162 Old Notes S97

Quiz 1, 4 Apr. 94

1. Granny gave you ~~$1,000~~ $823 when you were born. It grows at 8.3% comp. How much will you have when you are 23? ~~$6,258~~ $5,151.

$823 × 6.258 = $5,151

2. What if it grew at ~~4.15%~~ 2%. half the rate?

$823 × 2.575 = $2,265

Wendy Marcus & Wu

3. a 0-coupon bond, costs $700, grows at 5%, & matures in 12 yrs. You disc. future values at 4%. What ~~will~~ premium ~~will~~ you ~~will~~ pay for the bond ~~%~~ if you lived to ?.

$$700\left(\frac{1.05}{1.04}\right)^{12} = 700 \cdot 1.1217 = \$785.18$$

Premium = $85.18

4. What is the error in the Rule of 72 when n = 60 and i = .12

X credit?

Cheated?

$g = 100$

$g = .03$

i rises from .04 to .06

1. % rise of i
2. % rise of cap rate
3. % change in S

1. $\frac{.02}{.04} = 50\%$

2. $\frac{.02}{.01} = 200\%$

3. $\frac{3,333 - 10,000}{10,000} = -67\%$

2nd Quiz

Value: $q = 10$

~~$t = 50$~~ $R = 1,000$

$i = .06$

Use changes when $n = 14$

Quiz 2. Appl 11

1. Draw the locus of all points satisfying $F = I^n$ from $n = 0$ to $n = 3$ when $i = .08$. Use Rule of 72. (2 pts)

2. Use calc. to find F when

n =	F
13	2.72
31	10.87
59	93.76

3. Discount yr. answers in (2) to the present, when $i = .053$

$\frac{2.72}{1.96} = 1.39$ $\frac{10.87}{4.96} = 2.19$ $\frac{93.76}{21.05} = 4.45$

4. Illustrate Step 3 graphically

Mon. Sept. 14

1. Rate of exchange, intertemporal

A. 2 pts: $\frac{F}{P} = I^n$

$\frac{P}{F} = I^{-n}$

Rate changes w. I and n

B. Points & flow

$\frac{P}{a} = \frac{1}{i}$

Power of i.

C. Points & future flow

$P = \frac{a}{i} I^{-n}$

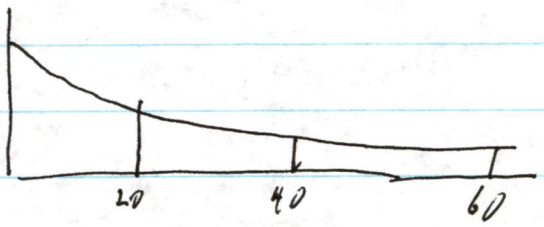
Even-filled tube of toothpaste.

Wed., Sept. 16

1. $n = 1$

$$ACFF = \frac{1 - I^{-1}}{i} = \frac{\frac{I - 1}{I}}{i} = \frac{\frac{i}{I}}{i} = \frac{1}{I}$$

Check

2. 

$$\frac{\frac{1 - I^{-40}}{i}}{\frac{1 - I^{-60}}{i}} = \frac{1 - I^{-40}}{1 - I^{-60}}$$

Use low i; then high.

3. Problem on inputing, p. 2

Then deduct K, for triple net PV.

Might also deduct xptn. cost, storage, etc.
"Netback" technique.
Thus, "the farmer" gets a small share.
Even the oil man does.

4. Orange grower

Quiz, 18 Apr. 97

1. You can double the useful life of the family from 7 yrs to 14 yrs by paying 40% more now (for extra room ~~a full back seat~~).
By what fraction do you raise the PV_0? $i = .16$

$$\frac{1 - I^{-14}}{1 - I^{-7}} - 1 = \frac{.875}{.646} - 1 = .354$$

2. A warehouse may last 75 yrs. $i = .03$
What fraction of DCF comes from last $\frac{1}{2}$ of life?

$$1 - \frac{1 - I^{-50}}{1 - I^{-75}} = 1 - \frac{.772}{.891} = .134$$

$$\left\{ \frac{DCF_{14} - DCF_7}{DCF_7} = \frac{DCF_{14}}{DCF_7} - 1 \right.$$

Fri. 18 Apr.

