



Relations Among Volume, Turnover, Capital, and Factor Shares

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M. Gaffney 12/8/71

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1. Simple model, inventory

- a. A storekeeper has \$100 capital. Each day he spends it on inputs and sells out.

Yearly volume (F) = \$36,500

Turnover (R) = 365 times a year

Capital (K) = \$100

Basic Relation:

$$F = K \cdot R$$

(Interest of \$5. is negligible, when R is so high.)

- b. At other extreme, he spends the \$100. planting trees, or finding unripe minerals. At 7%, money doubles every 10 years. In 60 years he sells for \$6400., thus making 7% yearly. Mean yearly volume is $\frac{6400}{60}$ or about \$107. $\frac{63}{64}$ of the volume is interest. Payroll of \$100. is negligible.

Between these extremes lie most cases.

2. Appreciating capital. The "Point-input point-output" model (PIPO).

Site is free.

a. Single Cycle

P = Payroll, time zero. (assumed only input)

m = period between payroll and sale of output, in years.

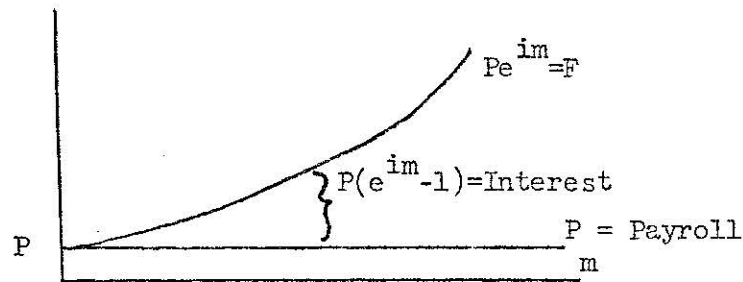
i = interest rate. (continuous)

In equilibrium, output after $\underline{m}$ years, (F):

$$F = P e^{im}$$

$$\text{Labor share} = \frac{P}{P e^{im}} = e^{-im}, \text{ a decreasing fn. of } \underline{m}.$$

$$\text{Capital share} = \frac{P(e^{im}-1)}{P e^{im}} = 1 - e^{-im}, \text{ the complement of labor share, and an increasing fn. of } \underline{m}.$$



b. Normalized or staggered operation

This is a mere matter of summation, but the Lutzes, Joan Robinson, Cassell, and many critics of Wicksell and Co. have thought there was something about normalization that changed basic implication of single cycle case, that longevity is the essence of capital-intensity.

Assume perfect normalization: plant and cut one tree yearly.

Number of trees growing equals $\underline{m}$: there is one of every age.

$$\text{Yearly output, } F = P e^{im}$$

$$\text{Yearly wage bill} = P$$

$$\text{Labor share} = \frac{P}{P e^{im}} = e^{-im}, \text{ as before}$$

$$\text{Capital, } K = \int_0^m P e^{in} dn = P \int_0^m e^{in} dn = P \frac{e^{im}-1}{i}$$

(Various proofs available for integral, but note it is simply the standard formula for the amount of an annuity.)

$$\text{Yearly interest} = Ki = P(e^{im} - 1)$$

$$\text{Capital share} = \frac{P(e^{im} - 1)}{Pe^{im}} = 1 - e^{-im}, \text{ as before.}$$

3. Depreciating Capital

Another summation, because PIPO is the math. building block from which all annuity formulae derive by summation. But many require convincing.

a. Single Cycle

A house yields steady service flow of \$1. over $\underline{m}$ years. It depreciates because of declining future life rather than decaying service flow.

Sum of service flows = \$ $\underline{m}$ over $\underline{m}$ years.

Warranted initial payroll outlay is present value of annuity of \$1. over $\underline{m}$ years.

$$P = \frac{1 - e^{-im}}{i}$$

This is equilibrium outlay for constructing building to endure $\underline{m}$ years.

Assume perfect vertical integration, so there is no outlay but payroll. (In practice, most depreciating capital requires primary products or raw material, the latter resulting from PIPO-type cycles, and a full model will integrate the two.)

$$\text{Labor share} = \frac{P}{\underline{m}} = \frac{1 - e^{-im}}{im}$$

a decreasing fn. of $\underline{m}$, due to more powerful influence of $\underline{m}$ in the denominator. To prove, show derivative of

