Finding Rates of Return on Investments After Taxes
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Finding rates of return on investments

Notes by Mason Gaffney, 7 May 87

For use in Economics 100B

1. Simplest case: no redemption, no salvage, no land-value (there is always land, of course, you can't invest in outer space, but some land is so cheap you may treat it as free).

$$RR = a\,\frac{1 - I^{-n}}{P} \qquad (1)$$

Trial-and-error method of solution: "Newton's Method".

RR = a/P when n = infinity. Otherwise, RR < a/P

2. Redeemable bond

D = Redemption value

$$RR = a\,\frac{1 - I^{-n}}{P - DI^{-n}} \qquad (2)$$

RR = a/P when D = P. What happens when D > P? Why? What happens when D < P? Why?

3. Depreciable capital (K), after income taxes at rate "t". No land-value; no resale.

$$RRAT = \frac{a(1-t)(1-I^{-n}) + tP/x(1-I^{-x})}{P} \qquad (3)$$

This assumes "straight-line" depreciation for tax purposes, meaning the investor can deduct the value of P in equal parts over x years.

4. Resale after n years; tax due on "capital gain", at rate "s".

F = Future sale proceeds, at time n. (This value consists of land value plus remaining value, if any, of depreciable capital at time of resale.)

s = effective tax rate on F - L , at time n. For simplicity we assume that all the value of depreciable capital has been written off in the right term of the numerator, i.e.

that $x<n$.[1]

q = land share of P. Thus, Pq = L ; and P(1-q) = K .

$$RRAT = \frac{a(1-t)(1-I^{-n}) + Pt/x(1-q)(1-I^{-x})}{P - [F-s(F-Pq)]I^{-n}} \qquad (4)$$

To simplify calculations:
-- divide top and bottom by P
-- rearrange terms in brackets in denominator

$$RRAT = \frac{a/P(1-t)(1-I^{-n}) + t/x(1-q)(1-I^{-x})}{1 - [F/P(1-s) + qs]I^{-n}} \qquad (5)$$

5. Findings on sensitivity of RRAT

These factors raise RRAT:

-- Low x: weak effect
-- Low n: strong effect
-- Low q: strong effect, when x is short
-- High F/P: strong effect, when n is short and q is low
-- High a/P; moderate effect, if n and x are long
-- Low t: moderate effect, if n and x are long; otherwise very weak
-- Low s: strong effect, if n is short.

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[1]Note for sticklers. The way the tax law works, actually the same tax rate, t, is applied to capital gains as to ordinary income like "a". But only a fraction of the gain has to be reported as taxable income. The effect is the same as a lower tax rate, and for simplicity I set up the algebra with $s<t$.

Land and Capital

DCFF broken into its Elements, with Land and Capital Separated

2. Level cash flows, with G and K treated separately: the Building Model

This "Building Model" applies to more capital than the PIPO model. Perhaps 20% of the world's capital is in growing forms that resemble livestock, or the tree model; 80% is in forms like buildings and machines that yield a flow of cash or service over life, and then have little salvage value except for the land they occupy.

You can apply Faustmann to buildings in two ways:

one) tear them down when b>CF. (CF is cash flow from the old junker.)

two) max G in two steps. First, capitalize future cash flows from each generation at its respective startup date. Second, apply Faustmann to max G.

We might pursue that approach here, but you already know about Faustmann, now, and may want to learn something else. We will therefore use the Redeemable Bond approach, which follows. It accomplishes the same ends, and gives yet another viewpoint, and has other applications as well. It is a little simpler, too. In the Redeemable Bond approach, we take all future cycles after #1 and express their value at one point, the end of Cycle #1.

b. Land and Capital

The analysis of most direct investing (i.e. actually making or owning real things, not just paper) involves the same math as (21,A), which is why I started you out with (21,A). Most direct investments consist of capital occupying and preempting land. When the capital is terminated, the land is released for another use. The land, therefore, is analogous to the Redemption Value, D, of the redeemable bond. The notation for land is G (remember? G is for Ground).

The other difference is that the original investment, P, is separated into two elements, K and G, for capital and land.

$$G + K = a\,\frac{1-I^{-n}}{i} + GI^{-n} \qquad (22,A)$$

(21,A) shows how the parts are tied together. Like all equations it can be solved for any element, so you can reason from what you know to what you want to find out.

Usually you treat known, fixed values as parameters, and solve for what you don't know. This is not just for recreation, but to help you maximize its value by adjusting the amounts of other inputs used. It is a judgment call what value is most fixed exogenously, and what you can manipulate: the answer varies with circumstances. In order to make good judgment calls, which is what managers are for, you have to know the techniques, the options, and the potential gains and losses. That's why you're learning this material.

VALUATION

i. Value of Land

$$G = \left[-K + a \frac{1-I^{-n}}{i} \right] \times \frac{1}{1-I^{-n}} \qquad (22,B)$$

The expression in brackets is the PV of Cycle One (C_1). To its right is the Land Value Multiplier (LVM)(16,G,).[1] What we have here is similar to Faustmann in its PV form (16,G). The difference is that Faustmann applies to PIPO, and (22,B) applies to flows.

Unlike Faustmann, (22,B) does not give us a way to solve for optimal life, n. The terminal n is just assumed. "a" goes along at a level value and suddenly self-destructs. This (the part in brackets) is therefore known as the "One-hoss Shay" model, as noted earlier. I repeat things sometimes, to help you remember, and tie things together.[2]

(22,B) can be adjusted (with added complexity) to show "a" declining, as is often true as capital ages. I have done so in a separate set of notes, "Declining Cash Flow Models." See Appendix II-9.

ii. Rate of Return

If G is exogenous (fixed, given), solve for i. Again, this is not pure recreation: you may want to maximize i.

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[1]This is discussed in section C,1,d,i.

[2]Repeating more, The One-hoss Shay is from a classic poem, The Deacon's Masterpiece, by Oliver Wendell Holmes, about a wonderful vehicle that ran perfectly for 100 years and a day until it abruptly fell all apart, something like Cinderalla's coach at midnight (except that turned into a pumpkin).

$$i = \frac{a}{G + KxLVM} \qquad (22,C)$$

You must be getting tired of having me find all these expressions for you: I'll let you figure out how I got from (22,A) to (22,C). As you do, I hope you'll appreciate how nice and simple (22,C) is compared to what I might have thrown at you.

Values of LVM are in Table 5, in Section C,1,d,i.

(22,C) shows how the presence of G depresses i, the ROR. Compare this to (16,D). Everything said there applies here.

If G=0, (22,C) collapses into (5,C). If it didn't, something would be very wrong. Run through the algebra for an exercise.

To explore scenarios about the effect of rising and falling G on ROR, start again from (21,A), only this time put subscripts on G_o and G_η. Your expression will be a little more complex, but not daunting. For an extended analysis of this case, showing the effects of different income tax provisions, see Appendix II-10, "Land Gains, Fast Write-off, and Incentives to Build." My students in Public Finance master this material without collapsing: so can you.

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Those are the basic tools. The applications and variations are endless. If the tools seem hard to learn, remember poor Mr. Carter. You can learn the cheap way from me, or the expensive way from your creditors. There are lots of folks out there eager to help you learn by taking your money.

May The Force be with you. Keep away from The Dark Side.

Following this paper, go to DCFF model with rise of Site Value.