

Opt. Life by Max. Rent

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Finding opt. n by max rent

$$a = a_1 \frac{1 - (I+d)^{-n}}{i+d} \frac{i}{1 - I^{-n}} = a_1 \frac{i}{i+d} \frac{1 - (I+d)^{-n}}{1 - I^{-n}}$$

Let $d = .03$, a const.

Find effect of i on $\frac{da}{dn}$

1. $i = .05$

n	$i = .05$					$.10$				
	$1PF$	PV	$1PF$	PV	Δ	$1PF$	PV	$1PF$	PV	Δ
4	.282	3.312	.934			.315	2.974	.938		
5	.231	3.993	.922	.012	.013	.264	3.517	.928	.010	.011
8	.155	5.747	.899			.187	4.799	.900		
9	.141	6.247	.879	.010	.011	.174	5.132	.891	.009	.010
16	.092	8.851	.817			.128	6.604	.844		
17	.089	9.122	.812	.008	.010	.125	6.729	.839	.005	.006
32	.063	11.435	.724			.105	7.938	.791		
33	.062	11.514	.720	.004	.006	.104	7.556	.790	.001	.001

~~b.r. 570 20~~

~~n 21~~

$$n \quad a_n = 1.03^{-n}$$

$$4 \quad .888$$

$$5 \quad .863$$

$$8 \quad .789$$

$$9 \quad .766$$

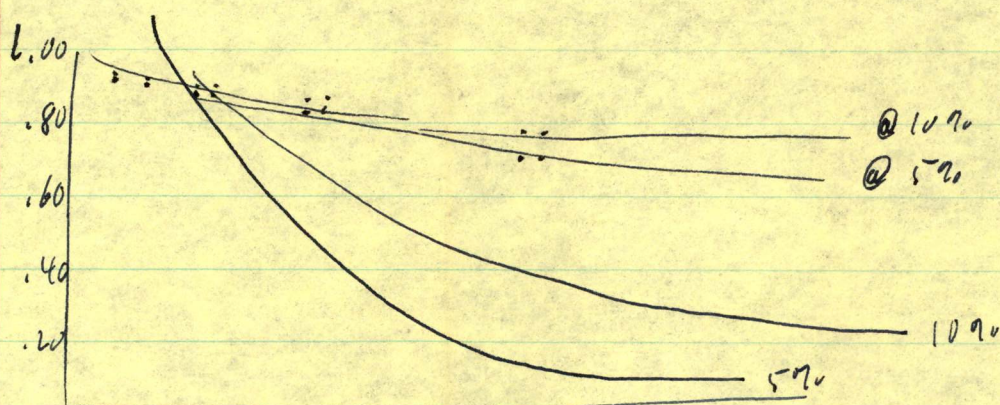
$$16 \quad .623$$

$$17 \quad .605$$

$$32 \quad .388$$

$$33 \quad .371$$

Fig. 1



Find Rent when $K_0 = 8$; $a_1 = 1$; $i = .05$; $d = .03$

<u>u</u>	<u>$PV - K$</u>	<u>IPF</u>	<u>$(PV - K) IPF$</u>	$a_n = 1.03^{-u}$
16	.851	.092	.078	.623
17	1.122	.089	.100	.605
32	3.435	.063	.216	.388
33	3.574	.062	.218	.377
40	3.925	.058	.229	.307
48	4.189	.055	.232	.242
50	4.233	.055	.232	.228
55	4.319	.054	.232	.197
60	4.377	.053	.231	.170

$$K_0 = 8; a_1 = 1; i = \frac{.08}{.10}; d = .03$$

<u>n</u>	<u>PV-K</u>	<u>IPF</u>	<u>(PV-K) IPF</u>	<u>$q_n = 1.03^{-n}$</u>
16	-1.896			
17				
22	.176	.098	.017	
25	.422	.094	.040	
28	.602	.090	.054	
30	.694	.089	.062	
32	.769	.087	.067	
33	.801			
40	.951	.084	.080	.307
45				
50				.228
55				
60	1.074	.081	.087	.170

So here, a higher i.r. makes bldg. last longer!
And this is normal — not an aberration. The
higher i.r. devalues the land.

In choosing between values of K_0 , higher i.r. will favor lower K_0 , I am sure. But for fixed K_0 — and fixed i and a , etc. — the geriatric afterlife is extended by devaluing the land.

