



$$L = \frac{1}{i} \frac{1 - I^{-n}}{1 - \left(\frac{G}{I}\right)^n}$$

RIPENING

$$L = d \left[ \frac{1 - I^{-n}}{i} + d(1+g)^n \frac{1 - I^{-n}}{i I^n} + (1+g)^{2n} \frac{1 - I^{-n}}{i I^{2n}} \dots \right]$$

$$= \frac{1 - I^{-n}}{i} \frac{1}{1 - \frac{(1+g)^n}{I^n}} = \frac{1}{i} \frac{1 - \frac{1}{I^n}}{1 - \frac{(1+g)^n}{I^n}}$$

$$= \frac{1}{i} \frac{\frac{I^n - 1}{I^n}}{1 - \frac{(1+g)^n}{I^n}} = \frac{1}{i} \frac{I^n - 1}{I^n - (1+g)^n} \quad (g < i)$$

if  $n=1$ ,

$$L = \frac{1}{i} \frac{1+i-1}{1+i-1-g} = \frac{1}{i-g} )$$

$$L = \frac{1}{i} \frac{I^n - 1}{I^n - (1+g)^n}$$

$$g = .03$$

| $n$ | $i = .04$ | $.08$ | $.16$ | $.32$ |
|-----|-----------|-------|-------|-------|
| 2   | 98.6      | 19.7  | 7.6   |       |
| 4   | 95.7      | 19.2  | 7.4   |       |
| 8   | 90.5      | 18.2  | 7.1   |       |
| 16  | 81.4      | 16.7  | 6.7   |       |
| 32  | 67.2      | 14.6  | 6.3   |       |
| 1   | 100.0     | 20.0  | 7.7   |       |

Morals: ① For  $n > 1$ ,  $L$  becomes slightly less sensitive to  $i$ . But  $L = \frac{1}{i-g}$  is a good approx; basics are not much changed by going to periodic renewals.

②  $L$  is a decr'g fn. of  $n$ . Argues in favor of earlier, not later renewal.

(Exception wd. be where  $g$  has spurts  $> i$ .)

③ The closer  $g$  is to  $i$ , the more sensitive  $L$  is to  $n$

$$L = \frac{1}{i} \frac{I^n - 1}{I^n - (1+g)^n}$$

$$g = .02$$

| <u>n</u> | <u>i</u> = | .03  | .04  | .08  | .16 |
|----------|------------|------|------|------|-----|
| 2        |            | 99.0 | 49.5 | 16.5 | 7.1 |
| 4        |            | 97.1 | 48.6 | 16.2 |     |
| 8        |            |      | 46.9 |      | 6.8 |
| 16       |            |      |      |      |     |
| 32       |            | 16.0 | 38.6 | 13.6 | 6.3 |
| 1        |            |      |      |      |     |

~~The constraint is~~

Sensitivity?

y is "up 370%" from x

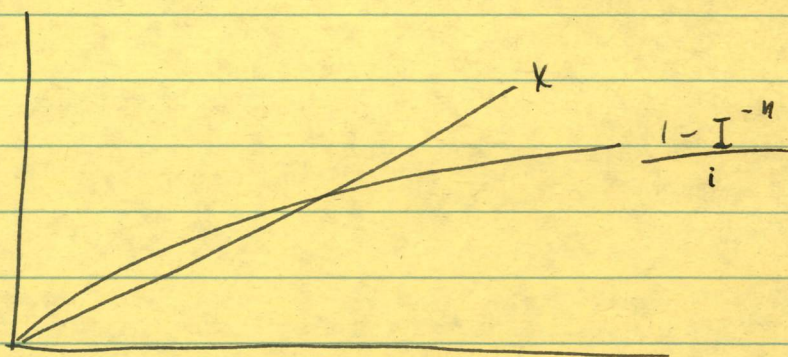
$$x + 3.7x \stackrel{?}{=} y$$

$$4.7x = y$$

Principal as share of ~~the~~ total payments

$$P = \frac{[F(1-s) + PLs] I^{-n}}{[F - s[F - PL]]}$$

| $x$ | $\frac{1 - I^{-x}}{x}$ | $(i = .08)$ |
|-----|------------------------|-------------|
|     |                        |             |
| 2   | .071                   |             |
| 4   | .066                   |             |
| 8   | .057                   |             |
| 16  | .044                   |             |
| 32  | .029                   |             |
| 64  | .016                   |             |



