

Calculating Mean Periods of Investment Using Normalized Models
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Calculating Mean Periods of Investment, Using Normalized Models

Notes by Mason Gaffney, 11/84; rev. 1/92

A. Managing Capital

Managers of individual enterprises (and of national economies) need to regulate the flow and ebb of their capital, which rotates in endless cycles. The enterprise remains solvent by recovering its invested capital quickly enough to "meet the payroll" (and other expenses). The national payroll, too, must be met regularly by recovering enough capital from prior investments to cover most of the national wage bill, plus other income payments. Reinvesting recovered capital is what "sets labor in motion," as Adam Smith put it. Until you recoup it, you can't reinvest it. Elementary? You'd think so, but people keep forgetting. Let's you and I remember.

The current flow of investing and reinvesting also hires the current services of good land, which are required to complement labor. You can't produce or market goods from the moon, or the north pole, or the Mojave Desert. Current investing also buys and activates the current services of fixed capital itself.[1] To activate fixed capital, someone must also buy materials and services to operate it. For example, to plow a field you must rent the field, rent the plow and tractor, buy gas and pay the plowman.

Failing a reserve of working capital for this, there is the possibility of macro-economic gridlock when a dearth of working capital precludes the recovery of frozen capital: a "liquidity crisis." It takes working or liquid capital to use or "thaw out"

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[1]To be sure the owners of fixed capital can and do lower their asking prices when demand falls, to keep their capital busy. This, however, downvalues their capital. The economic effect of that is the same as destroying part of the capital physically. See Addendum B to this Appendix. She who uses her own capital in her own business can even supply it free if she will, but then she receives no capital recovery allowance wherewith to replace it, and it is soon gone.

The owners of land can also lower their asking prices when demand falls. Land doesn't wear out and require replacing, so this adjustment has no social cost. Landowners generally enjoy more holdout power than others, however, so overpricing land can be and often is a serious barrier to making jobs. This set of notes does not address this problem; it is covered in notes used in my macroeconomics courses.

fixed or "frozen" capital. Lacking the former you cannot thaw liquid capital out of frozen capital. This can leave you with capital shortage in the midst of apparent plenty. Think about it: in this circumstance a rise of $\underline{i}$ (the interest rate) would not evoke more liquid capital, but less: a disequilibrating, possibly catastrophic effect. Half the trick in regulating capital is to guard against such a major "glitch."

The question addressed here is how to measure the true "payback period" of investments, so you can know when to expect your capital back. The naive payback period as sometimes used in business and in sales promotions is simply capital cost divided by cash flow, giving the number of years it takes to recover the whole capital invested, without interest. (For irregular cash flows it would be the time required for the sum of cash flows to equal capital cost.)[2] To make it look better, the average period is even shorter, because half your capital comes back after half the whole life.

The problem with that scenario is, it fails to allow that interest is charged on unrecovered capital, and taken off the top before there is any capital recovery at all. If you finance your investment, your lender will make that very clear. If you put in your own capital ("equity"), you pay yourself interest, too: otherwise you're wasting your own capital.

What is the "period of investment" during which capital is frozen or committed to particular forms or processes before it is liquidated or recovered? That is a question because units of capital are commingled during their respective periods of investment such that one cannot easily trace the course of individual units over a period. Many economists, indeed, have declared the whole concept of a period of production, as used by the "Austrian School" particularly, to be meaningless, even conceptually, and certainly therefore not measurable. This

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[2]In mineral industries, a standard concept is the "life index." This means the number of years before proven reserves are exhausted. It is basically the ratio of proven reserves to output. This is based only on physical, not economic, measurements. It is much longer than the time normally required for payback of the capital costs of discovery and development, because these are much less than what reserves later sell for.

position characterizes the Chicago School followers of Frank Knight.[3]

The question addressed is therefore important both to private and public managers. Yet it still wants a generally accepted solution.

B. The simple concept of a mean period

Picture a lake holding a constant amount or "fund" of water (K), with a steady flowthrough (F). Water here is "K" because water represents capital, as in an inventory. How long a period (P) does each unit of liquid spend in the lake? Currents and eddies will move some molecules through quicker than others. If the units were discrete and labeled we could still calculate a mean period. The general problem is that the units commingle and lose their identity: they are "fungible" (as opposed to merely "interchangeable"). This has boggled the minds of some writers - or been used to boggle their readers.[4]

However, generations of physicists, as well as storekeepers and college registrars have long since observed that the mean period of the units in the fund is the fund/flow ratio:

$$P = K/F \qquad (1)$$

where

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[3]Ironically this same School has helped preserve the concept of velocity of money, or deposit turnover. Yet the reciprocal of velocity is the mean period between exchanges of money, and is measurable as the ratio of the flow of money to the fund of money. We will see that investment periods can be measured in the same way.