Fig. 2

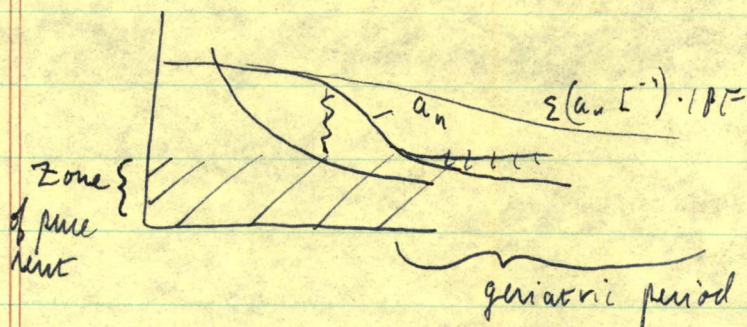


Fig. shows how long geriatric pd. can go on, or be cut short, by pressure of future rent.

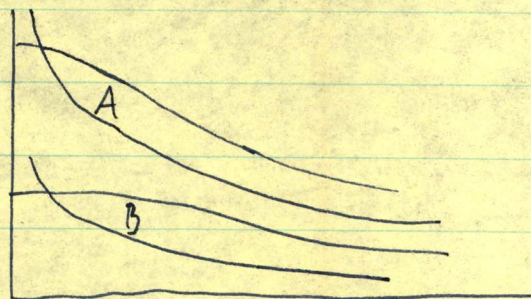
Future rent depends on future i.r. The landholder can wait for favorable rate. ∴ ∴

- Future rent is det. indep. of G_t rent.
 - " " " based on a low i.r.,
- bes. in the ups + downs of rates, owner will select a time of low rates
- This is what makes bldg. so sensitive to i.r. Otherwise, l.v. cushion wd. offset Δ i.r.
 - But in planning life of G_t you do not assume

short life bus. of hi future d.v. based on low i.r. —
 bus. you might have to wait 10-15 yrs. for it.
 The uncertainty of timing offsets the certainty that
 there will be a time.

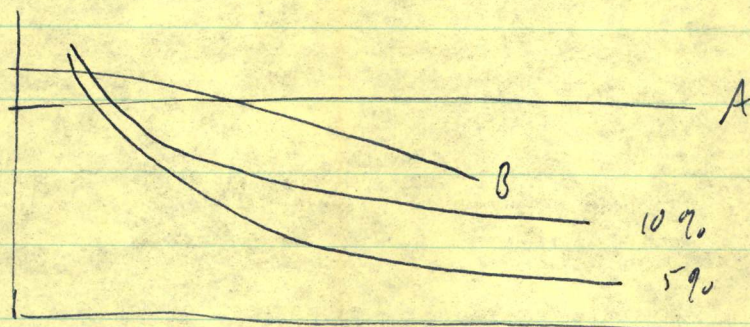
— The rent you can get from G, depends on your
 borrowing cheap. Grab long term at a good time.

Fig. 3



Obviously, A is more sensitive to i.r. Regardless of
 longevity. GTO p. 9

Fig. 4



See next 2 pp.

Based on Fig. 4, p. 6

A (p. 6) is unaffected by i.r. Has long life at any rate. But its net rent is more sensitive to i.r.

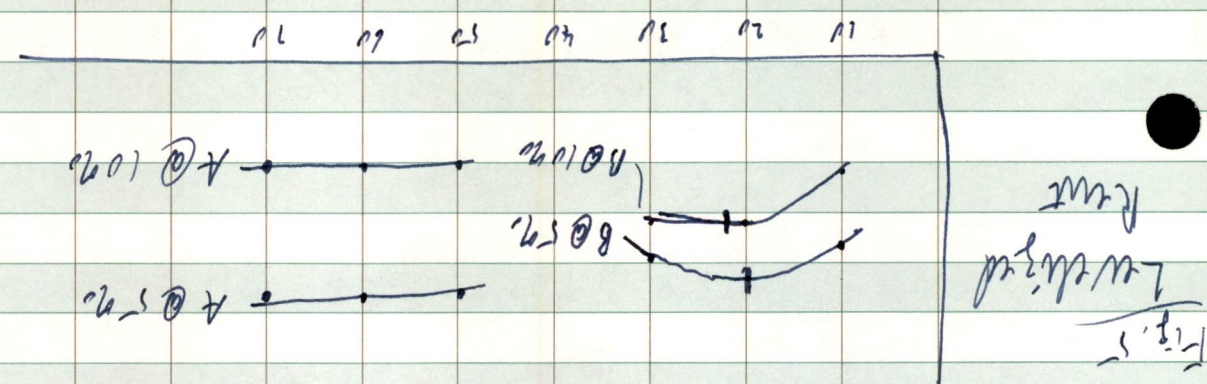
B's life is shortened by higher i.r. But its rent drops less. $\therefore$ higher i.r. causes shift from A to B.