$$S_n I^{-n} > S_0$$

$$\frac{S_n}{S_0} > I^n$$

# Mostly about Optimizing

New Ideas, April, 1995 (rev. Wed., 5 April)

## INTRODUCTION

Limitations of ec theory. It is a tool, not a dogma, creed, or doctrine. Pan "expanding domains of ec. theory." Pan inhumane attitude of some ecsts: Scrooges. Beware their arid theology of extremism.

- Milliman on empty bldgs same as unemployed labor;
- contempt for environment because nobody owns it; --
- contempt for homeless, hungry people;
- notion that you cannot compare subjective feelings.

Ecs. is never an excuse for moral blindness, greed, or cynicism. "Greed is good" - Ivan Boesky, Gordon Gecko; Michael Milken. Idea is not just to make money; it is to make money doing something socially useful. Can make money robbing banks: "Some folks do it with a six-gun; some with a fountain pen."

Min cost only refers to what we include and measure. Misuse of my work on forest maturity, ignoring environment.

### 1. Theme: Optimizing

But note two limitations;

- a, Beware suboptimizing (finding local max)
- b, Composing is better. Give examples, e.g. marriage.

Multiplication, Division, Subtraction, Addition

### Multiplication:

1.  $P \times Q$  to max revs. Show elasticity of demand. To get  $MR > 0$ , must have  $\epsilon > 1$ , meaning  $dQ/Q // dP/P > 1$ . Marginal Revenue: relation to elasticity of demand ( $\epsilon$ ). (See separate sheaf on rectangles.)

2. Another, similar e.g. is  $t \times Q$  to max revs of fisc. Currently called "dynamic revenue forecasting." Precedents in Laffer Curve; in tariff for revenue.

3. No. of stems vs. output per stem - meaning of MP

### Division:

1.  $\text{Cost/Output} = \text{AVC}$

This is often called "efficiency," an intimidating word. Also, its reciprocal is called "productivity," another intimidating word.

Don't let them bamboozle you! This is suboptimizing: min AVC is not enough. Must also consider fixed costs.

AVC, AFC, and ATC

To co-measure, use \$, but that is not enough.

To co-measure point values and flow values, need capital theory. Big part of this course. Need to annualize the point values, thus make them commensurable, so you may COMPARE them.

Comeasuring also needed to weigh land with capital. "User cost" of K a common concept. User cost of K includes interest on basis, plus depreciation, plus maintenance, plus operating costs, plus insurance vs. fire and accident, plus proptaxes. User cost of land is just interest and proptaxes.

But this is still suboptimizing: min ATC is not good enough if  $MC < Price$ . It shows you have undervalued part of AFC, the part that is price-determined. This part is usually land, because the other values have costs of production. Competing away vs. imputing away excess profits.

$$\Delta C = MC$$

Optimizing (or suboptimizing) is finding min value of ATC. Same as where  $MC = Demand$ , if demand is flat. If demand is not flat,  $MC = P$  is still socially best.

2. Discounting, e.g. value of growing tree.

*when you divide, watch the base.*  
Better, discount value of tree plus site ( $S+B$ ). Show how this leads to earlier max, because B is constant.<sup>1</sup>

### Subtraction

1. Revs. less Costs; or  $AR-AC$  (as you expand all inputs)

---

<sup>1</sup>If B is rising, this seems superficially to lead to later max than if B is constant, but I doubt this. Check this out, for material on ripening.

