

Mathematics Temporary

	11	12	2	3
Mon	Phillips	Caywood	Pickart	
Tues	A laoglu			
Wed.	Granados	Heritt	Norris	
Thurs.	Whiteman	Lorenz	Putnam	

Holyoke 43



$$\theta = \frac{s}{r}$$

$$s = \theta r$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$v = r \omega$$

radiants

~~radiants~~

Math. Review.

1. Exponents

Axioms (5)

$$(a^x \cdot a^y = a^{(x+y)}; (a^x)^y = a^{xy}; a^x \cdot b^x = (ab)^x)$$

Consequences.

$$a^0 = 1 \quad (a^x \cdot a^0 = a^{x+0} = a^x \cdot a^0; a^0 = \frac{a^0}{a^0} = 1)$$

$$\begin{aligned} (1 = a^0 = a^{x-x} = a^x \cdot a^{-x}; a^{-x} = \frac{1}{a^x}) \quad (8 \text{ proofs, of course}) \quad a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x \\ (a^x = a^{x+y-x} = a^x \cdot a^{y-x}; \frac{a^x}{a^x} = a^{x-x} = 1; a^x > a^x) \quad a > 1, y > x \rightarrow a^y > a^x \\ \left(\left(\frac{1}{a}\right)^y = \left(\frac{1}{a}\right)^{x+y-x} = \left(\frac{1}{a}\right)^x \left(\frac{1}{a}\right)^{y-x} = \left(\frac{1}{a}\right)^x \left(\frac{1}{a}\right)^{y-x} > 1; \left(\frac{1}{a}\right)^y > \left(\frac{1}{a}\right)^x\right) \quad a < 1, y > x \rightarrow a^y < a^x \\ (a^x \cdot 1^x = (a \cdot 1)^x = a^x; 1^x = \frac{a^x}{a^x} = 1) \quad 1^x = 1 \end{aligned}$$

2. Logarithms

If $y = a^x, \quad x = \log_a y$

1. $x = \log_a(a^x), \quad y = a^{\log_a y}$ (sub.)
2. $\log_a a = 1; \log_a 1 = 0 \quad (\log_a(a^1) = 1; \log_a(a^0) = 0)$
3. $\log_a xy = \log_a x + \log_a y \quad (\log_a(a^{\log_a x} a^{\log_a y}) = \log_a(a^{\log_a x + \log_a y}) = \log_a x + \log_a y)$
4. $\log_a(x^y) = y(\log_a x) \quad (\log_a(a^{(\log_a x)y}) = y \log_a x)$
5. $(\log_a b)(\log_b a) = 1 \quad (\log_b a^{\log_a b} = \log_b b = 1)$
(by #4)
6. $\log_a x = \frac{\log_b x}{\log_b a} \quad (= \log_b a^{\log_a x} = \log_b x \log_a a = \log_b x \log_a b)$
 $= \frac{\log_b x}{\log_b a} \quad (= \log_a x^{-1} = -\log_a x)$
7. $\log_a\left(\frac{1}{x}\right) = -\log_a x$
8. $a^x = b^{x \log_b a} \quad (= b^{\log_b a^x} = b^{x \log_b a})$

Con
version

3. Differentiation

A. $D_x \log x = \frac{1}{x}$
 $D_x e^x = e^x$

$$D_x a^x = \log a \cdot a^x \quad (= D_x e^{x \log a} = e^{x \log a} \cdot \log a = a^x \log a)$$

$$D_x \log a x = \frac{1}{x \log a}$$

B. $D_x \sin x = \cos x \quad D_x \sec x = \sec x \tan x$
 $D_x \cos x = -\sin x \quad D_x \csc x = -\csc x \cot x$
 $D_x \tan x = \sec^2 x$
 $D_x \cot x = -\csc^2 x$

Loomis, April 29

Problems -

$$v = (10^{1+t})^2$$

$$y = f(x), \quad x = g(t)$$

$$y = f(g(t))$$

How to find $D_t y$?