	<u>A</u>			<u>B</u>		
	$d = .01; K_0 = 7; a_1 = 1$					
	$i = .05$			$i = .10$		
<u>n</u>	<u>$PV - K_0$</u>	<u>$1/PF$</u>	<u>$(PV - K)/PF$</u>	<u>$P - K$</u>	<u>$1/PF$</u>	<u>$(P - K)/PF$</u>

	A				B			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\bar{n}$	P-15	P-15	P-15	P-15	P-15	P-15	P-15	P-15
$\bar{t}$	.05	.05	.05	.05	.05	.05	.05	.05
a_n	1	2	3	4	1	2	3	4
K_0	7	7	7	7	7	7	7	7
d	.01	.01	.01	.01	.01	.01	.01	.01

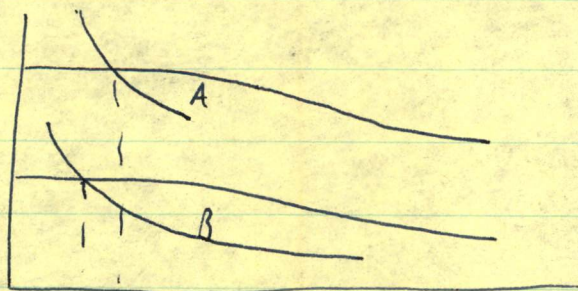
10	3.038	.130	.393	1.385	.163	.225	.771
20	5.519	.080	.443	2.739	.117	.322	.297
30	1.694	.106	.180	.742	6.132	.065	.399
40	1.951	.102	.200	.672	6.284	2.993	.044
50	8.762	.055	.480	2.042	.101	.206	.608
60	9.161	.053	.484	2.074	.100	.208	.550
70	9.385	.052	.485	2.085	.100	.209	.498

The means of i.v. from 5% to 10% water B rise from 2d choice to 1st choice; whereas from 70 or 50 down to about 20.



From Fig. 3, p. 6

A will last longer because its a_n is much higher and its rent is the same!

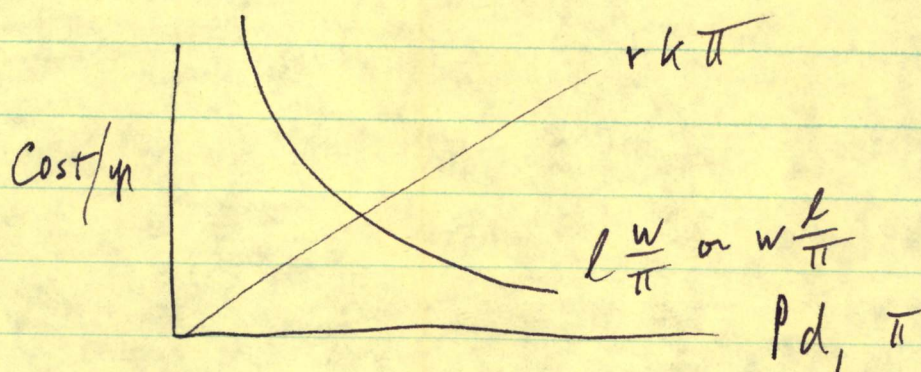


Seems likely, A will max at higher u , given symmetry.

Once we find opt. u ; and rent; then we can figure cap. recovery, which is resid. after rent, and int. on unrecovered K .

But if L.V. rises, during life, the cash for the Δ L.V. income comes right out of cash flow, just like int. + rent, bcs. it is income, + income comes out first. Thus, Δ L.V. prelates cap. recovery.

