

Ec. 162-A, F-875 Old Notes

Ec, 162 A

Thurs., 4/10/83

1. We are learning powers, ~~and~~ + electricity, use in prodn. eqs.

$$y = x^4$$

E.g. you have x cubes, so volume is $x \cdot x^3$
 Δx by one, + ~~there~~ there is one more cube, +
each cube is larger by $3x^2$, so the set is larger
by $x \cdot 3x^2 = 3x^3$.

But there is also one more cube at x^3 .
 $3x^3 + x^3 = 4x^3$

Then you could have x sets of x cubes @, etc.

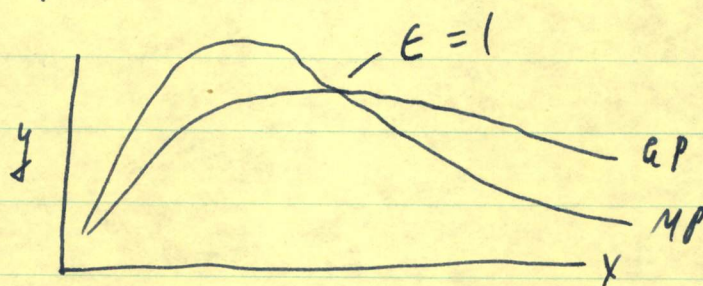
Or you could divide ea. cube into x parts,
+ y is the no of parts (rather than volume). E.g. as
a city grows + space is dear, you get more
bldgs, with more rooms per bldg., + larger bldgs.
E.g. as air travel Δ d, got larger planes, more
planes, ~~the~~ higher use factors, smaller seats. Let
 x be length of plane, and the other factors are
all prop. to x , then capacity $= kx^5$

2. When $y = x^2$, & x varies, 2 is E

$$y' = 2x$$

$$\frac{y'/y}{1/x} = \frac{2x/x^2}{1/x} = \frac{2x}{x} = 2 \quad (x' = 1)$$

3.
$$\frac{y'/y}{x'/x} = \frac{y'/x'}{y/x} = \frac{MP}{AP}$$



4. Can also do these backwards.
Often is more useful, e.g. if $y = x^2$

If $y = x^2$, then $x = y^{\frac{1}{2}}$

x is input, or Cost

$$E = \frac{MC}{AC} = .5$$



5. Engineering "6/10 rule" - of Thumb.

$$A = \pi r^2$$

$$r = \left(\frac{A}{\pi}\right)^{\frac{1}{2}} = \frac{1}{\pi^{\frac{1}{2}}} A^{\frac{1}{2}}$$

$$dL = 2\pi r = 2\pi^{\frac{1}{2}} A^{\frac{1}{2}}$$

$$\frac{dL}{dA} = \frac{1}{2} 2\pi^{\frac{1}{2}} A^{-\frac{1}{2}} = \frac{\pi^{\frac{1}{2}}}{A^{\frac{1}{2}}} = \left(\frac{\pi}{A}\right)^{\frac{1}{2}} = \left(\frac{1}{r^2}\right)^{\frac{1}{2}} = \frac{1}{r}$$

$$\frac{\frac{dL/dA}{L}}{1/A} = \frac{\pi^{\frac{1}{2}}/A^{\frac{1}{2}}}{2\pi r} = \frac{\pi^{\frac{1}{2}} A^{\frac{1}{2}}}{2\pi r}$$

$$= \frac{1/r}{2\pi r} \pi r^2 = \frac{1}{2}$$

(This is the hard way!)

Ec. 162

Wed. 10/9

1. The power rule.

a. As variant of the product rule

$$a = BH$$

$$a' = BH' + HB'$$

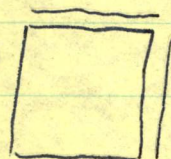
$$\text{Let } H = B$$

$$\text{Then } a' = BB' + BB' = 2BB'$$

$$\frac{a'}{B'} = 2B$$

b. Usually, as in B + P, ~~y~~ $y = x^2$
 $y' = 2x$

E.g. chessboard



add $8 + 8$

c. Cubes. Add 3 faces

$$y = x^3$$

$$y' = 3x^2$$

d. $y = x^4$
 $y' = 4x^3$

You have x cubes, each of base x , their total volume is $x \cdot x^3 = x^4$

It is important to be able to work in more than 3 dimensions, which is the limit of visual imagery. For many people, 2 dimensions is enough.

3-D shows a dep. vble. det. by 2 indep. vbles. Algebra can show more. Hence the importance of going beyond geometrical intuition.

e.g.
$$RRAT = \frac{\frac{a}{p}(1-\varepsilon)(1-I^{-n}) + \frac{t}{x}(1-L)(1-I^{-x})}{1 - \frac{F}{p}[(1-s) + Ls]I^{-n}}$$

e. Airplane example.

2. Elasticity.

ϵ by hand: E on printer.

$y = x^2$, x varies, 2 is ϵ

$$\frac{y'/y}{x'/x} = \frac{2x/x^2}{1/x} = \frac{2/x}{1/x} = 2$$

a. Exponents are elasticities

b. ~~Logs are exponents.~~ Log deriv.