Addition:

1. Add PV of successive generations of discrete land use, to get value of all.

A, Let the market determine salvage value, add it to DCF of  $G_1$ , thus:  $PVG_1 + PVR_n$

B, Assume repetition of  $G^1$  in perpetuity. Apply the land value multiplier, which is  $1/[1-I^n]$ , where  $I = 1+i$ , and  $i$  is the interest rate.

C, Assume constant annual cash flow; capitalize in simple way, divide by  $i$ .

D, Assume rising cash flow at annual rate  $g$ ; capitalize by dividing by  $[i-g]$  ( $g < i$ ).

2. Theme: co-measuring

To co-measure, use \$, but that is not enough.

To co-measure point values and flow values, need capital theory. Big part of this course. Need to annualize the point values, thus make them commensurable, so you may COMPARE them.

Comeasuring also needed to weigh land with capital. "User cost" of  $K$  a common concept. User cost of  $K$  includes interest on basis, plus depreciation, plus maintenance, plus operating costs, plus insurance vs. fire and accident, plus proptaxes. User cost of land is just interest and proptaxes.

But this is still suboptimizing: min ATC is not good enough if  $MC < \text{Price}$ . It shows you have undervalued part of AFC, the part that is price-determined. This part is usually land, because the other values have costs of production. Competing away vs. imputing away excess profits.

$$\Delta C = MC$$

Optimizing (or suboptimizing) is finding min value of ATC. Same as where  $MC = \text{Demand}$ , if demand is flat. If demand is not flat,  $MC=P$  is still socially best.

Now we see a second difference in measuring user cost of land and capital. First is depreciation of capital, (plus maintenance, insurance vs. fire and theft, etc.). Second is that the very measure of land must sometimes be by imputation, where the current use is better than any o.c. use. This is a big subject, only partially treated here, considering that land always has a meaningful, measurable o.c., but specialized capital does not. What the capital has that the land lacks is a reproduction cost.

- 2
3. Opt. combo of inputs
- a. Can't do this with min ATC
  - b. Needn't run thru all the tricks with MP, MNP, etc.
  - c. Use "Newark" model to show how to combine cash flow and K.
  - d. Link the Newark model with AFC and AVC.

- 2
4. Equimarginal principle
- a. Central Management as banking
  - b. Be sure your mps are mnps. Beware APs

- 2
5. Theme: fungible resources vs. land.

Msmt

It is harder to apply equimarginal allocation to land, because it is not fungible, not uniform, not even interchangeable.

- 3
6. Performance standards
- a. Fallacies about performance standards

- 4
7. Theme: sensitivity analysis
- Begin with PIPO only. Show use of  $\epsilon$ . Show use of logs.
- Show  $\epsilon$  of production as MP/AP.

Leverage: show graphically (by cutting off base).

Land Valuation: extreme sensitivity to i.r.

Note inverse relationship of land rent and i.r. Note how this requires distinguishing land and capital. Note misleading nature of micro theory on this: production functions as functions of two factors, labor and capital.

② 8. Theme: Fixed Costs

A. AFC curve shows ecs. of scale; AVC curve shows diseconomies of ~~scale~~ *crowding* *full use, spreading a'head; ①*

B. Efficiency, productivity: can lower AVC curve by raising AFC curve. Often loosely called "increasing efficiency." E.g., new air conditioner to save energy; new ATM to save labor.

Physicist defines efficiency solely in terms of energy output/energy input, which always argues for more expensive equipment. Ecst. includes cost of equipment.

Careless writers treat "productivity" solely in terms of labor cost, which always argues for substituting capital for labor. Ecst. includes cost of capital - and also cost of fuel that capital usually requires.

C. FC, once sunk, are immediately history (but leave legacy of debt).

1. They no longer bear on current optimization, except as they affect MC.

2. If market is limited, they pose important set of problems.

- If limitation is permanent, can never spread the overhead. Cut losses and leave. Milwaukee's Chinese restaurant - Mr. xxx.