Replacement

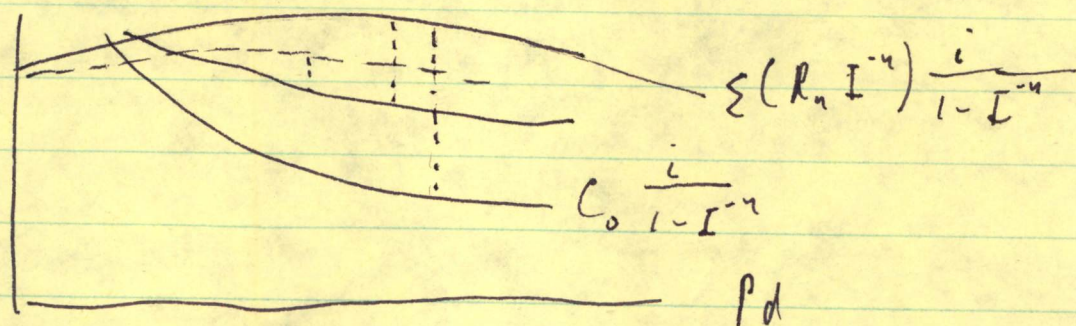


As Q in ea. order rises that part of cost causes $r k \pi$ to rise, in prop. to Q . Ecs. of scale are all subsumed in fixity of the cost of ordering, ("b" in Wonnacott).

You could also have b rise with Q , but with less than Q^1 -- say Q^b .

But how handle b , when order occurs every 20 years? Discount? You have to annualize. Then the model loses its simplicity. (also, mean stock no longer = $\frac{1}{2}$ max.) Now, interest cost appears in fac. factors

To annualize, go directly to a fish model?



But this has introduced a new vble: R . Whereas the inv. model is strictly cost min.

2

The inventory model is a simulated normalized model.

As a normalized model, it has a const. level of K (lower than actual, slightly) which remains fixed at its mean level. And an input of labor which is the same every year.

You lower the mean level by lowering the max. level, by Δ freq. of order, Q , which Δ labor input.

In regular normalized models, you hold work input constant -- plant one tree a year -- and K changes.

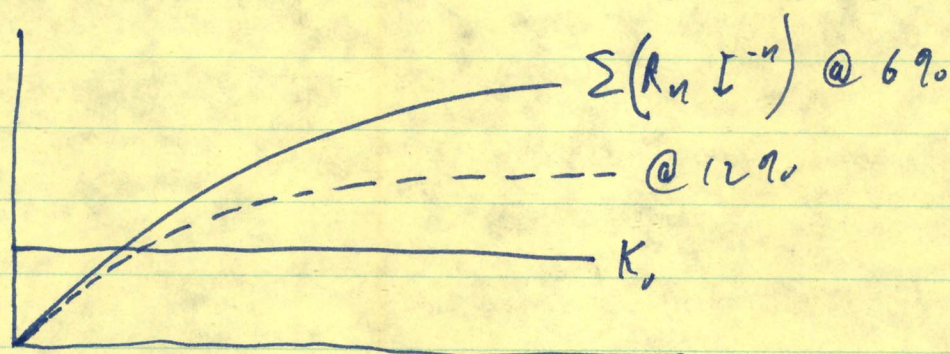
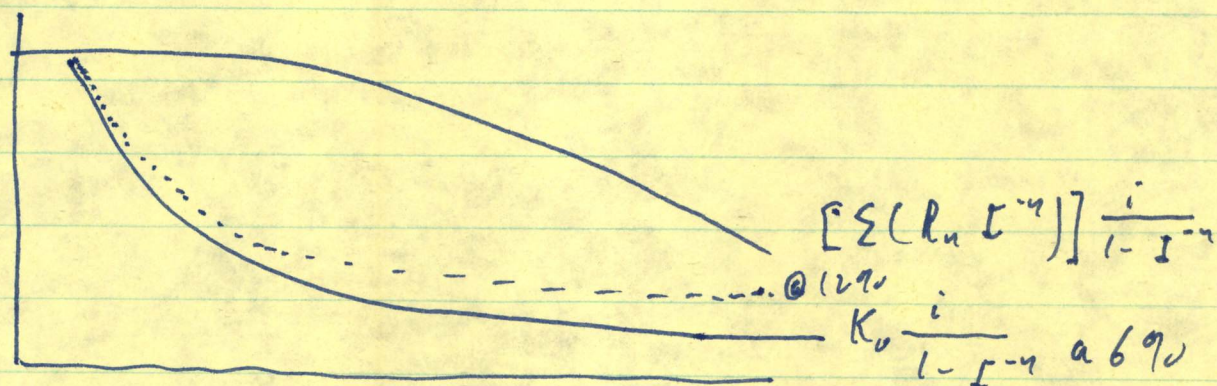
In inv. model you hold throughput constant: fixed sales, fixed income. And the inv. does not grow during its life: -- its productivity is expressed in saving b costs (action costs). There is no make-up.

