improves yields.

Knetsch's practicable method of findg optima is interesting & easy.

Get yield as fn 2 inputs, $=n$. Set derivatives of yield on each input

$= \frac{\text{price input}}{\text{price yield}}$, get $2 = ns$, solve simul.

This is no doubt s.o.p. somewhere, but good to know.

Knetsch, Jack L., and Smallshaw, James. "The Occurrence of Drought in the Tennessee Valley." TVA, June 1958

They analyzed data from sample plots thruout Valley, and weather stations, computed probability of so many days of drought per season. Also of consecutive drought.

Probability of drought increased toward west, despite average rainfall not increasing.

Defined drought as days on which soil moisture in root zone had fallen to zero.

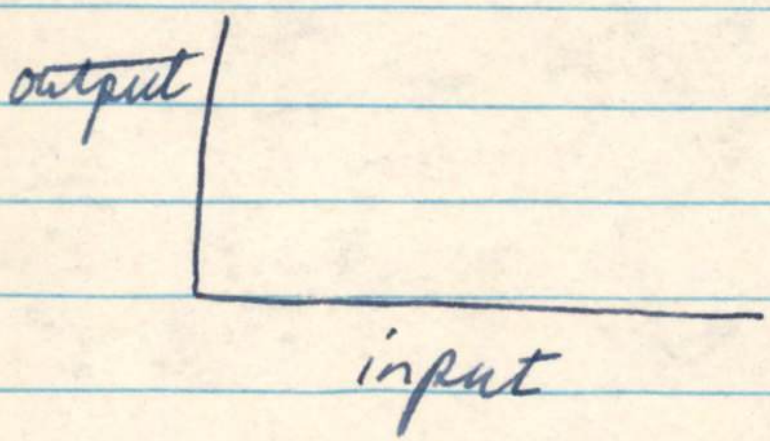
Don't do much else with it.

Heady, Earl O., and Shaw, Russell. RESOURCE RETURNS AND PRODUCTIVITY COEFFICIENTS IN SELECTED FARMING AREAS OF IOWA, MONTANA, AND ALABAMA. Ia. AES Research Bul. No. 425, April 1955

p. 355 In Ala. sample, for example, returns to labor and capital are very low on certain "low-labor" farms, but operator can afford to use them anyway, because he is not paying its mp for land, therefore "a portion of the marginal product of land can be used to reward capital services and labor."

Worth further study -- and quoting sometime

Heady open to suggestion. ask for opt. scale. Non-physical factors



Uses "pure resource" as input, or 2 resources in const. prop.

Most "biological" pr fns, speaks of, "pure physical" pr. fns — a basic concept of his, probly a no-no such.

elasticity ($\stackrel{?}{=}$ $\frac{\%\,\Delta \text{ output}}{\%\,\Delta \text{ input}}$?)

$e = 1$ where $MP = AP$

$e > 1$ " $MP > AP$

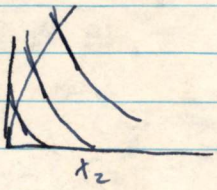
$e \stackrel{?}{=} \frac{MP}{AP}$

Where output = f(2 inputs)

$y = f(x_1, x_2) + C$

The mgnl rate of sub. betw. 2 inputs is ratio of their MPs: $\frac{\Delta x_1}{\Delta x_2} = \frac{\frac{\Delta y}{\Delta x_2}}{\frac{\Delta y}{\Delta x_1}}$

Cobb-Douglas fn won't apply unless one resource is limitational i.e. isoquants do not reach axes.

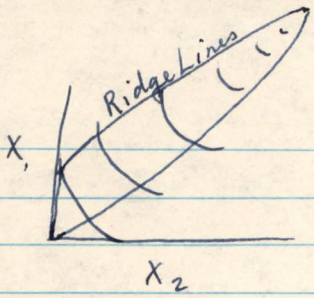


Isocline, path of $= \frac{\Delta X_1}{\Delta X_2}$,

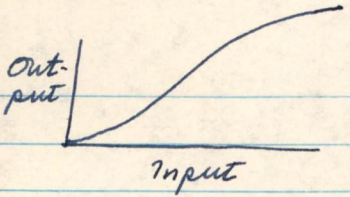
= mgnl rates of substn.

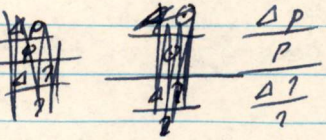
Get different opt. combo. of X_1 & X_2 for diff. scales. Term this the "expension path". (assuming const price ratios)

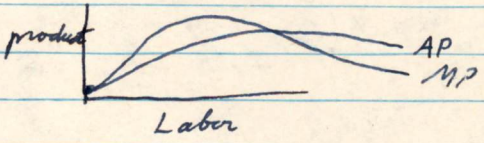
Draw family of isoclines



You get this when one resource is limitational (Presumably some 3d factor, X_3)



$e =$ 



$$MP = \frac{\Delta P}{\Delta L}$$

$$AP = \frac{P}{L}$$

$$e = \frac{MP}{AP} = \frac{\Delta P}{\Delta L} \cdot \frac{L}{P}$$

Simple AP of time has no useful meanj, bcs diff. yrs not comparable, while diff. laborers are. Ea. yr has position! ordinal, not just cardinal. (But so does ea. acre, & we have ~~MP~~ AP of land.?) (But w. land, we deduct xptn cost to figure AP - deduct them along w. all compl. costs.

W. time, my treatment is not quite ||, bcs am includj xptn-thru-time as cost, not deductj from MP. Suppose try discountj all back to yr 0. — next p.)

W. space, ea. looks at it from diff. base pt. W. time, all same. ~~Take~~ Refer MP of space back to base pt. of ea. individ., to cf. individ. MP. But time, can use any base pt., all individs. the same.

Also, to get Δg, must carry whole tree another yr. While add'l another acre doesn't add space cost to all others.