



Finding Optimal Life of Capital Affixed to Land

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Finding Optimal Life of Capital Affixed to Land

Mason Gaffney, 3/92

b = annual ground rent

K = capital cost of building, time zero

IPF_n = Installment Plan Factor over n years

a = cash flow

If a were level we would find b simply by levelizing K with IPF , and subtracting it from a . Here, a is not level, so to levelize it we first find its PV, then multiply PV by IPF .

Then we find the value of n that maximizes b .

Numerical example. A building yields \$2k/yr for 5 years, followed by \$1k/yr, diminishing by 9%/yr. $K=\$6k$. When is b a max?

(In the following equation and table, "k" is omitted.)

$$-K + P = -6 + 2 \times \frac{1-I^{-5}}{i} + I^{-5} \times \frac{1 - (I+.09)^{-(n-5)}}{(i+.09)} \quad (1)$$

Table 1 shows values when $i = .05$.

TABLE 1: P-K, IPF, and b, with $i=.05$

	[$b = (P-K) \times IPF_n$]					
n	5	8	9	10	15	20
P-K	2.66	4.48	4.94	5.35	6.75	7.47
IPF	.23	.15	.14	.13	.10	.08
b	.61	.69	.70	.69	.65	.60

(P-K) rises indefinitely. However, b peaks when $n=9$. Think of b (ground rent) as a kind of "time-average," i.e. an average per year, adjusted for the time value of money. P-K is the total value of cycle-1; b is the value/year.

A lot of economic principles flow from this simple demonstration.

1. A necessary feature of the solution is that cash flow, a , equals rent, b , in the year when b is a max. If it were

higher, it would raise the average, and if lower, lower the average.

Cash flow in this example is the value of the second annuity in (1), after it has been going for 4 years. It begins at 1, and decays 9%/yr, so after 4 years the value is 1.09^{-4} , or .71. (The slight excess over .70 is due to rounding, plus probably that the true max falls at 9+ years -- we only took values at discrete intervals of one year.)

2. The economic cost for missing the optimal year of life looks small, if we focus on the mean rent, $\bar{b}$. However, the marginal penalty is much larger, in the year of decision. In year 10, current $\bar{a}$ falls to 1.09^{-5} , or .65, some 9% below the max. You would then be getting 9% less from the site in that year than its levelized yearly value, which you could get by installing the next cycle of use.

Maxing life-cycle rent is a good tool for planning ahead, estimating optimal life in advance, looking forward from time-zero. You can be a little vague about it then, n years ahead of the event. In the marginal year, the moment of truth itself, the decision is much more critical.

Typically, at that time there is great inertia, both individual and governmental. Commonly, landowners defer replacement too long. The cost is more than a loss of potential rent. Sometimes there is a loss of entire industries, and whole cities.

3. $b = T_i$, where T (for *Terra*) is land value, and $\bar{b}$ is not expected to rise over time.

4. Another approach is possible if T is exogenous (given, known). In the first approach we determined optimal n and $\bar{b}$ (hence T) simultaneously (using a tabular method). Sometimes T is given because you know what another buyer will pay for your land. Then, optimal n is when current $\bar{a}$ falls below $T \cdot i$.

Let ET be exogenous T .

If $ET = .70/.05$, or 14, it is the same as your internal T , and will not change optimal n .

If $ET > 14$, it will shorten optimal n . At earlier years, current $\bar{a}$ is higher. At the same time, mean life-cycle $\bar{b}$ is lower (because you have spread K over fewer years). This says it

would be a mistake to repeat your first cycle, you can do better to sell. (The first cycle may have looked right when you started, but new data came to light in mid-life.)

If $ET < 14$, it will lengthen optimal n . However, if you can repeat cycle-1, the land is worth more to you than to the outside bidder.