[4]The bane of economics is that many practitioners represent this or that lobby, or privileged group, whose aim is not to empower you with truth, but divert and weary you by leading you into labyrinths of fruitless speculation. They get too much cooperation in this from some economists who make a mystery of the subject, like ancient high priests, to aggrandize and shelter themselves.

Influential leaders of "The Chicago School" (Frank Knight, George Stigler) have devoted themselves to obscuring the distinction between capital, which has a finite life, and land, which has infinite life. To this end they have made a mystery out of capital replacement and turnover, sowing confusion.

P = Period of containment
K = Capital (the fund in storage)
F = Flowthrough per time unit

Why college registrars? If there are 4,000 students enrolled, and 1,000 graduate each year, the mean period is four years. See how simple? You already knew that.

Sales represents outflow from a fund of capital. With a constant fund it also represents inflow. In what follows, inflow is to come from more than one source, so for simplicity we take outflow to represent inflow as well, and each equals flowthrough.

If the grocer's milk inventory equals sales per day, then K/F = 1 day. The oriental rug merchant may carry an inventory of 10 times his sales/month, giving a period of 10 months; and so on. We do not dwell on, but merely remind you of what is a standard concept in business. Many readers will find such a reminder useful, bearing in mind Samuel Johnson's saying that "men more frequently require to be reminded than informed." New ideas follow as well.[5]

The reciprocal of the period, P, is turnover, T. The period of investment is most often expressed that way in business: "volume equals capital times turnover." This is the easiest form of the equation to remember. Thus, the oriental rug merchant's volume/month is 1/10 of his inventory. The grocer's milk volume/day equals his inventory. The college's yearly volume of graduates is 1/4 of its inventory of students.[6]

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[5]Returning to lakes, the mean period of water in Wisconsin lakes is about five years, ranging from 2 to 9. In California reservoirs used for seasonal storage, the period is a lot shorter: they fill up in April and empty by September. That is busy water storage, but your toilet tank is busier yet - 'nuff said about that, but think about it next time you flush 7 gallons down the drain. Reservoirs that are big relative to streamflow, like Lakes Mead and Powell on the Colorado, have much longer periods and are used for "cyclical" storage from wet cycles to dry ones. Sewage in Riverside's Acorn Street Treatment Plant takes 28 days in process: think how much more land would be needed if it took 112 days instead, and how much less if it took 7 days. Money in a typical student's bank balance? You know more about that than I; I would guess about 10 days. For some more common periods of investment, see Addendum III to this Appendix.

[6]Interestingly, one of the earlier uses of "turnover" in the economics literature is in Karl Marx, Capital, Vol. III, Chap.

Note well that the volume of business one does with a given capital rises directly with turnover. Employing workers and producing goods are closely related to volume. Thus, a given capital can do a lot or a little depending on its turnover. Your business is not limited only by its capital, but by capital times turnover. The national volume, or GNP, is likewise not limited by its capital, but by capital times turnover. Individuals, businesses, and the whole economy can adapt to any amount of capital by adjusting the turnover.

C. The period of intermediate inputs

The question addressed here is how to modify the simple basic concept recounted above, to account for the value added to or taken from capital during its period in stock. Value is added to capital-in-stock routinely by the silent inputs of the services of the capital itself. This is an "autogenic"[7] capital input. The annual value of the land "complementing" - i.e. under and supporting and locating - capital is also added to the capital value. This is a "geogenic"[8] capital input, and may be major or minor, depending on the relative values of land and capital involved.[9]

The cost of these two inputs, land and capital, is often only imputed, i.e. not represented by overt cash payments. What is invisible is too easily overlooked. However, the value of

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VIII. We also find there that Marx used Adam Smith's concept and very wording about the turnover of capital "setting labor in motion." Methodologically, Marx tracked classical political economists in many ways, for better or worse: in this case it was for better.

[7]Autogenic means the same as endogenous, biologically. Endogenous has another usage in economics, however, and carries a lot of baggage not relevant here.

[8]Geogenic is a "neologism," i.e. a word I coined. The fact I had to coin it is a measure of how neglected this important factor has been in economic-financial thinking of the past.

[9]A good forest site ("Site I" means the best land) is flat, wet, warm, and accessible. It costs a good deal more than the cost of planting same; some sites, indeed, are allowed to restock themselves. A marginal forest site ("Site VI") has no value, and costs a lot to restock, and probably should never be logged over in the first place.

assets like timber, range cattle, or fruit trees consists largely of stored-up rent on land and interest on capital. When the money is borrowed, and/or the land rented, these are highly visible and are called "capitalized interest," and "capitalized rent." The value of slow-to-complete nuclear generating plants, which are built mostly with borrowed money, includes more capitalized interest than construction cost.[10]

Value is also taken <u>from</u> many kinds of capital, routinely and regularly, by depreciation. Funds set aside to cover depreciation are called "capital consumption allowances," or CCA. In addition, capital normally (and in equilibrium always) yields a return in excess of depreciation or replacement. This return is used to pay interest to lenders, and profit to equity investors. This return is "importunate":[11] it comes off the top before anything is available for recoupment. We will see how to determine mean periods of investment in spite of each of these complexities.

