Fnce Raw Mtrls

Installment #3 for Ec. 162A. F84, by M.G.

9. Practical influences, (cont.)

e. Depreciation and Amortization, level annuity

How much of annual payment is depreciation of asset? Same formula shows amortization of principal of debt.

Three approaches.

1) $D = \frac{1 - I^{-n}}{i} - \frac{1 - I^{-(n-1)}}{i}$

$$= \frac{I^{1-n} - I^{-n}}{i} = \frac{I^{-n}(I-1)}{i} = I^{-n}$$

(19)

2) Payment is divided between interest and amortization. (Amortization to the debtor means repayment of principal. He is unhappy because it is so slow. But the creditor is also unhappy it is slow: to the creditor it means recovery of his capital. Be ready to look at this from both sides.)

Interest is due at each passing year on the unrecovered principal P_n, where n is the number of years of life remaining (that is neither the original n, nor the number of years that have passed.)

$$\text{Interest} = Pi = \frac{1 - I^{-n}}{i} \times i = 1 - I^{-n}$$

$$\text{Deprec.} = 1 - Pi = I^{-n} \qquad (20)$$

3) Graphically:

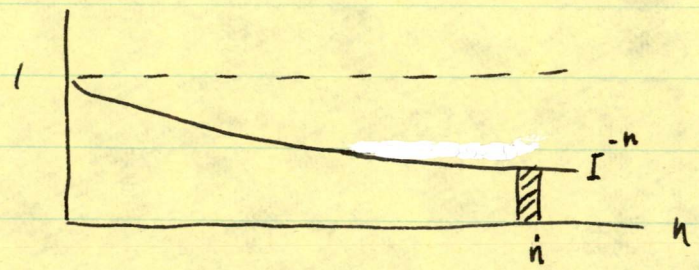


Fig. 6: Depreciation

Shaded column is Deprec. when n is $\dot{n}$. It is $I^{-\dot{n}}$, where $\dot{n}$ is the number of years of life remaining.

10. Appreciation: PV of a rising cash flow

Let C.F. $= 1 + (1+g) + (1+g)^2 + \ldots + (1+g)^{\infty}$

Then $PV = I^{-1} + I^{-2}(1+g) + I^{-3}(1+g)^2 + \ldots + \left(\frac{1+g}{I}\right)^{\infty}$

Sum the series.

$$PV = I^{-1}\frac{1}{1-I^{-1}(1+g)} = \frac{1}{I-1-g} = \frac{1}{i-g} \qquad (21)$$

(Hereafter, use P alone for PV)

Solving (21) for P_i:

$$P_i = 1 + P_g \qquad (22)$$

P_g is ΔP, (by obvious influence from the top line on this page, the definition of cash flow.)

1 stands for $\underline{a}$, any initial C.F. (22) may be rewritten:

$$P_i = a + \Delta P \qquad (22A)$$

Thus P_i equals the sum of current cf. plus current appreciation. Note the symmetry and consistency of this with (20).

11. Periodic payments and repeating cycles.

Solve (11) for a/i

$$\frac{a}{i} = \frac{P}{1 - I^{-n}} \qquad (23)$$

$\underline{a}$, as we have discussed in class, is the n-year annuity corresponding to P.

Dividing $\underline{a}$ by $\underline{i}$ "capitalizes" it in perpetuity. It gives the present value of a perpetual flow of $\underline{a}$. (See (7), p. 11.)

Thus the right side of (23) is the present value of a perpetual succession of P, received every $\underline{n}$ years. Label it Σ.

[If the P repeats, but not forever, we adjust Σ by subtracting the discounted value of the portion of Σ contributed by the later

years, the ones that are cut off. This is the same technique we used on p. 13 to find (11).

Let x = no. of rep.s of P. Then:

$$\Sigma = \frac{P}{1 - I^{-n}} - \frac{P I^{-xn}}{1 - I^{-n}} = P \frac{1 - I^{-xn}}{1 - I^{-n}} \qquad (24)$$

You need not "memorize" (24) for any exam. I just want you to see that and how it can be done.]

12. Positive NPV and Land Values

If investments normally yield positive net present values, NPV, after deducting the initial capital cost, that sounds too good to be true for long, and it is. It creates demand for the land on which these happy results can be achieved. Hence Land Values, which rise to soak up the surplus.

But Land lasts forever, so its value soaks

up the value of not just the first cycle of investment and return, but all succeeding cycles as well. These are called "Generations".

The NPV of one Generation (G) or cycle of investment and recovery is summarized as P at the time the initial outlay is made. We have called this the "present" when dealing with one cycle, alone. In dealing with a succession of Generations, each P has its own future date.

a string of future P may be summed up as a string of periodic payments, as in (23). If a cycle repeats (in perpetuity):

$$L = \frac{P}{1 - I^{-n}} \qquad (23A)$$

where L = Land Value

Take out $\frac{1}{1 - I^{-n}}$ from that and

we have the "Land Value Multiplier". See Table 6.