Orange Grove

$$PCF = a\frac{1 - I^{-15}}{i} + \frac{b}{i} I^{-15}$$

Let $b = 5a$; $i = .06$

$$PCF = 9.7a + \frac{16.7b}{2.40} = 6.96b$$

$$6.96b = 34.8a$$

Wed, 23 Apr. 97

Rising Rents

p. 1 1. Usufruct, p. 1

p. 1 2. Land yields perpetual c.f. if you apply K every 25 yrs. or so.

$$S = \frac{a}{i-f} - K_0 \frac{1}{1-I^{-25}} \quad \text{if land is bare}$$

$$S = \frac{a}{i-f} - K_{25} \frac{1}{I^{25}-1}$$

$$\frac{1}{1-I^{-25}} = 1.42 \qquad (i = .05)$$

$$\frac{1}{I^{25}-1} = .42$$

$$\left[\frac{1}{I^n-1} + 1 = \frac{I^n + I^n - 1}{I^n - 1} = \frac{1}{1-I^{-n}}\right]$$

pp. 1-2 3. Assume q in current \$, not const. \$
Use nominal i.r.
Explain – how to handle inflm.

p. 2 4. Let $t = 0$. Skip sections 2, 5, 6, 7, 9, 10

p. 2-3 5. $\hat{P}/e^q = \frac{1}{i-f}$

p. 4 6. $\Delta s = gS$

p.4 7. Inc. = $a + gS$
gS is part of current income.

pp.5-6 8. Inc after inc. tax = $a(1-t) + gS$

(gS largely escapes inc. tax)

(so does ~~a~~ $\frac{a}{3}$, if there is a bldg. to deprec.)

9. cc = Si (t=0)

p.5 10. $Si = a + gS > a$

You need to count gS as inc., to cover Si
(If you can't, don't buy land.)

p.10 11. $Si - a$ = Negative Cash Flow
(If you're 100% financed.)
If only s is not fin'd, $S(1-s)i - a$ = NCF
Here we introduce a new wrinkle in c.f.
Interest is a current expense.
This is personal, as vs. social ecs.

pp.6-10/12. Skip 5-7, 9-10.

p.12 13. Leverage of g
p.12 is too obscure. Use numerical e.g.

~~$S_1 = \frac{a}{.08 - .03} = 20a$~~

A. $S_1 = \frac{a}{.08 - .03} = 20a$

B. $S_2 = \frac{a}{.04 - .03} = 100a$

i falls by 50%; S rises by 400% $\left(\frac{80}{20}\right)$.

pp. 12-13. 14. Results of leverage

a. ~~Ris~~ Volatility of S

b. Power to strong hands; ↓ concen. of ownership

p. 13 15. Dilution

If $g < 0$.

Fri. 25 Apr. 97

Valuation of intermittent values.

$$S = G_1 \frac{1}{1 - I^{-n}}$$

a. $n = 1$

Then $G_1 = a$

$$S = a \frac{1}{1 - \frac{1}{1+i}} = a \frac{1}{\frac{i}{1+i}} = \frac{a(1+i)}{i} = \frac{a}{i} + a$$

extra "a"?

Why the ~~[illegible]~~. Bcs. G_1 is at $t = 1$.

b. $n = \infty$ $\quad S = G_1$

c. $G_1 = a \frac{1 - I^{-n}}{i} - K$

$$S = \frac{a}{i} - K \frac{1}{1 - I^{-n}}$$

To #35

Mon, 28 Apr.

1. DCF from intermittent series

$$S = \frac{a}{i} - K\frac{1}{1-I^{-n}} + \frac{a}{i}$$

$$S = \frac{a}{i} - K\frac{1}{I^{n}-1}$$

$$\frac{1}{1-I^{-n}} - 1 = \frac{1-1+I^{-n}}{1-I^{-n}} = \frac{1}{I^{n}-1}$$

2. Nos. – See #24, p. 5

3. #35. Office bldg. model. Linking cause & effect.

Wed. 30 Apr.

1. Purpose of Office Bldg. Model

a. Show how opt. K is determined.
b. " " S is determined
c. Introduce concept of ROR at the margin.

2. #42. Count Tillie

a. "Capd. income" from timber
1. Find ~~annual income~~. PV of G_1

$$G_n = -C_0 I^n + F_n = -100 \cdot 6.45 + \$900$$
$$= 254.66$$

$$S = \frac{G_n}{I^n - 1} = \frac{254.60}{6.45 - 1} = 46.7$$

2. Find annual value

$$S_i = \$2.80$$

3. Makes all time slots commensurable

4. Consump. ratio

5. Curr. value of prop. tax p. ℔

Mon., 5 May.