Capital is Protean,[12] assuming many material forms. To measure all these together economists of course use value. We describe capital as units of value up front, say $1,000 spent on inputs like labor and materials and land rent.[13]

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[10]It has taken about a median 13 years to complete nuclear generating plants in the U.S. Capitalized interest is added to the capital cost. This aggregate cost is the legal "rate base" on which utilities are guaranteed a "fair" rate of return. Thus you, you lucky future ratepayer, will pay for it over the years in your monthly utility bills. You will pay off the capitalized interest, plus also interest on the capitalized interest. That is not double-counting, but it surely mounts up, and "My Gosh! How the money rolls in."

[11]Is this a new word for you? Look it up. When I use a word in these notes I assume you will look up everything you don't already know, so I may use it in exam questions.

[12]From Proteus, ancient Greek sea God, son of Neptune, who could change his form and appearance at will. This made it very hard for mighty Odysseus to pin down little Proteus when they wrestled.

[13]The cash itself is not capital, but represents it by metonymy.

The hired inputs[14] are applied at the beginning of an investment process, "stored up" in some material form, and recovered at the end of the investment period. Recovery may represent sale of the material corpus itself, or else cash flow[15] from the services of a durable "carcass." Under equilibrium conditions (as postulated here, for simplicity) depreciation equals cash flow less the income portion of cash flow. Capital recovery, our present concern, is cash flow less the interest, (explicit or imputed), charged against unrecovered capital. Interest is importunate: it always takes priority over CCA.[16]

We adopt Adam Smith's usage, quaint but terse, and write "capitals" instead of "units of capital." "K" denotes an amount of capital, measured in units called "capitals."

D. The full salvage ("cow-sow") model

1. A Single Life or Cohort

A simple model is the full salvage, or "cow-sow" model. It is approached in nature by breeding, milking and laying stock like hens, sows, cows, bulls, boars and studs. A sow, for example, yields litters for some 10 years and then has a considerable slaughter value.[17] To keep the example simple, let the slaughter value equal the initial capital cost of the gilt

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[14]"Hired inputs" includes everything except working capital. When the manager owns the land and fixed capital, he still is hiring it from himself. If he defers his compensation, as he may, he is saving from income, and plowing it back or "capitalizing" it into the product. For simplicity we finesse all that, which is a collection of wash transactions, and write simply of "hired" inputs.

[15]Recovery usually takes the form of cash flow, but may also be a "service flow," e.g. the enjoyment of occupancy of an owner-occupied dwelling. Herein, for brevity, I let "cash flow" cover both kinds of flow. Remember, cash flow means revenues less associated current expenses.

[16]If you defy convention and give CCA priority it accomplishes nothing: the unpaid interest is simply capitalized, offsetting the drop of the unpaid balance. The lender's gotcha - that's how he stays in business.

[17] (for the hide, pet food, and things you don't want to know)

(young sow), purchased from outside. You and the sow nurse and wean the little piggies, then sell them. Cash flow from the litters is sales price less your current expenses. Prices and costs adjust[18] such that the flow of litters[19] just yields a market interest return on the capital value of the sow, which remains constant from beginning to end. There is neither appreciation nor depreciation over life. The mean investment period is the same as the carcass life from purchase to slaughter.

2. The Normalized Model

A "normalized" operation means a fully staggered operation such that you begin one investment cycle each year, and terminate another (you get to play Arnold Schwarzenegger). This maintains an even flow of activity, and also a steady level of capital on hand. For that reason, it is also called a "Going Concern" model.

If you are breeding hogs, you buy one gilt each year, and slaughter the oldest sow. The period is ten years, so your capital is 10 sows. Your slaughter volume is 1/10 of your capital each year. In addition, your volume includes [i x K] in the form of litters you wean and sell. Total volume or flowthrough, F, is K[1/P + i]. Recoupment of capital, R, is flowthrough less interest:

$$R = F - Ki = K(1/P + i) - Ki = K/P \qquad (2)$$

It remains true in (2), which accounts for interest and the productivity of capital, that R is an inverse function of P. F is a decreasing function of P. Models to follow are not so simple, but the same principles hold throughout.

3. The Fallacy of the Disappearing Inventory

A recurrent fallacy is that normalization eliminates the lapse of time between investment and recovery. "Once you set up a going concern," goes the fallacy, "investment and recovery occur at the same time, so recovery is instantaneous. The lag between investment and recovery is eliminated, regardless of the period of time involved." A transparent fallacy? Perhaps, but I

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[18](in equilibrium, as herein postulated)

[19](they come along every 6-8 months, if you manage the hogs' sex life optimally)

can show you many high-paid foresters, and a few famous economists (like J.B. Clark), who believe or believed it. Can you refute it?