$$D_t y = D_x y \cdot D_t x$$

ex. $y = \sin t$

$D_t y$ (let $t^2 = x$)
 $y = \sin x$

$$D_x y \cdot D_t x = \cos x \cdot 3t^2 = \cos(t^2) \cdot 3t^2$$

ex $y = \sin(3t+1)^{\frac{1}{2}}$

let $x = (3t+1)^{\frac{1}{2}}$

let $z = (3t+1)$

$y = \sin x$; $x = z^{\frac{1}{2}}$; $z = 3t+1$

$$D_t y = D_x y \cdot D_z x \cdot D_t z = \cos x \cdot \frac{1}{2} z^{-\frac{1}{2}} \cdot 3$$

$$= \cos(3t+1)^{\frac{1}{2}} \cdot \frac{1}{2} (3t+1)^{-\frac{1}{2}} \cdot 3$$

$$\rightarrow = 10^{2+2t} = (e^{\log 10})^{2+2t}$$

or $= 10^2 \cdot 10^{2t}$

now

$$D_t y = 10^2 D_t (10^{2t}) = 10^2 D_t (100^t) =$$

$$10^2 \cdot 100^t \log 100$$

$D_t v = D_y v \cdot D_t y$, let $v = e^y$, $y = \log_{10}(2+2t)$

$$D_t v = e^y \cdot 2 \log 10 = \frac{e^{\log_{10}(2+2t)}}{10} \cdot 2 \log 10$$

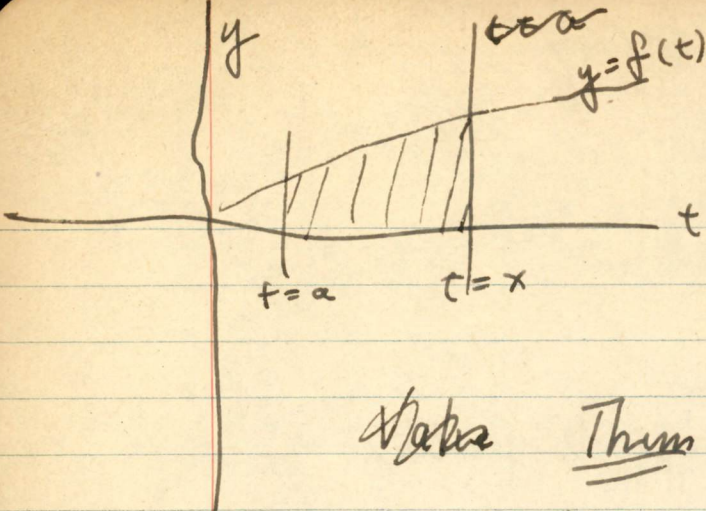
Now review integration

Def. $F(x)$ is an integral of $f(x)$ if $f(x) = D_x(F(x))$

Thm: If $f(x)$ is an " " " " , then every integral is of the form $F(x) + C$.

Geometric interpretation.

Let $A(x)$ be area under curve $y = f(t)$ between $t=a$ & $t=x$



Define Thm: $D_x A(x) = f(x)$

Corollary: $A(x)$ is an integral of $f(x)$

$$A(x) = 0 \bigg|_{x=a}, \text{ i.e., } A(a) = 0$$

Corollary $A(x)$ is the integral of $f(x)$ which $= 0 \big|_{x=a}$

Def.s.

Integral notation: an integral $\int f(x) dx$

The integral which $= 0$ at $x = a$: $\int_a^x f(t) dt$

rules,
conventions

$$\int_a^b f(t) dt = \text{area under } y = f(t) \text{ between } t = a \text{ \& } t = b.$$

$$\int_a^b + \int_b^c = \int_a^c$$

$$\int_a^b = - \int_b^a$$

$$y = \frac{1}{t} \quad \begin{array}{c} y \\ \hline t \\ \hline t=a \quad t=x \end{array} \quad F(x) = \int_a^x \frac{1}{t} dt$$

$F(x) + F(y) = F(xy)$, strangely amazing, looks like log.
Define $\log x = \int_1^x \frac{1}{t} dt$; $\log x + \log y = \log xy$.

$$F(x) = \int_1^x f(t) dt; \quad D_x F(x) = f(x); \quad D_x \log x = \frac{1}{x}$$

$$D_x \log x = \frac{1}{x} \log_e^{-1} a?$$

$D_x \log x = \frac{1}{x}$ only if $? = ?$ so corresponds to e function.
Define exponential function: inverse of log. - $y = e^x$ if $x = \log y$