1. Cinco de Mayo

#50 2. Annualizing + PF

$$a = P \frac{i}{1 - (1+i)^{-n}}$$

$$\frac{a}{P} = \frac{i}{1 - (1+i)^{-n}}$$

3. Extremes

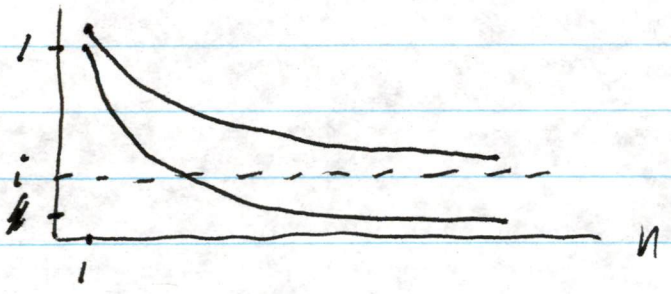
$n = 1$ $\qquad \frac{i}{\frac{i}{1+i}} = 1+i$

$n = \infty$ $\qquad i$

$i = 0$ $\qquad \frac{0}{0} = ? \quad \frac{1}{n}$

4. Table - make one as homework

5.

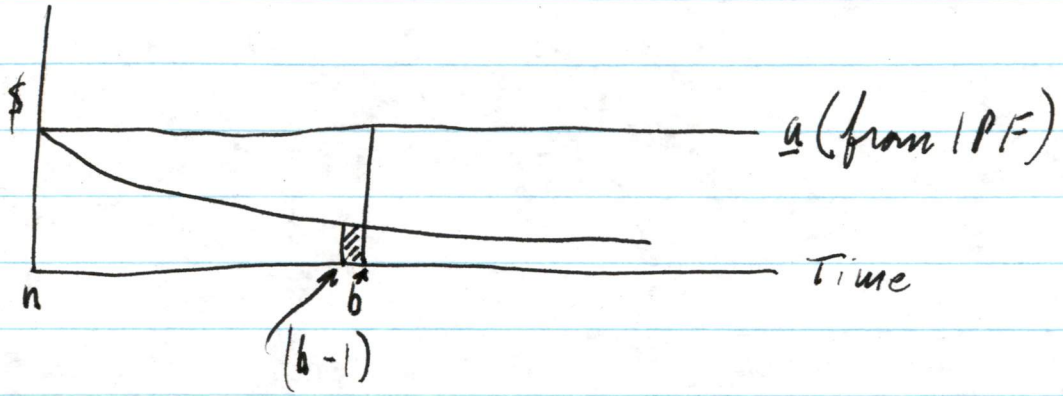


a "time-average," adjusted for the time-value of money.

a combo of dividing by n, & multiplying by i.

Tube of toothpaste; roll of paper

#5 | 6.



n = current year
6 = remaining life (balance of life)
(w = whole life = n + 6) 6 = w − n
D = Depreci. = shaded area
B_n = Balance = $P_1 - \sum_1^n [D_6]$
$D = a\, I^{-n}$

Wed. 7 May

1. How much higher than $\frac{1}{n}$ is IPF? ($i = .07$)

n	$\frac{1}{n}$	IPF	IPF/$\frac{1}{n}$
10	.10	.142	1.42
25	.04	.086	2.15
50	.02	.072	3.62

2. What fraction % do you lower annual pmt. by doubling n?

$\frac{1}{n}$ falls by 50%

$$\frac{.072 - .086}{.086} = \frac{.072}{.086} - 1 = -16.3\%$$

3. How about doubling from 5 to 10?

$$\frac{.142}{.244} - 1 = -41.8\%$$

4. Work #2 the other way: cut n from 50 to 25. Double i from .02 to .04.

By what fraction Δ IPF? $\frac{.086}{.072} - 1 = 19.4\%$

Moral: to cut down on interest, pay off your debts.

5. How much of total paid is interest? $\frac{P}{na}$ is % not interest

$$TP = na = nP\frac{i}{1-I^{-n}}$$

$$Int = na - P = P\left(\frac{ni}{1-I^{-n}} - 1\right)$$

$$\frac{Int}{TP} = \frac{na-P}{na} = 1 - \frac{P}{na}$$

$$\frac{P}{a} = \frac{1-I^{-n}}{i} \qquad \frac{P}{na} = \frac{1-I^{-n}}{ni}$$

$$\frac{Int}{TP} = 1 - \frac{1-I^{-n}}{ni} = 1 - \frac{DCFF}{n}$$

n	$\frac{DCFF}{n}$, $i=.07$	$i=.14$
5	.18	.31
10	.30	.48
25	.53	.73
50	.72	.86

6. Return to 1st year pmt: ~~last~~ how much do you lower Balance? By what %.

$$0 = aI^{-n}$$

$$P = a\frac{1-I^{-n}}{i}$$

$$\frac{\Delta}{P} = \frac{I^{-n}}{1-\frac{1}{i}} = \frac{i}{I^n - 1} \qquad (p.4)$$

6 (cont.)

