



Markup, Turnover, Overhead, and Return on Investment

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MARKUP, TURNOVER, OVERHEAD, AND RETURN ON INVESTMENT

(Notes by M. Gaffney, Feb. 3, 1978)

Return on investment is first perceived as return on sales, or "markup." The next step to wisdom is to perceive return on investment as markup times turnover. This is an improvement, but omits the compounding of interest, which worsens the return on slower turnovers by more than a simple linear factor. It also omits return on overhead investment. The following notes incorporate both these factors.

$F \equiv$ Future sale price

$P \equiv$ Present investment

Markup is $F-P$

% markup is:
$$\frac{F-P}{P} = \frac{F}{P} - 1 \equiv M$$

$r \equiv$ rate of return

$K \equiv$ Fixed or overhead capital and land occupied by the working capital, valued in dollars

$n =$ years from investment to recovery

$T =$ Turnover

$$T = \frac{1}{n}$$

$$r = \frac{\ln \frac{F+K}{P+K}}{n} \quad (A)$$

To begin, let $K = 0$, for simplicity.

$$r = \frac{\ln \frac{F}{P}}{n} = \frac{\ln(M+1)}{n} \quad (M = \frac{F}{P} - 1) \quad (B)$$

We want to know what happens to $\underline{r}$ when M and n both rise by the same %.

That is, does $\underline{r}$ fall when

$$\frac{dM}{dn} \times \frac{n}{M} = 1 \quad (1)$$

(Skip to p.3 unless you are concerned with mathematical proof.)

r falls if $\frac{dr}{dn} < 0$

Let $\frac{F}{P} \equiv f$

Differentiating (B):

$$\frac{dr}{dn} = \frac{\frac{n}{f} \frac{df}{dn} - \ln f}{n^2} \quad (2)$$

$dr/dn < 0$ when:

$$\frac{n}{f} \frac{df}{dn} < \ln f \quad (3)$$

$M = f-1$, so condition (1) is

$$df/dn \frac{n}{f-1} = 1 \quad (4)$$

$$n \frac{df}{dn} = f-1$$

Combining (3) and (4), $dr/dn < 0$ when:

$$\frac{f-1}{f} < \ln f \quad (5)$$

When $f = 1$ both sides are zero.

But for $f > 1$, let $f = 1 + d$. By Taylor's expansion, $\ln(1 + d) \approx d(1 - \frac{d}{2})$. Now (5) can be written:

$$\ln f + \frac{1}{f} > 1$$

$$d(1 - \frac{d}{2}) + \frac{1}{1+d} > 1$$

$$\frac{d(1 - \frac{d}{2} + d - \frac{d^2}{2}) + 1}{1+d} > 1$$

$$1 + \frac{d(1-d)}{2(1+d)} > 1$$

$$\frac{d}{2} \frac{(1-d)}{(1+d)} > 0$$

True. Q.E.D.

The table below shows a numerical example of how $\underline{r}$ falls with $\underline{n}$, under the assumption that $T \cdot M$ is constant at .20.

$\underline{n}$	T ($1/\underline{n}$)	F/P	M ($=F/P-1$)	$\ln (M+1)$ ($\ln F/P$)	$\underline{r}$ ($=T \cdot \ln(M+1)$)	MT $=M/N$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
.25	4.00	1.05	.05	.049	.196	.20
.50	2.00	1.10	.10	.095	.190	.20
.75	1.33		.15			
1.00	1.00	1.20	.20	.182	.182	.20
2	.500	1.40	.40	.336	.168	.20
3	.333		.60			
4	.250	1.80	.80	.588	.147	.20
<hr/>						
8	.125	2.60	1.60	.955	.119	.20
<hr/>						
16	.0625	4.20	3.20	1.435	.090	.20
<hr/>						
32	.0312	7.40	6.40	2.001	.062	.20
<hr/>						
100	.01	21.00	20.00	3.040	.0304	.20

Here, markups increase in step with recovery periods ($\underline{n}$), so M/n is always the same, .20 (Column 7). But the compounding effect lowers $\underline{r}$ below .20, and increasingly so for higher $\underline{n}$.

