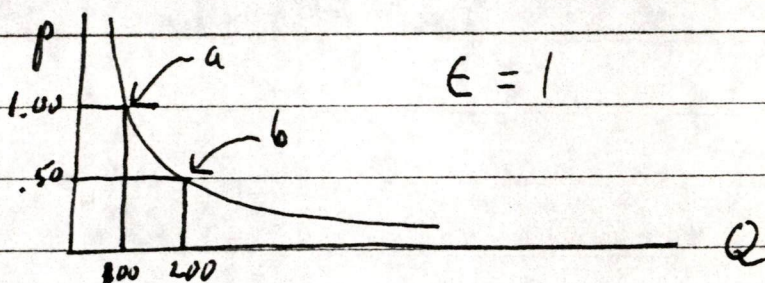


Ec. 162, Exams, Hand-
outs

"Arc-elasticity"

This illustrates why we need arc-elasticity when there are large changes in P and/or Q .

Consider a demand curve of "unit elasticity", $E = 1$. Every 1% drop in P causes a 1% rise of Q . Thus $P \cdot Q$ remains constant. The curve is a hyperbola:



Moving down the curve, a -1% ΔP gets smaller; a 1% ΔQ gets larger (because Q gets larger).

at a, $P = \$1.00$; $Q = 100$; $PQ = \$100$

at b, $P = \$0.50$; $Q = 200$; $PQ = \$100$

If you move from a to b in one hop, $-\frac{\Delta P}{P} = \frac{.5}{1} = 50\%$

$\frac{\Delta Q}{Q} = \frac{100}{100} = 100\%$. Simple $E = \frac{100\%}{50\%} = 2$. The

cumulative change in the bases distorts the value. To adjust, use arc- E .

$$\text{Arc-}E = \frac{\left(\frac{100}{150}\right)}{\left(\frac{50}{75}\right)} = 1$$

(over)

If you don't arc the ϵ you'll think you need $\epsilon = 2$ to accomplish a ΔQ which in fact only needs $\epsilon = 1$.

Or you'll think, after a sale with 50% off, you have $\epsilon = 2$, when in fact it is only $= 1$.

Thus you'll become biased against large price cuts, and in favor of small ones.

Qu. on 3d Quiz: Answer below

$$\text{Given: } -\frac{\Delta P}{P_1} = .3 \quad ; \quad \frac{\Delta Q}{Q_1} = .4$$

For ease, $P_1 = 1$, $Q_1 = 1$. Any other constant will cancel out anyway.

$$\text{Simple } \epsilon = \frac{.4}{.3} = 1.33, \text{ too high.}$$

$$\text{Arc } \epsilon = \frac{.4/1.2}{.3/.85} = 1.33 \times \frac{.85}{1.2} = .94$$

A quicker way to get $\text{atc} - \epsilon$ is with logs.
Try this:

$$\frac{\ln 200 - \ln 100}{\ln 1 - \ln \frac{1}{2}} = \frac{.6931}{.6931} = 1$$

$$\frac{\ln 140 - \ln 100}{\ln 100 - \ln 70} = \frac{.3365}{.3567} = .94$$

ϵ is the ratio of the changes in the natural logs, however large or small the change.
This also works with log base 10, but I recommend you stick with natural logs (LN), in general, because the change in the natural log is the fractional change in the number.

For some insight into why this works, try log base 2.

$$\frac{\frac{256 - 128}{128}}{\frac{8 - 4}{8}} = \frac{\frac{2 - 1}{1 - \frac{1}{2}}}{1} = 2$$

$$\frac{\log_2 256 - \log_2 128}{\log_2 8 - \log_2 4} = \frac{8 - 7}{3 - 2} = 1$$

Marvel & ponder! I leave it at that.