The answer is that the longer the period of investment, the more capital must be kept on hand at any given time. The number of sows equals the life of one sow. Cutting the cycle from 10 years to, say, 8 years, means the going concern keeps 8 sows at once, instead of 10. If each sow is financed at the bank, that cuts your interest bill by 20%.

A partial fallacy sometimes found is to recognize that the number of capital units equals m, but to overlook their housing and land requirements: more sows, more land. Confined animals may not occupy much land directly, but if you are growing their feed, that takes land. At this point you might shift the metaphor from sows to range cattle, which occupy land directly and obviously. To pasture more Hereford cows and calves, you need more pasture.

4. Growing and Flowing Kinds of Capital

Other forms of capital may be divided into those that appreciate over life, like trees, and those that depreciate, like buildings, furnishings, fixtures, machinery, vessels, aircraft, refineries, rails, pipes, fences, paving, wires, poles, pipes, rolling stock, etc.. Appreciating capital is "growing" capital; depreciating capital is "flowing." Why "flowing"? Because it yields returns as flows, which compensate for its loss of value.

In these cases the mean investment period is shorter than the carcass life. In the first case, Growing Capital, capital is added regularly as each year's growth is reinvested automatically into next year's growing stock. In standard financial usage, it is "capitalized." This is an "autogenic" input of capital. The tree automatically reinvests in itself annually. Each such increment of capital is a new investment, with its own period, shorter than the whole biological life of the tree.

In the second case, Flowing Capital, (buildings, machinery, etc.), cash flow in excess of interest cost begins from an early date (although sometimes with construction delays during which interest accrues and is "capitalized," as with the nuclear generating plant). When cash flow does exceed interest on the accumulated capital, that excess represents capital recovery. In equilibrium, capital recovery is synchronized with depreciation.

In practise there are many variations and combinations of these stylized prototypes.[20] The simple models are useful to help students get their bearings, and to help all of us keep our bearings at all times. Very few people can keep highly generalized fully comprehensive models in their heads much of the time. The stylized models actually resemble a large share of all real capital goods, which track them fairly closely.

E. The tree model of appreciating capital

Trees grown for timber make the basis for another simple model. It is called "PIPO", for "point-input point-output." The name implies that all capital is put in at one point, the beginning, and recouped at another point, the end of the investment cycle.

Actually, it is a little more complex. It's true that all capital is recouped at one point, the end, but the planting cost is not the only capital input. Instead, the tree plows its annual growth back into itself each year, like a corporation with earnings but no dividends paid out. (Intermediate service flows mostly take the form of externalities and non-salable amenities, not considered here.)

A simple approach to finding the mean period of investment is afforded by first introducing a complication, that of a "normalized" forest. A normalized forest is one in which tree ages are perfectly staggered, such that each year one acre is planted, and another acre logged, with an acre for every age group in between. Thus there is an even flow of input and output each year, providing constant employment and supply, even though the individual trees provide employment and supply only once every $\underline{m}$ years ($\underline{m}$ stands for age of maturity).

The number of acres equals the harvest age of the trees, since there is one acre for every age from one to $\underline{m}$. The unit of capital is the cost of planting one acre, such that k_o equals one. Annual sales are the maturity value of an initial investment of one capital compounded over $\underline{m}$ years. That value we

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[20]For example, managed timber is pruned, thinned, sprayed and fertilized from time to time in midlife. Each of these intermediate capital inputs has its own period, shorter than the whole life. They are not entered in the present model.

denote as I^m, where I = 1+i, and i is the relevant rate of interest.[21]

Yearly sales of I^m exceed yearly cash outlays of 1. That excess is "imputed" (credited) to some input, and the only other input here is the capital in the growing stock. From this we can infer the total value of the growing stock, thus:

$$Ki = I^m - 1$$

$$K = \frac{I^m - 1}{i} \qquad (3)$$

(The right side of (3) is our old friend, the Future Value Factor. It is the area under a curve of the compound interest factor, from 1 to [m-1].)

Knowing the total number of capitals in the forest inventory we can now calculate the mean period of investment, which is the inventory value divided by sales outflow, thus:

$$P = K/F = \frac{I^m-1}{iI^m} = \frac{1-I^{-m}}{i} \qquad (4)$$

(4) is the same as the present value of an annuity of m years. We know how this function behaves. Its value is always less than m, which is the whole carcass life of the tree: slightly less than m when m is small, and much less when m is large.

In the full salvage model (cows, sows, etc.) the investment period equals the carcass life. In the present tree model, the investment period of the initial planting cost, k_o, considered separately, also equals the whole life. k_o is assumed equal to one; K = m, F-Ki = 1, so P=K/1=m. For some purposes we want to know the payback period of the original k_o; it is m. However, the overall payback period is less than m (the carcass life), because increments of supplemental investment are added to the

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[21]For simplicity we enter ground value (G) at zero -- otherwise maturity value would include accumulated interest on the land value, although not the land value itself, and would be $[I^m + G(I^m-1)]$.

carcass during its midlife. These increments consist of capitalized imputed interest.

