Choosing Harvest Date to Maximize PV of Timber
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Econ 162 Notes by M. Gaffney

CHOOSING HARVEST DATE TO MAXIMIZE PV OF TIMBER

Supplement to class discussion of Wednesday, 28 Sept 88

Our time was too limited fully to deal with the excellent points raised in discussion. Mr. Holmes and Mr. Dingwall and others had a good instinct which, for time's sake, I had to put aside in order to get to the syllabus-timely point about maximizing present value.

However, they were quite rightly thinking about the value of the site for growing the next tree. A fuller solution takes that into account, along with (not instead of) compound interest. This is a great example of optimizing -- remember that means defining an objective function (goal) that combines two or more legitimate considerations.

To bring the future of the forest site into the picture, a good way is simply to maximize PV in perpetuity, looking at a series of future cycles, rather than simply the first, as in our class example. Here is how to do that.

First, to unclutter the notation, I always use the following definition:

$$I = 1+i \qquad (1)$$

That makes algebra much easier to write and read. "i" is the rate of interest expressed as a decimal: that is, 6% is written as .06.

Hewing to Einstein's advice to "make things as simple as possible, but not moreso", we assume that all generations after the first are to have the same life (n) as the first. That's the good news. The bad news is that "n" is what we are solving for, so we have to find them all at one shot. Sounds like a real teaser, but not to worry, that's what math is for.

$$PV = \frac{g}{I^n} + \frac{g}{I^{2n}} + \ldots + \frac{g}{I^{\infty n}} \qquad (2)$$

(2) is a geometrical progression, infinitely long, whose first term is g/I^n, and whose rate of progression is $1/I^n$. Plug those into the formula for the sum of a geometrical progression and we have:

$$PV = g\left[\frac{1}{I^n} \times \frac{1}{1 - I^{-n}}\right] = \frac{g}{I^n - 1} \qquad (3)$$

To max (3) set the derivative w.r.t. n = 0. But now remember our "poor man's calculus", to make it easier. The quotient is max when the % growth rates of top and bottom are the same. That is when:

$$\frac{g'}{g} = \frac{iI^n}{I^n - 1} = \frac{i}{1 - I^{-n}} > i \qquad (4)$$

Note that this solution for optimal harvest time requires that the tree's growth rate (g'/g) > i, which means that g'>gi. (If that is not clear, pause, scribble on a clean sheet, and figure it out -- be sure you really understand.) Compare that with the solution in class of the simpler problem (the one-generation problem) where the solution was g'/g=i, or:

$$g' = gi \qquad (5)$$

Now, in (4), we are requiring a higher growth rate, meaning a shorter life for the tree before harvest.

Clearly in (5), compared with (4), we have added something to the marginal cost of time. We can express that as the cost of deferring future harvests, which is formally correct, but a little hard to visualize. We will therefore show next here that what we have added is the annual value of the forest site, or ground rent. (Foresters use the German name, Bodenrente, because this was originally worked out by a German forester, Martin Faustmann.)

We now ask, what is added to gi in (5) to get the result in (4)? (I leave it to you to rearrange (4) to see what g' equals there.) That unknown extra something is "x":

$$\frac{gi}{1 - I^{-n}} = gi + x \qquad (6)$$

Solve (6) for x (an algebra exercise which I leave to you). You will find that:

$$x = i\frac{g}{I^n - 1} \qquad (7)$$

But the quotient in (7) is PV, from (3), so x is interest on the site value.

$$x = PV \times i \qquad (8)$$

Interest on site value is also ground rent. So "x" turns out to be ground rent. Ground rent is denoted "a", for annual income. So in the future you will see "a" where in (8) you see "x".

So-o-o-o-o, what (6) and (8) boil down to is that tree growth has to pay interest on tree value (gi) plus interest on site value (PV x i), and when the tree stops doing so it is time to cut and cash out, freeing both your capital and your ground for re-use.