- If limitation is temporary, try to observe  $P=MC$  during growth period. Hotelling's bridge.

D. Concept of crowding as a cost, and consumption. Reaching capacity of bridge, roads, wires, canals, restaurant seating, etc.

① But not long prodn. runs.

2  
9. Students are prepped on cost curves. Are they also on production curves? I doubt it, but check.

A. Review dim rets with fixed land, one variable (labor, stems, grazing cattle, fishing boats).

Reasons for taking land as fixed:

1, is likely to be so, to firm, in practice, esp. in short run. For many reasons.

2, is not homogeneous nor fungible, so cannot treat it as variable without standardizing the units somehow, which would be complex.

3, is fixed in space. Adding land to fixed labor would entail relocating labor, but adding labor does not entail relocating land.

4, is permanent in time, likely to be permanent, fixed base of firm operations, while labor and capital come and go. This lets us deal gracefully with replacement.

B. Next, introduce idea of MNP. Each new worker needs floor space, a workbench equipped with latest tools and machines, working capital, allocation of supervisory effort, paperwork, etc. They each need an allocation of the fixed land supply: parking space esp., causing crowding, some have to walk further, etc.

Example: compare math profs with agronomy profs., physicists with their labs, etc.

C. Make  $K$  the independent variable. The "Newark" model. Measure input in  $K \times i$ ; measure output in c.f. Thus, have comprehended many variables -- all except depreciation of  $K$  and appreciation of land.

Note how land rent is spun out as by-product.

D. Divide both  $Ki$  and c.f. by  $K$ , put it in % terms.

E. Introduce  $d$  in simplest way: add it to  $i$ . User cost is now  $K[i+d]$ . C.f. must cover both  $i$  and  $d$ .

F. Modify by using IPF instead of  $i+d$ .

③ 10. Alternatively, can compare point values: compare  $K$  with DCF.  
Carry this to marginal increment of  $K$ :  $\Delta K$ , compared with  $\Delta f$   
(where  $f$  is flow of cash or service).

② 11. Land valuation with successive discrete improvements. Good  
place to begin; also, later, get into progressive obsolescence,  
replacement.

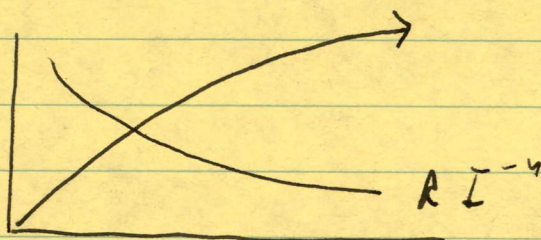
12.

# Ideas for Syllabus, S95

1. Ground Rules; Intro  
(Use K as input & DCF as output - thus use MB thy. - p. 12, "Ground Rules."  
(Maybe can also adapt AC curves this way?))
2. Optimizing - Intro to  
~~to~~ Need for <sup>co-</sup>measurement; opt. combos. of inputs; trade-offs
3. Performance Standards - true & false
4. Sensitivity Analysis - finding out what really matters.
- 5.

# Performance Standards

1. Max PV



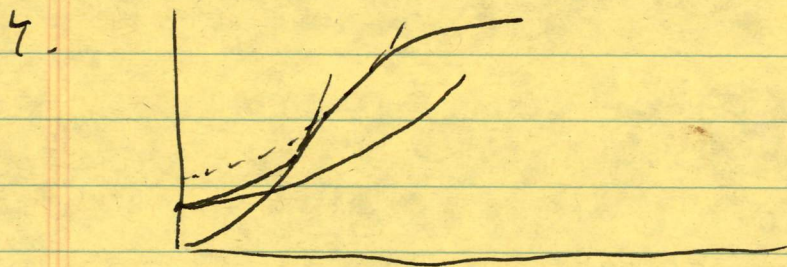
Long invt. always looks better.  
 $R I^{-\gamma}$  changes that