The meaning of this result depends on the context. The investor concerned about capital payback will not receive his initial investment back any sooner than m years by virtue of this shortened mean period. It is his later investments, the capitalized imputed interest, which shorten the mean period. They do so only by adding to the total capital inputs, not by speeding the return of the initial ones. He has to keep pouring in more capital in order to recover the initial outlay. So the ratio of K to initial cash outlay (k_o=1) ends up higher here, not lower, than in the cow-sow model. The same process that lowers the mean period of investment actually adds to his capital needs.

Again, from the viewpoint of "setting labor in motion" the investment period calculated here does not set labor in motion any oftener than once in m years. On the contrary, the tree soaks up and sequesters more and more capital as it grows, capital which might otherwise have gone to set labor in motion elsewhere. The foregone interest which is automatically reinvested each year goes to employ capital, not labor. It is an investment outlet that employs no workers.[22]

To get a quantitative grip on this relationship it is useful to distinguish outside capital (planting cost) from autogenic capital, then figure the ratio of all capital to outside capital. From (3), $K=(I^m-1)/i$. Outside capital sums up to m, i.e. 1/year for m years. The ratio is K/m:

$$K/m = \frac{I^m - 1}{im} \qquad (5)$$

We have worked with (5) before: the right side is the same as the ratio of compound interest to simple interest (the "com/simp ratio"). It is an increasing function of m, starting from one when m=1. Beyond that, with i=.08, it rises to 2.29 when m=20, 6.48 when m=40, 20.9 when m=60, and 73.6 when m=80. That means if it takes 80 years to grow a tree, the total capital is about 74 times the sum of all outside capital used for planting costs. Autogenic capital as a share of all capital is

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[22]This is an Achilles' heel of traditional Keynesian analysis. In Keynes, all investment makes jobs. This notion has been adopted whole-hog in standard macro-economics.

(K-m)/K = 1-m/K = 1-1/73.6 = 1-.014 = .986. The autogenic capital is nearly as pure as Ivory Soap.

Thus, with an 80-year life cycle it takes a towering buildup of autogenic capital to recover each unit of outside capital.[23]

To some economists, all investing is the same. To refrain from cutting a tree in 1992 is simply to invest the capital in the tree to produce another year's growth. Likewise, to hold coal reserves unmined during 1992 is to invest the opportunity cost (the 1992 sales price) to secure the higher future price. These economists have a point, but are thinking narrowly and leaving turnover out of the equation. In terms of making jobs, this kind of "passive investing" makes none; in terms of volume of production, it generates little.

Today we are hearing we should build public works to spend our way out of the depression. "It seems to me I've heard that song before, it's from an old familiar score." It was widely believed during the great depression, when public works, including planting trees, were considered the way to go. The trouble with such investing is that income from autogenic capital goes entirely to capital, and employs capital. Should capital be oversupplied, and labor in short supply, then tree growth might add to national income by putting idle capital to work. The normal social problem is the reverse, with labor oversupplied relative to capital. In depressions this chronic problem becomes acute. (Are we in a depression now? If you think not, you probably have a good job lined up, unlike most.)

In the normalized forest with an 80-year rotation, where payroll (planting cost) is one/year, K/payroll=K=FVF=5,887 (with i=.08). That is not $5,887 per job, but per dollar of payroll. If one job pays $30,000/year, the complement of capital per job is $30,000 x 5,887 = $177 millions.[24] That is a lot. By

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[23]When we add geogenic capital we build up even more capital. In the present example, however, land that takes 80 years to mature a tree isn't worth much. It takes shorter cycles to support high rents: that is where geogenic capital becomes a weighty factor.

[24]This refers only to jobs in planting timber, since we are looking just at the timber management stage of production, excluding logging. Later stages are more labor-using. For a full, vertically integrated picture we would follow the wood through topping, felling, bucking, skidding, perhaps rafting, loading,

contrast, consider the working capital in a busy restaurant where it turns almost daily, say 300 times a year in round numbers. Here, it takes only $100 of capital to complement one $30,000 job. Thus, the 80-year forest requires 1,770,000 times as much capital per job as the restaurant.

The example is perhaps overdrawn, to make a point. You may whittle it down as much as you please, however, and not escape its message: to make jobs with limited capital, put the capital to work turning fast, not into timber or public works of slow payout.

The period of wage-paying investment may be figured separately. It is K/W (where K is the FVF, and W (wage-bill) is one), rather than K/F, which was originally calculated. W = 1, so K/W = K. We know K from (3):

$$K/W = K = (I^m - 1)/i \qquad (3, \text{rpt.})$$

(3) is greater than m, showing that the mean period of wage-paying investment is longer than m, the whole or biological life of the tree. That is because so much of the investment is not wage-paying. (3) is greater than (5), which we have already seen tower so high. When m=80, K=5,887, or 74 x m. That is the number of years it takes the average dollar of capital to get around to paying wages.