$$\frac{1 - e^{-im}}{m} < 0.$$

$$\frac{d}{dm} \frac{1-e^{-im}}{m} = \frac{m i e^{-im} - (1 - e^{-im})}{m^2}$$

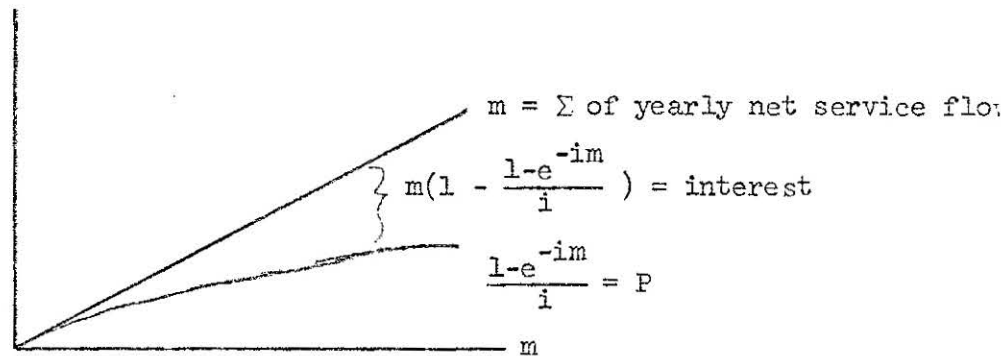
Derivative negative if

$$m i e^{-im} < 1 - e^{-im}$$

$$m < \frac{e^{im} - 1}{i} = \text{Amount of an annuity of one per annua,}$$

always greater than $\underline{m}$, if $i > 0$.

So labor share is a decreasing fn. of $\underline{m}$. Graphically,



$$\text{Interest is residual, } m - \frac{1-e^{-im}}{i} = m - P$$

$$\text{Capital share} = \frac{m-P}{m} = 1 - \frac{1-e^{-im}}{im}$$

This is the complement of labor's share, and is an increasing function of $\underline{m}$.

It so appears on the graph.

Note graphical demonstration that P/m is a decreasing fn. of $\underline{m}$.

As $\underline{m}$ doubles, P cannot double, because curve is concave downwards.

b. Normalized or staggered operation

Build and demolish one house yearly. Number of houses equals number of years of life (m). This entails double-summation of the PIPO building block, but may be simplified.

Yearly service flow = m (\$1 @ from m houses)

$$\text{Yearly } P = \frac{1 - e^{-im}}{i} \quad (\text{cost of one house})$$

$$\text{Labor share} = \frac{1 - e^{-im}}{im} \quad \text{as before, a decreasing function of } m.$$

$$\text{Yearly interest bill as residual} = m - \frac{1 - e^{-im}}{i}$$

Capital share is $1 - \frac{1 - e^{-im}}{im}$, as before, the complement of labor's share and an increasing function of m .

To confirm, calculate interest bill directly by finding amount of capital, K .

$$K = \int_0^m V_n \quad \text{where } V_n \text{ is remaining present value of the house}$$

whose remaining, unexhausted future life is n . (if n were elapsed life we would want $V_{(m-n)}$.)

$$V_n = \frac{1 - e^{-in}}{i}$$

$$K = \int_0^m \left[\frac{1 - e^{-in}}{i} \right] = \frac{1}{i} \left[m - \int_0^m e^{-in} \right] = \frac{1}{i} \left[m - \frac{1 - e^{-im}}{i} \right]$$

(Note the integral performed is merely the standard case of the present value of an annuity.)

$$\text{Interest, yearly amount,} = Ki = m - \frac{1 - e^{-im}}{i}$$

$$\text{Share of capital} = \frac{Ki}{m} = 1 - \frac{1 - e^{-im}}{im} \quad \text{as before, an increasing function of } m.$$

4. The Brems-Wicksell macro model.

Note in every case examined, $K = +f(m, P)$

Let $P = M \cdot W$

M = Man-hours (including females)

W = Wage-rate per man-hour (no discrimination)

$$M = f(K, \frac{1}{m}, \frac{1}{W})$$

Implication: to increase employment, increase capital, reduce $\underline{m}$ (speed turnover), and/or lower wage-rate.

In specific cases, sensitivity of M to m is very high, suggesting attention to this factor.

a. PIPO example.

$$K = P \frac{e^{im} - 1}{i}$$

$$P = M \cdot W = \frac{K \cdot i}{e^{im} - 1}$$

$$M = \frac{K \cdot i}{W[e^{im} - 1]}$$

b. In simple inventory model (p. 1), it is clearer, although over-simplified and omitting interest.

On p. 1, $F = KR$.

But $F \propto M$

$$\text{and } R = \frac{1}{m}$$

$$\therefore M \propto \frac{K}{m}$$

5. Effect of taxes on land use

Method: define effect of taxes on present value of site. FIP0 case.

Begin with pretax equilibrium where

$$S = \frac{F - Pe^{im}}{e^{im} - 1}$$

S is independent of m for all $F = (P + S)e^{im} - S$ (an identity)

a. Income tax

π $\equiv$ Site value after income tax. Assume tax lodges 100% in lower

Site value, i remaining fixed.

$$\pi = \frac{F(1 - t) + tP - Pe^{im}}{e^{im} - 1} = \frac{S(e^{im} - 1)(1 - t) - tP(e^{im} - 1)}{e^{im} - 1} = S - t(S + P)$$

The percentage decline of S is:

$$\frac{S - \pi}{S} = t \frac{S + P}{S} = t \left(1 + \frac{P}{S}\right)$$

This is not a neutral tax. It favors low values of $\frac{P}{S}$, thus motivating less intensive use.

b. Property Tax on Capital

θ is site value after absorbing property tax on tree value. Tax on site value is not included, being assumed neutral.