The model does not handle compound interest; nor does it handle rent (although we could enter storage costs other than interest.)

Thus it is a crude model.

In a proper model:

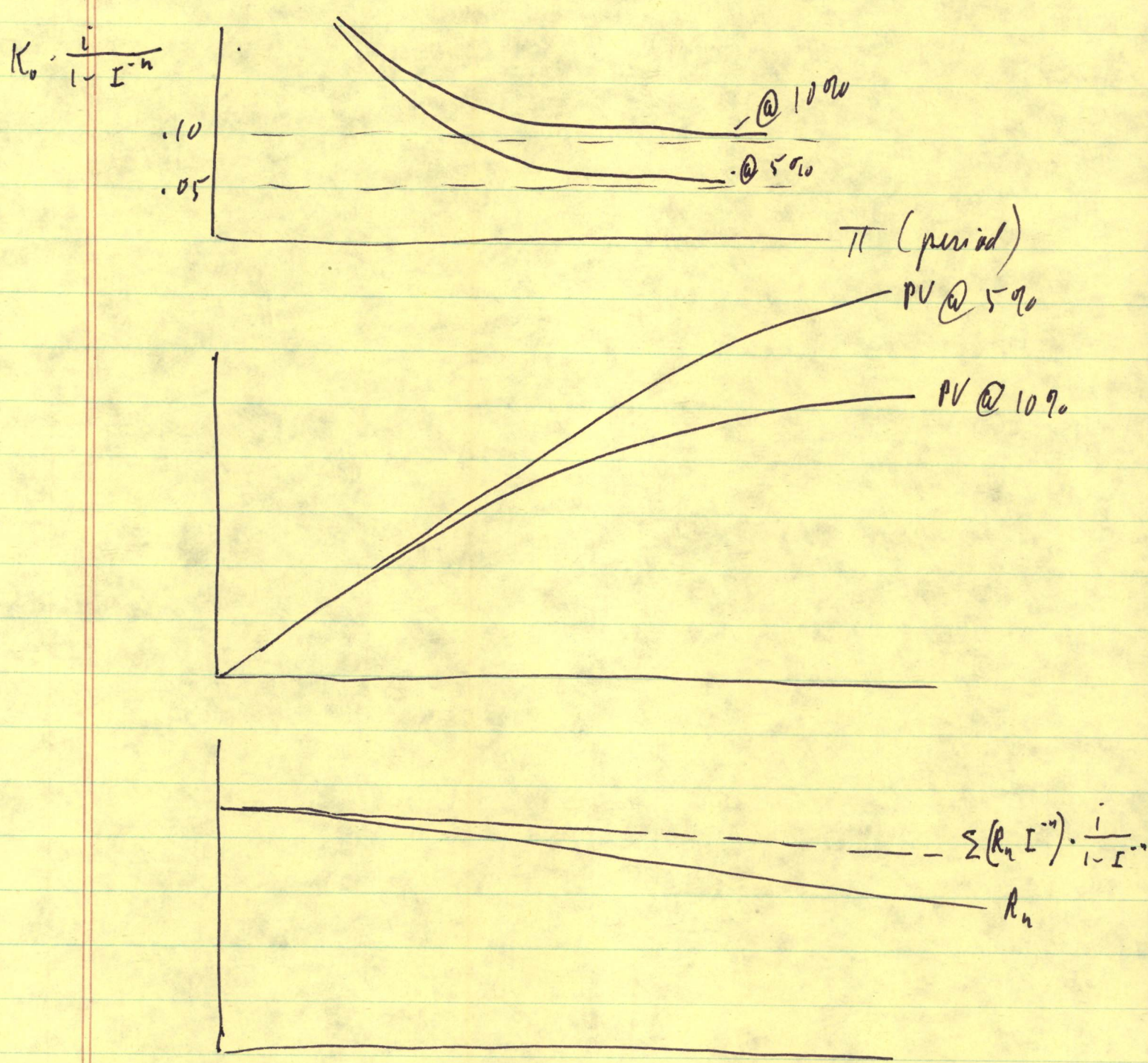
$$\text{Max} \left[\sum (R_n I^{-n}) - K_0 \right] \frac{i}{1 - I^{-n}}$$



As ~~top~~ PVR falls, IPF rises, offsets. E.g. if α doesn't decay,
~~the~~ levelized $R = \alpha$.