The reciprocal of (3) is, as before, the frequency with which capital employs labor. It is much less than 1/m. Investments of this kind are very capital-intensive and unfavorable to employment. The "job-creating efficiency" (to coin a concept) of capital in forms like trees is very low. The volume of business per unit of capital is very low.

Returning to the mathematics, note that Eqn. 3 shows the area under a curve showing the growth of money at compound interest. It is, that is, the value of a tree just planted for $1 ($I^0$), plus that of a one year old sapling (I^1), plus the value of a two year old sapling (I^2), and so on up to (I^{m-1}), the value

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hauling, ponding, decking, marketing, milling, drying, treating, packing, shipping, retailing, carpentering, and another 70 years or so in service in a building. Our present aim is to stick with a simple model we can work with easily.

of a tree that is (m-1) years old.[25] This constitutes a series of columns which the equation adds up or "integrates" to get the area under the curve.

We can express the same area as the sum of a set of horizontal strips or rows. This is "inverting the order of integration." Each such row is an input of capital times its own proper period. If we add up all the rows, and divide by the number of rows, we get the mean period. That is basically what we do when we divide Eqn. 2 by the ordinate at $\underline{m}$, (I^m). Each row may have different depth or "thickness" than the others, but this procedure weights the periods by the thickness of the rows. Alternatively, it weights the amounts of capital by their respective periods. Either way we perceive it, it gives us the mean period, viewed as a weighted average.

6. The machine model of depreciating capital

The last model needed to complete our trio of models is the machine that yields a positive cash flow from its first year and continues to do so until it expires in year $\underline{m}$. Then it is scrapped and replaced. The mathematics is simpler, without losing adaptability, if we let the cash flow be one, and continue at that level until year $\underline{m}$.[26]

As with the trees, we now assume a normalized operation with one machine of every age from 1 to $\underline{m}$. While few industrial firms

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[25]The series begins with I^0, rather than I^1, and ends with I^{m-1}, rather than I^m, as a matter of mathematical convenience, to simplify the factor. The reason for this is a fine mathematical point that need not delay us here.

[26]The assumption of level cash flow, followed by total demise, would seem to make this a "one-hoss shay" model, where cash flow remains level without any decline until the very last year, when the "shay" collapses totally. That is the way lenders finance much capital on the installment plan, and therefore relevant enough. But we have a simple device to make the model more general without complicating the mathematics. The device is simply this: add any decline rate to the interest rate. Thus a cash flow that declines at d%/year has a present value found by adding d% to the interest rate %. Simply use the higher interest rate and proceed as you would with a level flow. While mathematically less than perfect, this method yields a result precise enough so that any error may usually be ignored in practise.

would operate just this way, a banker rotating his portfolio would come a lot closer, and the whole economy comes closer yet. So bear with us, the model represents reality in the aggregate. We will also see that the mean period calculated for the aggregate is also the correct mean period for the individual machines, just as in the tree case. We use the normalized model because the mathematics is simpler. More important, this is a way of integrating sequential analysis with simultaneous or "coexistential" analysis of factor proportions.[27]

Total cash flow is m, since each of m machines yields annual cash flow of one. Each year we retire one machine which has no scrap value. We replace it with a new one whose cost is the present value of one over m years. Cash flow goes to pay interest on all the machines plus the outlay for one new machine each year. As before, we find K by dividing the interest flow by the interest rate. The mathematics follows.

The yearly capital input is k. This is the present value of one over m years, which is the warranted cost of a new machine yielding that cash flow.

$$k = \frac{1-I^{-m}}{i} \qquad (6)$$

Interest charges are the excess of cash flow (m) over replacement (k). They are also equal to the grand sum of all the capital in the machines, K, times the interest rate.

$$Ki = m - k \qquad (7)$$

From (7) we can figure the value of K, without having to go through the hammer and tongs process of summing the present values of m machines, each of a different age from 1 to m.

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[27]A major difference between micro and macro, as ordinarily expounded today, is this: micro deals with relations of coexistence; macro deals with relations of sequence. It is an unfortunate, artificial, and divisive compartmentalization of thought, that blocks communication between the subdisciplines and weakens both. Normalized models break down the compartment walls and integrate micro and macro in one stroke.

$$K = \frac{m-k}{i} \qquad (8)$$

Having K, we only need to find R, the annual recovery of capital, in order finally to find the period. No problem: we know R must equal cash flow less interest payments.

$$R = \text{cash flow} - Ki = m - (m - k) = k \qquad (9)$$

As expected, R is exactly enough to replace the machine being scrapped. (It would be quite disturbing if it were otherwise.) (9) may also be deduced as follows. There is one machine of every age, so the sum of their yearly depreciations (cash flow - Ki) must equal the full depreciation of one machine (k).