2. IQR

OK, but only if include cost of bond

3. Max a.

Avoid probs - e.g. reswitching.



$$\frac{(F I^{-\gamma} C) i}{1 - I^{-\gamma}}$$

Wed June 1  
Ec. 162

$$i = \frac{a(1 - I^{-n})}{P - D I^{-n} + L_0 - L_0 I^{-n}}$$

$$= \frac{a(1 - I^{-n})}{P - D I^{-n} + L_0(1 - I^{-n})}$$

Normally  $D = 0$ ,  $L_0$  lowers  $i$ .

~~SI~~

Finding Max of  $\frac{n}{I^n}$  w.r.t.  $\underline{u}$

(or min.)  
1. Max. of a ratio is where  $\frac{u'v - v'u}{v^2} = 0$

This is same as  $\frac{u'}{u} = \frac{v'}{v}$

$$\left( \begin{array}{l} \frac{u'}{v} = \frac{v'u}{v^2} \\ \frac{u'}{u} = \frac{v'}{v} \end{array} \right)$$

2.  $\frac{u'}{u} = d \ln u$

3.  $d \ln n = \frac{1}{n}$

4. ~~the~~  $I^n = e^{pn}$ , where  $p = \text{ln}(\text{I})$

$$\left( \begin{array}{l} \text{ln} I = p \\ p = \text{ln} I \end{array} \right)$$

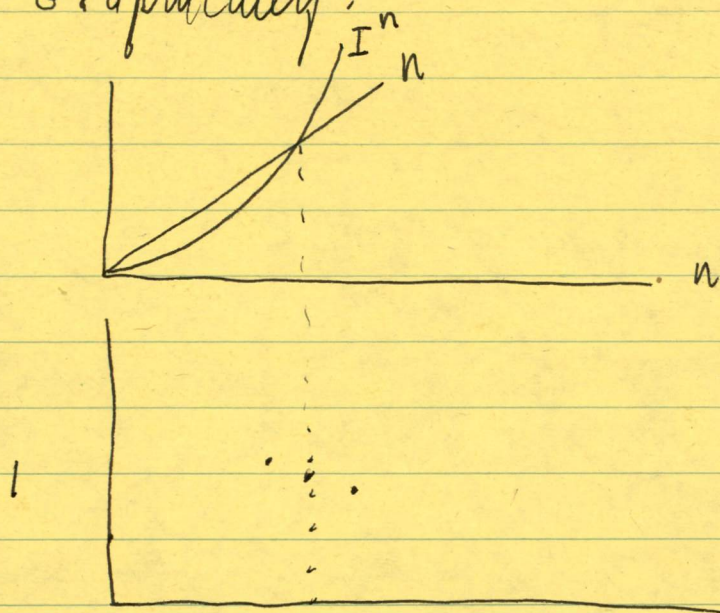
5.  $\frac{d p n}{d n} = p$

6. Max is where  $\frac{1}{n} = p$

$$n = \frac{1}{p} \approx \frac{1}{i}$$

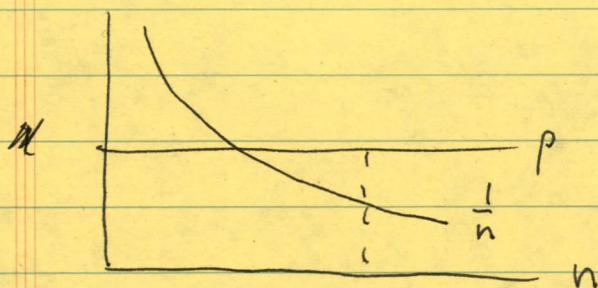
$$(pn = 1, \text{ so } e^{pn} = 2.71828 \dots)$$