The table below shows the effect of adding overhead capital, K.

<u>n</u>	<u>T</u>	<u>F/P</u>	<u>M</u> <u>(F/P-1)</u>	<u>$\frac{F+K}{P+K}$</u> [≠]	<u>$\ln \frac{F+K}{P+K}$</u>	<u>r</u> <u>$(=T \cdot \ln \frac{F+K}{P+K})$</u>
.25	4.00	1.05	.05	1.025	.0247	.0988
.50	2.00	1.10	.10	1.050	.0488	.0976
.75	3.33					
1.00	1.00	1.20	.20	1.10	.0950	.0950
2.00	.500	1.40	.20	1.20	.1820	.0910
3.00	.333					
4	.250	1.80	.80	1.40	.3360	.0840
8	.125	2.60	1.60	1.80	.5880	.0735
16	.0625	4.20	3.20	2.60	.9560	.0597
32	.0312	7.40	6.40	4.20	1.4350	.0448
100	.01	21.00	20.0	11.00	2.398	.0240

Here, we add the effect of adding overhead (k) to the investment base. This lowers r still further (and makes r less sensitive to n).

[≠] where $K = P = 1$

Conclusions:

1. Turnover times Markup (TxM) is not a sufficient definition of rate of return (unless T be defined more sophisticatedly than simple $1/n$).
2. It is better for $\underline{r}$ to speed Turnover by a given % than to raise Markup by the same %.
3. Simple Markup, or return on sales, overstates $\underline{r}$ three ways: by neglecting T, by neglecting compound interest, and by neglecting return to overhead capital (k).
4. Modifying 3, above, if $T > 1$, neglecting T will understate $\underline{r}$.
5. The % drop of $\underline{r}$ with a % increase in $\underline{n}$ rises with $\underline{n}$, that is, each doubling of $\underline{n}$ results in a progressively greater % drop of $\underline{r}$. Otherwise put, the elasticity of $\underline{r}$ with respect to $\underline{n}$ rises (in absolute value) with $\underline{n}$.

Extension of example to Sensitivity analysis, using elasticity.

It is useful to know how much $\underline{r}$ is reduced by raising $\underline{n}$ (for any given F/p); and how much $\underline{r}$ is raised by raising F/p (for any given $\underline{n}$). The most uniform general way of reporting the findings is in terms of "elasticity"; a ratio of percentages that has no units and can be applied as needed to specific values.

Let $f = F/p$ (same as assuming $P = 1$)

$$M = f-1$$

Let's begin with an easy one, the elasticity of $\underline{r}$ with a change in $\underline{n}$. This is easy because we can see the answer directly, (or almost so).

$$\underline{r} = \frac{\ln f}{n} \quad (6)$$

Whatever we multiply $\underline{n}$ by we divide $\underline{r}$ by to keep this equality.

For elasticity (E) of $\underline{r}$ with respect to $\underline{n}$:

$$E = \frac{dr/dn}{r} n \quad (7)$$

$$dr/dn = \frac{\frac{n}{f} \frac{df}{dn} - \ln f}{n^2} = \frac{\frac{df}{dn} - \frac{\ln f}{n}}{nf} \quad (8)$$

$$E = \left[\frac{\frac{df}{dn} - \frac{\ln f}{n}}{f} \right] \frac{n}{\ln f} = \frac{1}{\ln f} \frac{df}{dn} \frac{n}{f} - 1 \quad (9)$$

With $\underline{f}$ constant, $\frac{df}{dn} = 0$ and $E = -1$

From (7)

$$\frac{dr}{r} = \frac{dn}{n} E = - \frac{dn}{n}$$

So an x% rise of $\underline{n}$ causes an x% drop in $\underline{r}$. (This is true only for small changes, and absolutely true only for infinitesimal changes).