Ratios of log. deriv. are ϵ .

$$\frac{d \ln y}{dx} = \frac{y'}{y} \quad \frac{d \ln x}{dx} = \frac{1}{x}$$

$$\epsilon = \frac{2x/x^2}{1/x} = 2$$

~~$$\ln y = 2 \ln x$$~~

~~$$\frac{d \ln y}{dx} = 2 \frac{d \ln x}{dx} = \frac{2}{x}$$~~

~~$$\frac{y'}{y} = \frac{2}{x}$$~~

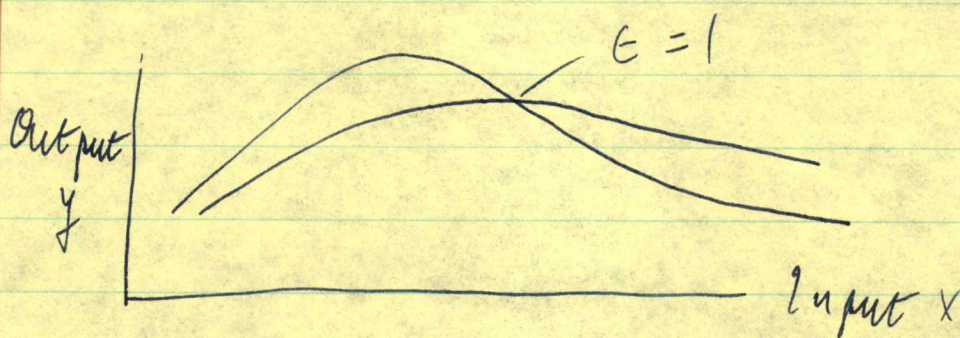
c. Can also invert

$$x = y^{\frac{1}{2}}$$

$$E = \frac{1}{2}$$

A 10% Δ area, a 5% Δ farming.

3. E and $e.s.$ of scale, Δg returns + Δx returns.



$$\frac{y'/y}{x'/x} = \frac{y'/x'}{y/x} = \frac{MP}{AP}$$

Ec. 162, Mon, 10-7

1. Quotients

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vu' - uv'}{v^2} = \frac{u'}{v} - \frac{u}{v} \frac{v'}{v} = \frac{u}{v} \left(\frac{u'}{u} - \frac{v'}{v} \right)$$

$$\frac{u}{v} = R$$

$$u = T$$

$$v = B$$

$$R' = R(t - b)$$

$$\frac{R'}{R} = r = t - b$$

E.g.

1. Timber case.

2. Real i, r.

Money in bank grows at nominal rate, i

$$\bar{F}_c = P(1+i)^n$$

Value of \$ falls as prices rise at inflation rate, ~~\$~~.

Constant \$, K , is Current \$, $C \dot{\bar{F}}_c (1+f)^n$

$$K = \frac{C}{(1+f)^n} \quad C = \frac{K}{(1+f)^n}$$

Real

$$F_k = \frac{F_c}{(1+f)^n}$$

E.g. if $(1+f)^n = 2$

$$F_h = F_c / (1+f)^n$$

$$= P \frac{(1+i)^n}{(1+f)^n} = P \left(\frac{1+i}{1+f} \right)^n \approx P(1+i-f)^n$$

3. Exchange rates

Peso falls 23%

Yen rises 8%

Yen rises vs. peso by 31%.

2. Discrete quotient As

$$R = \frac{T}{B}$$

$$R(1+r) = \frac{T(1+t)}{B(1+b)}$$

$$1+r = \frac{1+t}{1+b}$$

$$r = \frac{1+t-1-b}{1+b} = \frac{t-b}{1+b}$$

E.g. exchange rates

$$\frac{1 + .08}{1 - .23} = 1.403...$$

$$r = \frac{.08 - -.23}{1 + -.23} = \frac{.31}{.77} = .403...$$

E.g. Infln. rates

$$F_k = P \frac{1.99}{1.94} = P \times 1.026$$

$$\frac{.99 - .94}{1.94} = \frac{.05}{1.94} = .026$$

So, when the % Δ s are large, beware the simple rules!

3. The product rule. The chess board.

$$y = x^2$$

$$y' = 2x$$

$$y = x^3$$

$$y' = 3x^2$$

$$y = x^4$$

you have x cubes ^{of x base,} so $\text{vol} = x \cdot x^3$

4. $\in$

1. Quiz: Ratio for optimizing.
Relate to Mafj wealth.
Show impl. meaning.

2. Calc.

$$a = BH \quad (1)$$

$$a' = BH' + HB' \quad (2)$$

Chess board

Burkeley's ghosts, and discrete analysis.

3. 7. approach.

$$(2) \div (1) \text{ says } \frac{a'}{a} = \frac{H'}{H} + \frac{B'}{B} \quad (3)$$

$$a = h + b$$

4. Log derivatives.

$$\frac{d \ln A}{d x} = \frac{A'}{A}$$

$$\text{E.g. } \begin{array}{l} A' = 1 \\ A = 10 \end{array}$$

$$\frac{A'}{A} = .1$$

$$\ln 10 = 2.3026$$

$$\ln 11 = 2.3979$$

$$\Delta \ln \approx .1$$

It is slightly less -- that is Berkeley; murdered ghost.

5. Log deriv. of x

$$\frac{d \ln x}{dx} = \frac{x'}{x} = \frac{1}{x}$$

6. Finite changes. Discrete changes.

$$A(1+a) = B(1+b) \times H(1+h)$$

$$\div A = BH$$

$$1+a = (1+b)(1+h) = 1 + \cancel{bh} + b + h + bh$$

$$a = b + h + bh$$