Papama Game

5 yrs.

$$260 \text{ wks} \times 40 \text{ hrs.} = 10,400$$

$$260 \text{ wks} \times 2\frac{1}{4} \dots = 585$$

$$10,400 \times ~~780~~.075 = 780$$

$$585 \times .1125 = 65.8125$$

$$\underline{845.81}$$

10 yrs.

$$1691.92$$

20 yrs.

$$3383.25$$

$$\text{Monthly: } 845.81 / 60 \text{ mos.} = 14.10 / \text{mo}$$

5 yrs.

@ 12%

$$FV = \frac{1.01^{60} - 1}{.01} = 81.67$$

$$\frac{81.67}{60} = 1.36$$

$$1.36 \times 845.21 = 1150$$

$$10 \text{ yrs.} \quad \frac{1.01^{120} - 1}{.01} = 230$$

$$\frac{230}{120} = 1.92$$

$$1.92 \cdot 1691.92 = 3,243.39$$

$$20 \text{ yrs.} \quad \frac{1.01^{240} - 1}{.01} = \cancel{1089} 989.25$$

$$\frac{\cancel{10.89} \quad 989.26}{240} = 4.12$$

$$4.12 \cdot 3383.25 = 13,945.41$$

<u>Yrs</u>	<u>Mos.</u>	<u>Pg Calc.</u>	<u>Correct Calc.</u>
5	60	852.74	845.81
10	120	1605.48	1691.62
20	240	3411.96	3383.25

For ^{some} Wage Workers, 2% / mo. is more ~~relevant~~ fitting than 1%.

<u>Mos</u>	<u>FVF</u> <u>@ 2%</u>	<u>FVF/n</u>
60	114.05	1.90
120	488.26	4.07
240	5,744.44	23.94

Mgmt. problems

1. Marie's invt. goal is to double her accum. savg. every 8 yrs. by saving ~~\$1,000~~^a / yr. What i.r.? (to 2 figs)

$$F = a \frac{I^n - 1}{i}$$

$$F = 2 \times 8a = 16a$$

$$n = 8$$

$$16a = a \frac{I^8 - 1}{i}$$

$$i = \frac{I^8 - 1}{16}$$

<u>i</u>	<u>Left i</u>	
.01	.041	
.03	.133	.017
.08	.425	.053
.16	1.139	.142
.12		.157
.18		.172
✓ .14		.189

2. Begin paying SST age 18. Employer matches.

By age 65:

a. How much will have ~~in yr acc~~ put in?

Assume 15% of \$45k, = 6,750

$$65 - 18 = 47$$

$$47 \times 6750 = 317,250$$

b. At 9%, how much int. accum?

$$F = a \frac{1.09^{47} - 1}{.09} = 626.86 a$$

$$627a - 47a = 580a = \$3,914,074$$

c. At age 65 you may receive benefits of \$1,000/mo.

i) If you live to 80, how much ~~for~~ accum. bns. Will you ^{have} received? $15 \times 12,000 = 180,000$

$$P = a \frac{1 - 1.09^{-15}}{.09} = 8.06 a = 96,730$$

ii) a) What is their PV as of yr. age 65?

b) What % is this of value of yr. contribs. at that time?

$$\frac{626.86 a}{8.06 a} = .0769$$

iii) What is their PV as of now?

$$8.06 a / 1.09^{47} = \frac{8.06 a}{57.42} = .1404 a = \$947,53$$

$$b) \frac{PV}{15a} = \frac{8.06}{15} = .54$$

Quiz. 18 Nov. Answers.

$$1) a) \quad i = \frac{970}{6964} \frac{1 - I^{-10}}{1 - I^{-n}} = .139 (1 - I^{-n})$$

$$i = .066$$

$$b) \quad P = a \frac{1 - I^{-n}}{i}$$

$$\frac{Pi}{a} - 1 = -I^{-n}$$

$$I^{-n} = 1 - \frac{Pi}{a}$$

$$-n \ln I = \ln \left[1 - \frac{Pi}{a} \right]$$

$$n = \frac{-\ln \left[1 - \frac{Pi}{a} \right]}{\ln I}$$

$$\frac{Pi}{a} = 1 - I^{-n}$$