Now we are tooled up to find the mean Period (P), using the principle that P=K/R:

$$P = K/R = \frac{m-k}{ik} = m/ik - 1/i = \frac{m}{1-I^{-m}} - \frac{1}{i} \qquad (9)$$

To make (9) easier to remember, we could also write the right side as (m x IPF -1)/i, where IPF is the familiar Installment Plan Factor valued over m years.

A table is now useful to show us how (9) behaves for different values of m and i. Table 1 shows the mean period before recovery of capital in the case modeled above. It is derived from the case of a normalized bank of machines, as explained; but it applies as well to a single machine.

Eqn. 9 would give a misleading result for small values of m because it is based on the assumption that cash is received only in a lump at the end of each year. For example, when m=1, (9) gives a period of one, when the true period is nearer to 1/2. To correct for this we change it to the continuous form in Eqn. 10, and derive numbers in the table from that.

The continuous form of compounding is explained in Appendix II-1. Continuous compounding is a way of assuming that cash comes from the machine in a steady flow, rather than once a year. Its effect on the numerical results is for practical purposes the same as shifting the time unit from one year to one month or one

week, which is what the practical world normally does. We assume that the reader either knows this, or accepts it, or will study Appendix II-1.

$$P = \frac{m}{1-e^{-pm}} - \frac{1}{p} \qquad (10)$$

where e is the base of natural logarithms and p=LN(1+i).[28]

Table 1: Mean period of investment of capital in asset yielding level cash flow over m years[29]

i=	.001	.01	.04	.08	.12
m					
2	1.0003	1.003	1.01	1.03	1.04
4	2.001	2.01	2.05	2.10	2.15
8	4.005	4.05	4.21	4.41	4.60
16	8.02	8.21	8.83	9.60	10.3
32	16.09	16.8	19.3	22.0	24.1
64	32.3	35.4	44.2	51.5	55.2

The first column, where i = .001, is to show what happens as i approaches zero: the mean period approaches m/2. This is a check, which gives us a feel for what Equations 9 and 10 are doing. If there were no interest, all of cash flow would be capital recovery and the mean period would be half the whole life, m/2. Those who make this claim are implicitly assuming an interest rate equal to zero - not very realistic.

The top three rows likewise display mean periods which are not much above m/2. For low values of m, interest is not much of a factor, even at high rates.

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[28]In standard notation, the Greek rho is used for this value, called the "force of interest." I use the Roman p because it looks more like the Greek rho than anything else in my font. I might use r, because that is how rho transliterates, but many writers use r where I use i, and that could be confusing. If it helps your memory, think of it as the "phorce" of interest.

[29]My normal convention is to use three significant figures. The accuracy of most economic data does not warrant more. There are exceptions to the convention. In the upper left of this table I extended the figures far enough to show you the number is not followed by an endless string of zeroes.

However, where there is substantial interest to pay on (or otherwise charge against) unrecovered capital, it becomes a weighty matter that the interest charge comes off the top, so there is less left for capital recovery. The lower right quadrant of Table 1 shows mean periods which rise appreciably above m/2. For example a building whose whole life is 64 years yields the investor back his average dollar of capital not in 32 years but in 55 (at 12% interest). Many a bank and S&L has learned to its sorrow that money tied up in building loans amortized over something like 32 years is not recovered in an average of 16 years, but more like the 24 years shown in Table 1 (again at 12%). It gets stickier yet when the borrower gets in trouble and misses a few payments. This unpaid interest is "capitalized" (added to the unpaid balance); payout is stretched out longer yet.

Rather than the mean period some might prefer the median for some purposes. This is the "half-life," the period in which half the capital has been recovered. This is calculated by solving the present value equation for m and setting the present value equal to k/2, where k is the initial present value. This gives the number of years of life remaining when the value is k/2. To get the age at which this occurs we subtract this from the original m. The mathematics involves logarithms; it is in the main text, pp. 25-29.

Table 2 compares three different concepts of the period of investment, as described above:

Col. 2 shows m/2;

Col. 3 shows the median or "half-life" when half the capital has been recovered;

Col. 5 shows the mean period calculated from Eqn. 10, and already tabulated in Table 1;

Cols. 4 & 6 compare Columns 3 & 5 with Col. 2, and thus also with each other.