7. Graphically:



$$n = I^n \quad \text{where } ?$$

$$p > \frac{1}{n}$$



So max is left of  $n = I^n$

~~the~~ Max comes when  $n > I^n$

8. Values of max, and of  $n$ , various  $i$

| $i$ | $\text{Max}$<br>$n/I^n$ | $n - \text{at max Value}$ |           |
|-----|-------------------------|---------------------------|-----------|
|     |                         | using<br>$p$              | using $i$ |
| .01 | 37.0                    | 100.5                     | 100       |
| .02 | 18.6                    | 50.5                      | 50        |
| .04 | 9.4                     | 25.5                      | 25        |
| .06 |                         | 17.2                      | 16.7      |
| .08 | 4.78<br><del>2.62</del> | 13.0                      | 12.5      |
| .10 |                         | 10.5                      | 10.0      |
| .12 |                         | 8.8                       | 8.3       |
| .14 |                         | 7.6                       | 7.14      |
| .16 | 2.47                    | 6.74                      | 6.25      |

9. Note that  $\text{max } \frac{n}{I^n} \propto \frac{1}{i}$

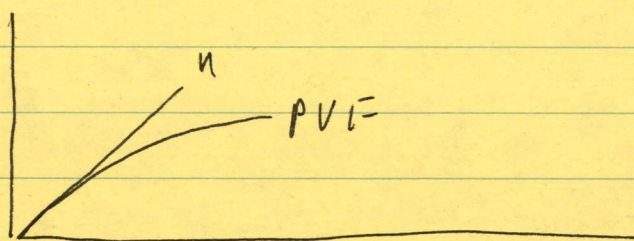
$$\text{or } \text{max } \frac{I^n}{n} \propto i$$

$$\text{Max } \frac{e^{p^n}}{n} = ki$$

10. When use these fr?

a. PV of annuity (not quite same)

$$PV = \frac{A}{n} \frac{1 - I^{-n}}{i} = \frac{A}{i} \frac{1 - I^{-n}}{n} = A \frac{PVF}{n}$$

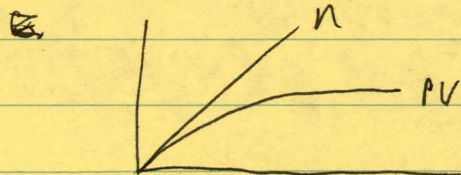


$$NPV = -\frac{K}{1} + \frac{A}{n} PVF = \frac{1}{n} (A PVF - K)$$

b. Compounding ratio =  $\frac{I^n - 1}{ni}$  or  $\frac{I^n}{1 + ni}$

Simple comp ratio =  $\frac{1 + ni}{I^n} = I^{-n} + \frac{ni}{I^n}$

c. PV as fraction of CF



$$\frac{PV}{n} = \frac{1 - I^{-n}}{ni}$$

d. CF as fraction of FV

$$\frac{n}{FV} = \frac{n i}{I^n - 1}$$

e. PV of fixed deprecn. allowance

$$\frac{K}{n} \frac{1 - I^{-n}}{i}$$

Values of  $\frac{1 - I^{-n}}{n i}$

| i   | n = 2                   | 4                       | 8                       | 16                                   | 32  |
|-----|-------------------------|-------------------------|-------------------------|--------------------------------------|-----|
| .02 | <del>1.97</del><br>1.94 | <del>3.95</del><br>3.81 | <del>7.92</del><br>7.73 | <del>15.85</del><br><del>15.58</del> | .73 |
| .04 | <del>1.84</del><br>1.84 | <del>3.61</del><br>3.63 | <del>6.84</del><br>6.73 | .73                                  | .56 |
| .08 | <del>1.88</del><br>1.78 | <del>3.83</del><br>3.31 | <del>5.72</del><br>5.75 | .55                                  | .36 |
| .16 | <del>1.80</del><br>1.61 | <del>3.70</del><br>2.80 | <del>4.54</del><br>4.34 | .35                                  | .19 |

