



Finding the Future Value of a Given Flow: the CCFF

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Article citation:

[Gaffney, Mason]. "Finding the Future Value of a Given Flow: the CCFF." N.d. Study guide.

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INTRODUCTION (repeated): Solving DCFF for its different elements

In Reading III, D we saw how the Compound Growth Factor (CGF) may be solved for its different elements. We saw that equations do not show causation: causation must be inferred from other information. Equations do show necessary relations, so you can find the unknown from the known.

That was just a warmup. Here we begin the process of doing the same thing for the DCFF. Up to now, we have focused mainly on valuation, where present value (P) is the unknown. From here on we focus on new uses for the DCFF, where P is known, and we use it to find something else.

As a reminder, here is the DCF equation:

$$P = a \frac{1 - I^{-n}}{i} \quad (1)$$

Converting P to F

Sometimes you want to know a future point value. To find the factor, just multiply the DCFF times the Compound Growth Factor (CGF). This has the effect of multiplying each of the many individual terms in the finite series by CGF, converting each to a future value: quite a labor-saving procedure.

$$F = PI^n = aI^n \frac{1 - I^{-n}}{i} = a \frac{I^n - 1}{i} \quad (2)$$

The ratio on the right of (2) is the Compound Cash Flow Factor (CCFF).

Graphically, you can relate this to the curve of exponential decay (Doonesbury's nose), which you learned about in Reading III, F, "The Discounted Cash Flow," p.6. Take the lowest point on that curve (far right), and raise it up to the ordinate for "1". Then draw the rest of the curve up left from there, duplicating and parallel to the original curve. The area under this curve (and above zero) is the value of F.

You can also relate it to the "cume curve" showing how the DCFF rises with n . This present cume curve, the CCFF, doesn't flatten out, but rises at an increasing rate. To graph it, you add up

and stack up the columns under the curve you just drew, but **beginning from the right**, and working back to the left.

Merrill Lynch, the broker, naturally wants you to invest regularly, so in their recent ads they show some of these cume curves, in bright and glowing colors, showing how rich you may become in 40 years by a regular savings plan. It's quite impressive, and grabs your attention. Control those spending impulses today, and you can be rich some day -- if nothing goes amiss, and if no one raids your fund, and if the dollar is still worth anything, and if you survive, and if you can still enjoy it, and if you won't regret all the youth you missed, and you're sure you're going to like your new son-in-law who may blow it all some day, etc. You can't take it with you when you go, so there are two sides to the question.

Algebraically, the annual growth of this cume curve is easy to find, and remember. Its growth has two parts. First, you contribute \$1 each year from outside; second, there is interest on the accumulated amount from prior years:

$$\Delta \text{CCFF} = 1 + i(I^n - 1)/i = 1 + I^n - 1 = I^n$$

$$\Delta \text{CCFF} = I^n \quad (3)$$

(For comparison, the annual growth of the basic PIP0 compounding factor (CGF) is iI^n , a lot less. It helps that you keep feeding new outside money into this kitty.)

(3) should also help you remember that the first term of this series is 1, rather than I^1 . That is, it begins with your putting in \$1 at the **beginning** of the first year. So the increase of the CCFF during year one is $(1+i)$.

The CCFF is used when you want to know how much a stream of payments in or out will come to at some future date, with accumulated interest. It may be money coming in: good news. Say your generous grandmother is putting \$5,000/year into a trust fund for you to control when you reach age 40, and the fund is earning 9%, how much will you have then?

Suppose you put aside \$100/month for your retirement, and can earn 8%. How much will you have at age 65? $1.08^{45} = 31.92$. You take it from there.

Often it is bad news. Say it takes \$13 millions to construct a plant, but the cost is stretched out over 13 years, financed at

14%. How much will you have tied up in the plant by the time it goes on stream? (2) will give you the bad news, and it is a shocker: \$32,088,654. Check it out. Some \$19 millions of that, or 59.5%, is accumulated interest.

When your plant finally goes on stream its cash flow must earn a return on the whole \$32+ millions, not just the \$13 millions. The plant won't last forever, either, so the cash flow must not only earn a return on \$32+ millions, but also a return of \$32+ millions: i.e. enough more to recover the capital, which now includes the accumulated interest. That's why it is called "capitalized interest": it's added to the capital cost.¹

Review problem: how much cash flow is needed to pay off \$32 millions over an assumed plant useful life of 25 years? Use IPF, and $i=.11$. Your answer is ???

How do I come off lowering i from the original .14 to .11? Because construction loans generally come higher than "take-out" loans you can get after going on stream.

Compound review problem. After 25 years of throwing back cash (at the rate you just found), your plant retires. Summing up at that time, how much, and what share, of its gross cash flow has gone to pay interest? Who got more of the loot, the hard-hats or the 3-piece pin-stripe attaché-case brigade?

The "Comsimp Ratio"

To find the ratio of total cost to construction cost, divide (2) by na . The a 's cancel out, leaving you with the $CCFF/n$. This is also the ratio of compound interest to simple interest, for which reason I call it the "Comsimp Ratio."² It and its reciprocal

¹Regulated utilities, like SCE, are guaranteed a fixed return on their invested capital, which is called the "rate base." AFDC is an acronym for Allowance for Funds During Construction: it is capitalized interest on plants that take a long time to build. It goes into the rate base when they go on stream. It is one reason why the richest economists are those who work on utility rate cases.

²"Comsimp" is a mnemonic, i.e. a name that helps you remember something. During the name-calling era of the 1950s, comsimp was a pejorative used by various inflammatory speakers and journalists to mean communist sympathizer. Symp was

crop up in managerial problems from time to time, so remember The Comsimp Ratio (3):

$$F/na = a \frac{I^n - 1}{ni} = \frac{I^n - 1}{ni} \quad (3)$$

The Comsimp ratio = 1 when $n=1$, and then rises, slowly at first, then ever faster. For an exercise, figure out the ratio for the French Canal du Midi, linking the Mediterranean with the Bay of Biscay: it took 200 years to complete. (Don't complain that I forgot to give you an interest rate; pick one yourself.) Closer to home, try the Tenn-Tom Canal, linking the Tennessee River and the Tombigbee in Alabama. It was started when Andrew Jackson was President (1829-37), and completed in about 1985.³

In spite of the great accumulative power of compound interest in the CCFF, it can be overstated. For an example, see the next reading, on California's forest tax law.

misspelled simp either to suggest simpleton or because the speller was one: which, is still a matter of opinion and dispute. Either way, remember the Comsimp Ratio.

³Many structures still yield low or negative cash flows after they are "completed," so the capitalized interest keeps building up. We will draw the curtain of charity over the rest of these horror stories so we can move on.