Values of SFF $\frac{i}{I^n - 1}$

n \ i =	.04	.07	.11
5	.185		.161
10	.083	.072	
25	.024	(.016)	
50	.007		

#52 Capital Sinks
Success stories.

Quiz, Fri, May 9

1. You borrow $42,560 at 8.7%, to be repaid in level annual payments over 25 yrs.
   a. What is the annual pmt.
   b. How much of the first pmt goes to retire the principle?
   c. How much int. will you have pd. after 25 yrs?

a. $\$42,560 \times \frac{.087}{1-1.087^{-25}} =$

$\frac{.087}{1-.124} =$

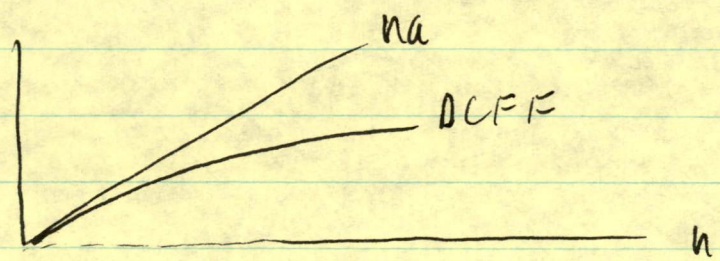
$.099 = \$4,228$

b. $4,228 \times 1.087^{-25} =$

$.124 = \$525$

c. $\frac{Int}{TP} = 1 - \frac{DCFE}{n} = 1 - \frac{10.07}{25} = 1 - .403 = .597$

$Int = .597 \times na = .597 \times 25 \times \$4228 = \$63,140$



Quiz 23 May

~~94 Honda Sedan~~ Sport-utes

$n = 3$

$R_1 = 18k$

$R_2 = 15.5k$

$r_{rent} = \$400/mo = 4800/yr.$

$P = \$30{,}000$

$i = .09$

#1. $a = \$30{,}000 \times \frac{.09}{1-1.09^{-3}} - 18{,}000 \times \frac{.09}{1.09^{3}-1} =$

$= P \cdot .345 - R \times .305$

$= 11{,}852 - 5{,}490 = 6{,}362$

#2. $11{,}852 - 15{,}000 \cdot .305 = 7277$

3. Is lessor using higher or lower than 9%?

3. Fractional change in $R = \frac{-3}{18} = -.167$

4. " " " $q = \frac{7277 - 6362}{6362} = 14.4\%$

5. $E = \frac{.144}{-.167} = -.861$

6. $P = 100k \qquad i = .07$

$R_{25} = 120k$

$K_{11} = 55k$

$a = P \cdot .086 + K_{11} \cdot .086 \cdot .475 - R_{25} \cdot .016$

$= 8600 + 2247 - 1920 = 8927$

Wed, 28 May

1. $i = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1$

E.g. $\frac{F}{P} = 2$,

$n = 25$,

$i = .028$

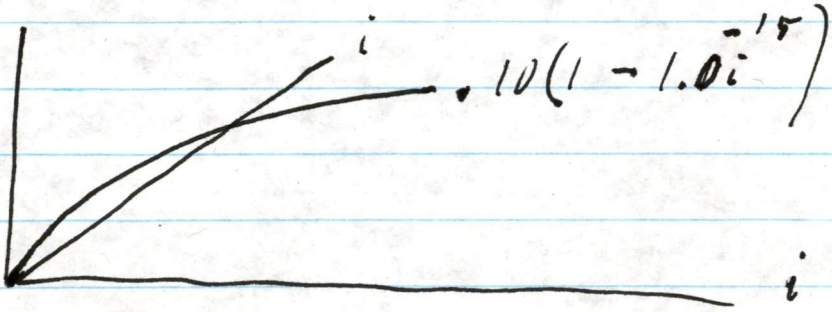
(Note, $ni \cong .72$)

$I^n = \frac{F}{P}$

$n \ln I = \ln 2 = .693$

$n = \frac{.693}{\ln I} \cong \frac{.72}{i}$

2. $i = \frac{a}{P}\left(1 - I^{-n}\right)$



If $\frac{a}{P} = .10$, then $i < .10$

Quiz, 30 May

1. $P = \$153k$
   $a = 17k$
   $n = 12$
   $i = ?$
   .095, 0.00, .049,

2. Let $R = \$100k$

   11, .095, 0.00, .049

Adding Headweight

$$i = \frac{a}{(P+X) - (F+x)F^{-n}}(1 - F^{-n})$$

$$= \frac{a}{P - FF^{-n} + X(1 - F^{-n})}(1 - F^{-n})$$

Lowns ROR

Monday

63. Carter
64. Rise of Site Value

June 2, '97

1. #63 Control Cut: "No disassemble"!