Tax base is $F_n = (P + S)e^{in} - S$. This is an idealized base, the investment value. In practice, base is usually lower.

$$\begin{aligned}
\theta &= S - t \int_{h=1}^m \frac{F_n}{e^{in}} \, dn \frac{1}{1-e^{-im}} \\
&= S - t \left[\int_0^m \frac{(P+S)e^{in}}{e^{in}} \, dn \frac{1}{1-e^{-im}} - \int_0^m \frac{S}{e^{in}} \, dn \frac{1}{1-e^{-im}} \right] \\
&= S - t \left[(P+S) \frac{m}{1-e^{-im}} - \frac{S}{i} \right]
\end{aligned}$$

The percentage decline of S is:

$$\frac{S - \theta}{S} = t \left[\left(1 + \frac{P}{S}\right) \frac{m}{1-e^{-im}} - \frac{1}{i} \right]$$

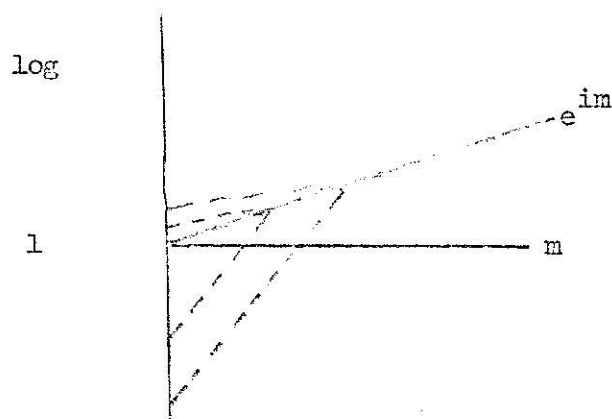
This is not a neutral tax either, under present assumption of the tax's lodging in lower site value (it is neutral if tax lodges in lower rate of return).

Direction of non-neutrality is different from usual emphasis, however. Sensitivity analysis shows the following:

1. $\frac{S - \theta}{S} > t$
2. $\frac{S - \theta}{S}$ is highly sensitive to P . To avoid tax, one holds down P -- i.e. fails to reforest. This is the paramount effect, rather than the one usually stressed, i.e. the tendency to hasten cutting of mature wood.
3. $\frac{S - \theta}{S}$ varies inversely with S , i.e. tax is harder on marginal land.
4. $\frac{S - \theta}{S}$ varies with m , but weakly, and at the lower level of stocking caused by (2).

6. Effect of changes in wage and interest rates on durability.

Graphical demonstration.

Figure 1

Let e^{im} be locus of future values having present value of unity at discount rate = i .

At a lower rate, $h < i$, present values from the locus curve are $\frac{e^{im}}{e^{hm}} = e^{(i-h)m}$, an increasing fn. of m ($i - h > 0$), shown by ascending y-intercepts above unity.

At a higher rate, $j > i$, present values from locus curve are $\frac{e^{im}}{e^{jm}} = e^{(i-j)m}$, a decreasing fn. of m ($i - j < 0$), shown by descending y-intercepts below unity.

So if capital is scarce, interest rates rise, causing shorter investments to look better. This speeds capital turnover, substituting labor for capital (cf 2, 3) and vice versa.

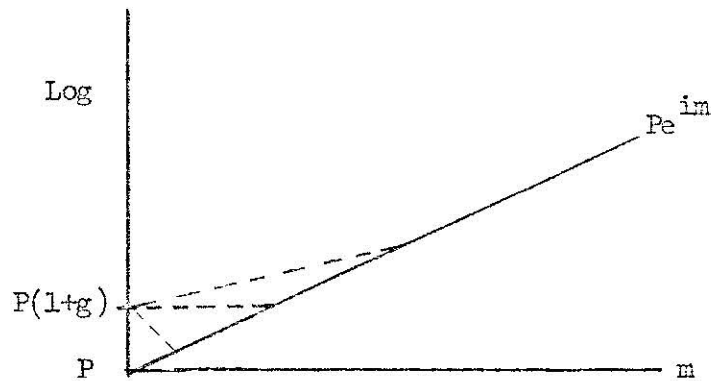


Figure 2

Figure 2 shows rise of wage rate with no change in value of product. Pe^{im} is locus of values yielding interest of i when payroll cost is P . Higher wage rates raise cost to $P(1+g)$. Dash lines show growth of investment at internal rate of return. Higher slopes mean higher rates.

Let r = internal rate

$$e^{rm} = \frac{Pe^{im}}{P(1+g)}$$

$$rm = im - \ln(1+g)$$

$$r = i - \frac{\ln(1+g)}{m}, \text{ an increasing fn. of } m.$$

So higher wage rates cause longer lives to be preferred, thus substituting capital for labor (cf 2, 3). And of course vice versa.

Thus lower wage rates and higher interest rates lead to full employment, in a free market (if we had one).

But it is the wage and interest rates as perceived and experienced by the entrepreneur that are controlling. Here is where institutional bias intercedes. Labor costs employers much more than labor takes home in pay. But capital and land cost employers less, at least comparatively.