Table 6: Land Value Multipliers

n \ i =	.04	.08	.12
1	26.00	13.50	9.33
5	5.62	3.13	2.31
10	3.08	1.86	1.47
20	1.84	1.27	1.12
40	1.26	1.05	1.01

To find Land Value, multiply P from Generation #1 by the appropriate figure.

To adjust for anticipated inflation, use a <u>real</u> interest rate — i.e. subtract the inflation rate from the nominal i.r. Thus, .04 or .08 is more realistic than .12.

13. Maturity and Replacement

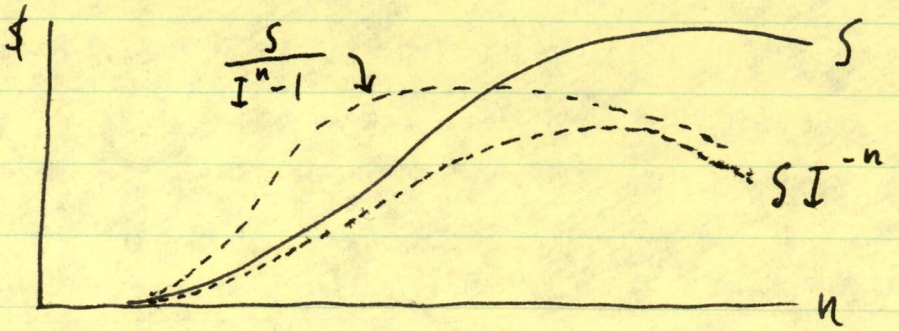


Fig. 7: Growth of Stumpage, Discounted Stumpage, & Land Value

$S \equiv$ Stumpage, the market value of a tree "on the stump", before cutting.

Alternative maximizing solutions.[‡]

1) Max S
2) Max S/n, "Mean Annual Increment"
3) Max SI^{-n}; PV of G_1

4) Max $\dfrac{SI^{-n}}{1-I^{-n}}$, PV of all G_j same as

Land Value, L.

Clearing the fraction:

$$L = \frac{S}{I^n - 1} \qquad (25)$$

as shown on Fig. 7

The economic solution is to max L, using (25), known as Faustmann's Formula. This advances the date of maturity over the other 3 criteria of maturity.

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‡ Solutions to the problem "When to cut the tree?"

[In (25) we have left unspoken the implicit assumption that all costs and revenues in the cycle are made commensurable with S by compounding them forward to year n. See p. 24, first full para., first sentence.]

Every maximizing criterion is also a marginal criterion. Let us see what marginal conditions correspond to the 4 maximizing solutions.

1) $S' = 0$ (S' is shorthand for dS/dn, or annual growth.)

2) $S'/S = \frac{1}{n}$

3) $S'/S = i$

4) S'/S = log derivative of the denominator, $(I^n - 1)$. The log derivative is:

$$\frac{i I^n}{I^n - 1} = \frac{i}{1 - I^{-n}} > i$$

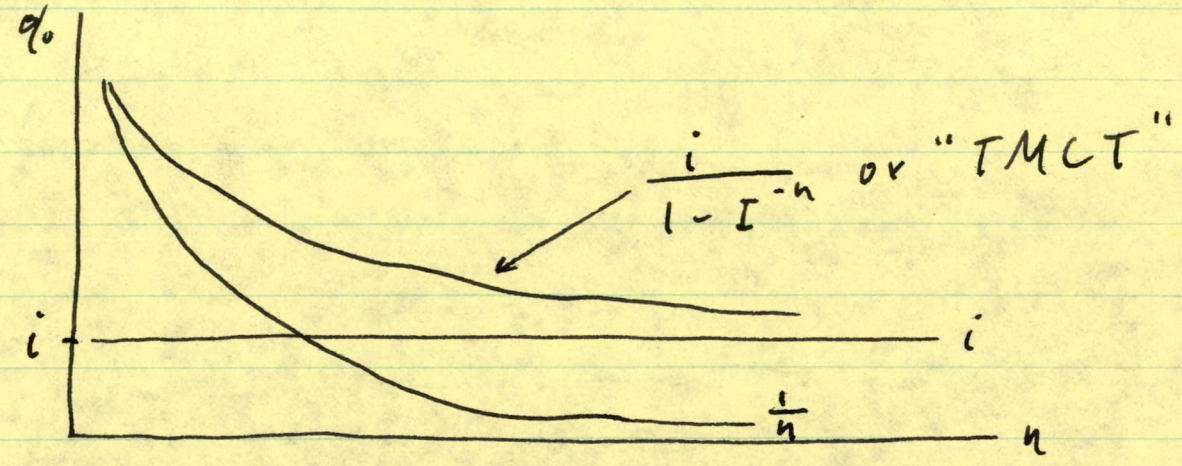


Fig. 8: Marginal Costs of Time Expressed as %

Fig. 8 shows how the true marginal cost of time, TMCT, relates to i and to $\frac{1}{n}$.

- TMCT is always higher than either i or $\frac{1}{n}$.

- For low n, TMCT $\rightarrow \frac{1}{n}$

- For high n, TMCT $\rightarrow i$

To use Fig. 8, overlay one more curve, showing the decline of s'/s as n grows. Cut when s'/s cuts TMCT.