2. #64. LV increments

a. p. 2

$$T = \frac{a}{i} - K\frac{1}{1 - I^{-n}}$$

p. 5 b. T lowers ROR

$$ROR = \frac{a(1 - I^{-n})}{K + T(1 - I^{-n})}$$

Table 4 shows how ~~the~~ T lowers ROR

Tendency of users to undervalue T.

c. ΔT offsets the lowering.

p. 6 When $\Delta T = K$, over life, ROR = [illegible]

$$\frac{a}{K + T_0}$$

p. 7 ΔT helps some, then hurts others.

3. 66A. RORAT

a. PVAT (cos)
Moral: learn how to program!

b. RORAT
appears twice:
66A, p. 2, Eqn. 5
66A App., p. 3, Eqn. 1

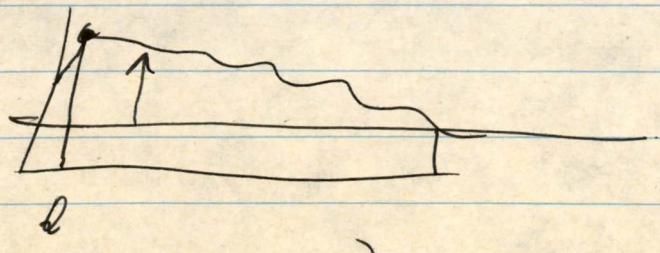
Inspection: p. 5

4. Performance Stds. for K budgeting

a. Max PV
Choose any inv. whose NCF > K_0
PVP = $\frac{a}{}$

b. Min payback time
Choose any inv. whose payback < X years.
i) W/o considering int.
ii) Subject to specified i.r.

c. Max ROR
Choose any inv. w. ROR > X %



d) Max a ("net")

June 4

Perf. Stds.

1. NPV, Payback Time, Net Ann. Value, ROR

2. Maximizing vs. "hurdle" or "threshold" approach.
E.g. of ROR
Mgt involves fine tuning, continuous variation, or "reasoned response" — the kind of thinking that lost the VN war.
Here, we look more at discrete alternatives. a CE needs simple directions for his Div. Chiefs.

3. Reject single-factor standards — e.g., Max. AP of labor or land.
Optimal combo. is the idea, + must "Bottom line" expresses it: but one final no., summarizes a long string of calculations.

4. A. NPV
Any invt. where DCF > K.
Problems:
1. How rank them, if are mutually exclusive?
If rank by size, are biassed toward bigger projects; also toward more durable ones. No replacement criterion. Not commensurable model, causes hdp to be too small. By $\frac{DCF}{K}$? No rationale for that. In office bldg.
2. If i.r. is very low, wd. fill in most of SF Bay, & chunk of Gulf of Mex.
3. K has no effect on ec. life. K means cost of replacement.

B. Payback time – pick any $n < 5$ years.

1. Be sure to define payback time right. Int. comes first. (So does rent, we haven't even handled that.)

2. Fences you out of all longer invts., which may have higher ROR.

Limits you to the most competitive fields.

May raise your volume higher than you can handle. Is good for those w. little K – not for those w. much.

~~Use Market Value. It's best~~

C. ROR

1. Be sure to enter catalyst values, at market. However, to est. those values, you must use Valuation based ~~Diff~~ on market rate of interest.

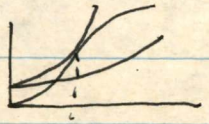
Land values are hard to take from mkt. bec. each parcel is unique – while K is fungible

2. Mkt. i is usually supplied exogenously. E.g., in office bldg. model, $i = 10\%$. Is no gain in stopping where $\frac{CF}{K} = 30\%$.

3. The danger is of "aggregating."

E.g. of tree harvest.

premature



To get the higher ROR, you apply too little K, and

impute 0 to land, so you can't justify using any at all.

D. Annual Value

Max. $\left(\underline{F(n) - K_0 I^n}\right)$ SFF

1. All costs are considered.
2. Future gen. are considered, b/c $\frac{SFF}{i}$ = Perp. Mult.
3. Land value is found from w/in the solution — and land is hardest to value from the mkt. No exogenous l.v. is required.
4. Note what happens when $i \to 0$.

$SFF \to \frac{1}{n}$

So you max $\frac{(F - K)}{n}$

Cf. NPV, where you max F.

$$\left(\frac{1.062}{1.076}\right)^{14}$$