Now for a harder one: change of $\underline{r}$ with $\underline{f}$. Actually, this one is too easy and not very useful. Actually, this one is too easy and not very useful. We know that raising $\underline{f}$ from 1.05 to 1.10, a 5% increase, raises $\underline{r}$ by 100%. The tighter question is, does $\underline{r}$ rise in step with $\underline{M}$? ($\underline{M}=\underline{f}-1$). Or does $\underline{r}$ march to a different drummer? We'll see that $\underline{r}$ lags behind $\underline{M}$.

$$E = \frac{dr/dM}{r} M \quad (10)$$

$$\frac{dr}{dM} = \frac{dr}{df} = \frac{1}{fn} - \frac{\ln f}{n^2} \frac{dn}{df} \quad (11)$$

$$\begin{aligned} E &= (f-1) \frac{n}{\ln f} \left[\frac{dr}{dM} \right] = \frac{f-1}{f \ln f} \approx \frac{M}{(1+M) \left(\frac{1-M}{2} \right) M} \\ &= \frac{1}{1+M-\frac{M}{2} + \frac{M^2}{2}} = \frac{1}{1+M + \frac{M^2}{2}} < 1 \end{aligned} \quad (12)$$

$\frac{dr}{r} = \frac{dM}{M} E$, so an x% increase of $\underline{M}$ causes a less than x% increase of $\underline{r}$.

For example, when $\underline{M}=1$, $E = \frac{1}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$, so a 10% increase in $\underline{M}$

means a 5% increase in $\underline{r}$. So, at this point, a 10% speedup of turnover helps you twice as much as a 10% increase in Markup.

"At this point" in the previous paragraph means at $\underline{M}=1$, a markup of 100%. Normally, this means $\underline{n}$ of several years (remember, we are talking net markup, after deducting associated costs.) From (12), E varies inversely with $\underline{M}$. Since $\underline{M}$ normally varies as an increasing function of $\underline{n}$, E also varies inversely with $\underline{n}$. An x% increase of a low markup in a high Turnover operation helps your $\underline{r}$ much more than the same x% increase of a high markup in a slow Turnover operation.

Going back to the table on p. 3, the algebra we've gone through adds more dimensions to it. The table limits us rigidly to one set of numbers, and one assumption to wit $\frac{\underline{M}}{\underline{n}} = \text{constant}$.

The algebra tells us what will happen if:

- a) For any n, we increase M
- b) For any M, we increase n.

Using our values for E in conjunction with the table, we get a good feel for the total range of possible combinations of M and n. Thus, at n=8, and M=1.60 or 160%, r=11.9%. E of r with respect to M,

$$E = \frac{1}{1 + \frac{1.6}{2} + \frac{2.56}{2}} = \frac{1}{3.08} = .325$$

This tells us that an x% rise in M gives you a .325 x% increase of r (n remaining constant at 8 years). Thus, raising M to 1.8 is an increase of $\frac{.2}{1.6} = .125$; this raises r to:

$$11.9\% \times (1 + .325 \times .125) = .119 \times 1.041 = 12.4\%$$

Where elasticities are negative, as between n and r, and you look at large changes, another adjustment is needed to apply elasticity. Doubling n, a 100% increase, does not reduce r by 100% but by 50%. The % change of r is not -x, but $\frac{-x}{1+x}$

For complex positive elasticities, analogous adjustments are needed for large changes, but are less urgent, and we stop here.

Why is all this pain necessary? So you can deploy your limited capital to the best advantage. You need a uniform rule you can apply to all investments, in place of a sales pitch from the partisans of each, playing up certain strong points. Try buying real estate some time for investment. Each salesman has a different way of calculating yields. You need one uniform method--yours--and plug in the data, to compare alternatives fairly.

Compare the bond market, highly organized and rationalized. Each bond has a "yield to maturity" calculated daily by solving for i in the equation:

$$\text{Price} = \text{coupon} \times \left[\frac{1 - e^{-in}}{i} \right] + \frac{\text{Face Value}}{e^{in}}$$

Optional

P.S. Mr. Bouslog asked why we use natural logs when conversion (compounding) is continuous.

Let f = frequency of conversion, e.g. 365 times per year.

$$F = \left(1 + \frac{i}{f}\right)^{fn} = \left[\left(1 + \frac{1}{f/i}\right)^{f/i}\right]^{in}$$

As $f \rightarrow \infty$, the part in brackets becomes e , by definition, so:

$$F = e^{in} \quad \text{and} \quad in = \ln F.$$