Table 2: Comparison of alternative measures of mean investment period for normalized machines with whole life of m. i = .08.

m	m/2	Half-life	(3) / (2) - 1	Mean Period	(5) / (2) - 1
(1)	(2)	(3)	(4)	(5)	(6)
2	1	1.04	.04	1.03	.03
4	2	2.15	.08	2.10	.05
8	4	4.61	.15	4.41	.10
16	8	10.32	.29	9.60	.20
32	16	24.06	.50	22.0	.37
64	32	55.1	.72	51.5	.61
30	15	22.2	.48	20.3	.35
40	20	31.6	.58	28.9	.45
50	25	41.3	.65	38.1	.52

The main findings of Table 2 are these:

a. For low values of m, the half-life and the mean period of investment are only slightly greater than m/2.

b. For higher values of m, say 8 and over, the half-life rises significantly above m/2. Column (4) shows the fractional excess of (3) over (2). It rises to .72 (72%) when m=64.

c. The half-life slightly overshoots the true mean period, but is tolerably close for most purposes. We use it because it is easier to visualize and to calculate.

As indicated earlier, the results of the normalized model apply as well to individual investments. We chose the normalized model to achieve our result without invoking the integral calculus, thus simplifying the mathematics and widening the audience. Even more important, the normalized model is a way of integrating sequential analysis with simultaneous or "coexistential" analysis.

Table 1 makes clear (and Table 2 confirms) that investors seeking rapid payback will do better investing in machines than investing either in livestock or trees of any given whole life. The true payback period of the cow or sow investment in our model

is simply the whole life. The true payback period of the outside capital used for planting trees is more than the whole life, using Eqn. (3, rpt.) in preference to Eqn. 4, for reasons there discussed. On the other hand, the true payback period of the machine is less than the whole life, especially if the whole life is short anyway, and the interest rate low.

Investing in depreciating capital is no guarantee against a liquidity crunch, however. Beware of what happens as the whole life of the machine rises, or interest rates rise. Beware of what happens if the investor assumes that investments in long-lived buildings or public works have depreciation paths comparable to those of machines or vehicles. Beware of what can happen to banks that put too much of their money into 40-year loans, even though fully amortized. In Tables 1 and 2 when the whole life doubles, the true payback period more than doubles. (More generally, the elasticity of true payback with respect to whole life is greater than one.)

That lays a foundation for understanding the greater perils that beset investors in long maturities when interest rates rise. They cannot accelerate the recovery of their capital to take advantage of the higher rates. Instead they are victimized: the present value of their investments falls. That drop is much more than a "paper loss" or an accounting fiction. On the contrary, accounting fictions often conceal the loss. The drop in present value is nothing less than a loss of capital, a real loss to them and to the whole economy. It represents the real truth that the investor will never recover his original capital, but only the reduced amount represented by the new present value.

Tables 1 and 2 do not show the last effect. Table 1 makes a rise in interest rates slow down recovery only a little. But Table 1 does not show the devaluation of investments that occurs when $\underline{i}$ rises in the mid-life of a long-term investment. This event actually cuts the amount to be recovered, having the same effect as destroying capital.[30]

All these effects together raise the specter of macro-economic gridlock. It takes working capital to utilize fixed capital and thus to recover the capital fixed in it. Let there

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[30]No wonder the supply of investable funds does not respond positively to a rise of $\underline{i}$. Many economists have wrongly taken this as evidence that savings do not respond to $\underline{i}$ - an implausible proposition. It is more plausible that recoupment of CCA from fixed capital is frozen by higher $\underline{i}$.

be some kind of mass error, a herd movement toward hedging against inflation, let us say, followed by a rise of real interest rates, and one can see the possibilities for a liquidity crisis. One can also read some history, starting with the South Sea Bubble, and appreciate that these are more than hypothetical possibilities. If you don't believe history, check the current economic scene, it will repay your careful study.

7. The neglected role of land pricing.

We have left out of this appendix the role of land rents and values, and will merely suggest here what their role may be. First, when land gets overpriced during a boom, it artificially stimulates particular kinds of investing, which we can label generically as "land-saving."[31] Land-saving investments are well above average in their periods of investment, and thus below average in turnover and recovery of capital. In addition, land rent is a prior claim on cash flow, even prior to interest. A rise in rents, or failure to lower them when demand drops, will interfere with capital recovery.

An important ancillary factor is that a rise of i tends to deflate inflationary land prices which have served increasingly as collateral for bank loans in the last few years. Already we see that the sour loans of banks are mostly domestic, and mostly in the fields of energy, agriculture and real estate, all land-based. Thus the role of land values is to help convert a liquidity crisis into a banking collapse. This is a long story, treated in another set of notes used in my macro-economics course.

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ADDENDUM A: Finding the half-life (Repeated from Book I.)

Solving the present value formula for n, we get:

$$n = \frac{-LN[1-Pi/a]}{LNI}$$

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[31]Five kinds of land-saving investments are distinguished and defined in a set of notes used in my course in Urban Economics. The notes are entitled "Land as a Distinctive Factor of Production."

Here, a=1. Let $P=k_o/2$.

$$n = \frac{-LN[1-k_o/2}{LNI} = \frac{LN2 - LN[1-I^{-m}]}{LNI}$$

Subtract n from m (the whole life) to get the half-life.