$$\frac{n}{I^n}$$

$$\text{Max when } \frac{d \ln n}{dn} = \frac{d \ln I^n}{dn}$$

$$d \ln n = ?$$

$$\frac{d \ln n}{dn} = \frac{1}{n}$$

$$\text{i.e. } \frac{dn}{dn} = 1$$

$$\frac{d I^n}{dn} = \frac{d e^{p n}}{dn} =$$

$$I^n \equiv e^{p n}$$

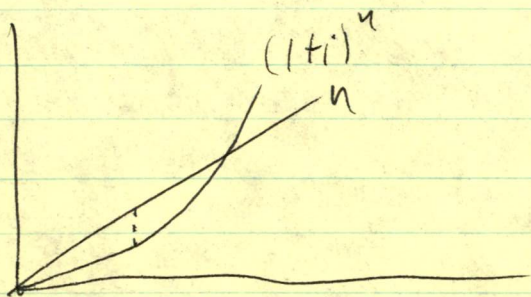
$$\ln e^{p n} = p n$$

$$\frac{d \ln e^{p n}}{dn} = p$$

$$\frac{(1+i)^n}{n}$$

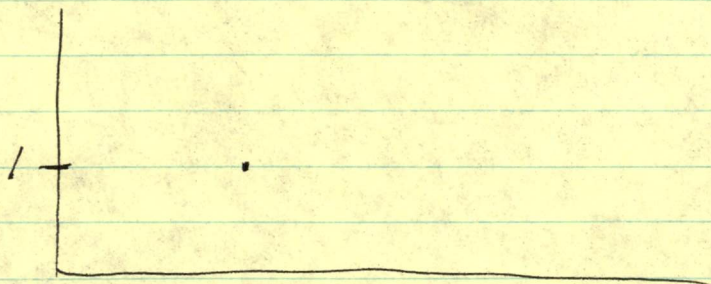
$$\frac{n}{(1+i)^n}$$

$$\frac{2}{i} \neq 0$$



$$0 > \frac{n}{(1+i)^n}$$

$$\frac{n}{(1+i)^n} > \frac{n+1}{(1+i)^{n+1}}$$



Max where

$$p = \frac{1}{4}$$

$$n = \frac{1}{p} = \frac{1}{0.05}$$

$$= 20.496$$

1100 | 800

$$P = 700$$

$$a/P = .214$$

$$a = 150$$

$$L = .4$$

$$x = 2010$$

$$u = 711$$

1200 | 900

$$F = 1000$$

$$F/P = 1.429$$

$$\frac{w}{L} = \frac{1.513}{.4}$$

$$t = .29$$

$$s = .50$$

.118 | .155

$$RAAT = 170 .166$$

With  $s = 0$ ,  $E$  of  $RAAT$  to  $t$  is ~~about~~  $-15\%$ .  
If  $s = .50$ , will  $E$  change? If so, in what direction?  
Explain, either verbally or by ref. to an eqn.

A new Tax Reform Act lets bldgs be deprec. only once (in my dreams). This lowers resale values of used bldgs., ~~but~~ <sup>and</sup> also lowers ~~of~~ land prices on which to ~~cost~~ <sup>put</sup> new bldgs. Show the likely possible effects on  $RAAT$  by analyzing the basic  $=u$  for finding same

$$P = a \frac{1 - \frac{1}{1+i}^n}{i} ; \quad a = P \frac{i}{1 - \frac{1}{1+i}^n}$$

$$\sum \text{pmts} = na = \cancel{n P \frac{1 - \frac{1}{1+i}^n}{i}} = n P \frac{i}{1 - \frac{1}{1+i}^n}$$

$$\frac{na}{P} = n \cdot \text{PFF} =$$

$$\frac{na}{P} = n \cdot \text{PFF} = \frac{n}{\text{ACF}} = \frac{20}{.14235} = 140.5$$

$$= 20 \times 2.025 = 40.5$$

$$\$14,378 \times 2.847 = 40,934$$