Put this on Lotus and play what-if.

In fish model, higher i leads to higher ϕ (freq.)
by flattening the $K_0 \cdot 10i$ curve; and speeding falloff
of R curve.



5
Fall off of R curve is a more subtle effect, much.

Let a = levelized current cash flow

Let d = annual % decay.

$$a = a_1 \left[\frac{1 - (I+d)^{-n}}{i+d} \right] \frac{i}{1 - I^{-n}}$$

How is sensitivity of a to n affected by i ?

Ripening

Assume ^{annual} rents begin tomorrow.

$$PV \text{ of land} = 1 + \frac{1}{i} = \frac{1+i}{i}$$

If skip 1st yr's rent, loss = 1

$$\% \text{ loss} = \frac{1}{\frac{1+i}{i}} = \frac{i}{1+i} \quad (1)$$

If 1st rent is at end of yr. one.

Loss is $\frac{1}{1+i}$

$$\% \text{ loss} = \frac{\frac{1}{1+i}}{\frac{1}{i}} = \frac{i}{1+i} \quad (2)$$

If rent rising.

$$\% \text{ loss} = \frac{\frac{1}{1+i}}{\frac{1}{i-r}} = \frac{i-r}{1+i} \quad (3)$$

Here, the % loss to owner is slight — even tho the absolute loss is same; and losses to others (employees, customers) are the same.

2

if land taxed

$$\text{Loss is } \frac{1}{1+i}$$

$$\text{PV} = \frac{1}{i+t-g}$$

$$\% \text{ loss} = \frac{i+t-g}{1+i} \quad (4)$$

From (1) + (2), you lose $i \cdot P$ by wasting one year. So a must have a permanent rise of $P \cdot i$ to recoup.

Actually, only G_i will be higher, unless this is a continuing rise.

Suppose an inf. series of 10-yr. leases

$$PV = a \frac{1 - I^{-10}}{i} + a(1+g) \frac{1 - I^{-10}}{i} I^{-10} \text{ extra}$$

$$+ a(1+g)^2 \frac{1 - I^{-10}}{i} I^{-20} + \dots + a(1+g)^{\infty} \frac{1 - I^{-10}}{i} I^{-\infty}$$

$$= a \frac{1 - I^{-10}}{i} \left[1 + \frac{(1+g)}{I^{10}} + \frac{(1+g)^2}{I^{20}} + \dots \right]$$

$$= a \frac{1 - I^{-10}}{i} \left[\frac{1}{1 - \frac{(1+g)}{I^{10}}} \right] = \frac{a}{i} \frac{1 - I^{-10}}{1 - I^{-10}(1+g)}$$

Instead of lease, cd. buy 40-yr. time-slot of land, pay up front.

How much reduce P ?

1. Const. rent.

$$\frac{\frac{1 - I^{-40}}{i}}{\frac{1}{i}} = 1 - I^{-40} = .86 @ 5\%$$

2. Rising rent

$$\frac{\frac{1 - (1+i-g)^{-40}}{i-g}}{\frac{1}{i-g}} = 1 - (1+i-g)^{-40}$$

$$\text{Let } g-i = 2\% \rightarrow \downarrow = .55$$

3. Steady rent ff. by Δ rent.

$$\frac{\frac{1 - I^{-40}}{i} + \frac{3}{i} I^{-40}}{\frac{1 - I^{-40}}{i}} = \frac{1 - I^{-40}}{1 - I^{-40} + 3I^{-40}} = \frac{1 - I^{-40}}{1 + 2I^{-40}}$$
$$= \frac{.86}{1 + 2 \times .142} = .67$$

Kari: Median voter model is not useful.

Parkinson's Law here

✓ You don't rent this yr; but you might rent
for more next yr. Assuming long term contract.
How much will you forego to get more

Call Bill V. 914/478-0577
w. Wild's no.

phone: 312/587-9890

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