Replacement Policy

PV of 1st replmt, (K_n)

$$PVK_n = K_n I^{-n}$$

$$\frac{PVK_{40}}{PVK_{100}} = \frac{I^{-40}}{I^{-100}} = I^{(100-40)} = I^{60}$$

Cf. ~~PVK~~ PV of replmt. in perp.

$\frac{A}{B}$, where $\alpha = 40$, $\beta = 100$, $A_0 = B_0$

$$\frac{A}{B} = \frac{(I^{+40}-1)^{-1}}{(I^{100}-1)^{-1}} = \frac{I^{100}-1}{I^{40}-1} > \frac{I^{100}}{I^{40}}$$

Thus, using PV of 1st replmt. only will understate advantage of deferring replmt.

T-2: PV of 1st replmt

i	$n =$ 20	40	60	—	100
.03					
.04					
.05					
.10					

T-3: PV of replmt in perp.

i	n = 20	40	60	–	100
.03					
.04					
.05					
.10					

E.g. at $i = .03$, $PV\,A_{40}$ is $\frac{30.66}{5.20}$ or 5.9 x higher than $PV\,B_{100}$

But ~~PV~~ A is $\frac{.44}{.05}$ ~~or~~ 8.8 x higher than B.

Cf. at $i = .10$

$$a = K \frac{i}{1 - I^{-n}}$$

Cost of replacements in perpetuity, $\bar{K}$

$$\bar{K} = \frac{K_0}{1 - I^{-n}}$$

T-1

Values of $\frac{i}{1 - I^{-n}}$

i	n = 20	40	60	100
.03	2.24	1.44	1.20	1.05
.04	1.84	1.26	1.11	1.02
.05	1.60	1.17	1.06	1.01
.09				
.10	1.17	1.02	1.00	1.00
.11				

Annual deposit in sk. fund: $F \frac{i}{I^n - 1}$

Answer in %

T-2

i	n = 20	40	60	100
.03	3.61 ~~3.56~~	1.29 ~~1.25~~	.66	.16
.04	3.23	1.01	.40	.08
.10	1.75	0.21	.03	.0007

T-3 PV of 1st replmt, in %

i	n = 20	40	60	100
.03	55.37	30.66	16.97	5.20
.04	45.64	20.83	9.51	1.98
.10	14.86	2.21	0.33	0.01

Richard.

1. Why most people overlook ~~repl. cost timing~~ longevity past 40 years.
   a) IBF
   b) ~~Hi i.r.~~ Look only at 1st renewal
   c) Hi i.r. (overlook inflm)
   d) Assume depn. reinvd. at same i.r.
2. How to emphasize diffs. in replmt cost (R)

A. Focus solely on R, bcs K_0 dilutes diffs.
~~i.~~ 1st R only. T-3

B. ~~ii.~~ R in perp.

$$\frac{\frac{i}{1-I^{-n}}}{1-I^{-n}} - 1 = \frac{i - 1 + I^{-n}}{1 - I^{-n}} = \frac{i - I^{n} - I^{n} + 1}{I^{n} - 1}$$

~~$i - 1 + \frac{1}{I^n} =$~~ I-1 less 1

From (i), can say the PV of R_{40} is $\frac{30.66}{5.20} = 5.9\times$ higher than PV of R_{100} $(i = .03)$

At $i = .1$, PV of R_{40} is $\frac{2.21}{.01} = 221\times$ higher.

From (ii) $\frac{R_{40}}{R_{100}} = \frac{.44}{.05} = 8.8$ $(i = .03)$

or $\frac{R_{40}}{R_{100}} = \frac{.0226}{.0001} = 226$ $(i = .1)$

Case is stronger ~~at lower i.r.~~ using (ii).

c) Use lower i.r.

a higher i.r. actually Δs the relative advantage of deferring renewal.

But also makes PV small, relative to current cost.

d)

a

$$\frac{i}{1-I^{-n}} - i = \frac{i - i + iI^{-n}}{1-I^{-n}} = \frac{i}{I^n - 1}$$

$a -$

$$a - Ki = K(IRR) - Ki = K(IRR - i)$$

$$a\frac{1-I^{-n}}{i}\frac{i}{I^n-1} = a\frac{1-I^{-n}}{I^n-1} = a\frac{1}{I^n}\frac{1-I^{-n}}{1-I^{-n}}$$

$$= aI^{-n}$$

Present Value of Replacement Cost

A. Conventional rationale for underplaying value of longevity > 40 years or so

1. Paying off a capital outlay (K_o) with interest in level annual instalments ($\underline{a}$) gives little advantage to stretching out payments, > 40 yrs.

$$a = K_o \frac{i}{1 - I^{-n}} \qquad (1)$$

where:

i = interest rate

$I = 1 + i$

n = no. of years of ec. life of structure or term of loan

"IPF" $= \dfrac{i}{1 - I^{-n}}$

Table I: Values of IPF (%)

i	$n =$ 20	40	60	100
.03	6.72	4.33	3.61	3.16
.04	7.36	5.05	4.42	4.08
.10	11.75	10.23	10.03	10.00

IPF → i as n → ∞, but n = 100 is close to ∞ for this purpose.

~~The elements of the case against longevity (i.e. against higher K_0 today to gain longer service life) are these:~~

2. Reinvestment of depreciation

$$IPF - i \equiv k = \text{return of capital} = (PF)_{\overline{\ell}|}^{-1}$$

where ℓ is life remaining. Sum of k_ℓ from

$$\sum_{\ell=0}^{n} I$$

$\ell = 1$ to $\ell = n$ equals K_0.

When K_0 is debt-financed, all k go to retire debt. You end up debt-free, where you began. This automatically "reinvests" k in an asset "yielding" the same i you are paying.

If K_0 is financed internally, and k are invested in low-yielding accounts, you could end up with too little to replace the facility in year n.

You might piece out a case on grounds that part of the initial K_0 is from equity, and part of that will be lost by non reinvestment. But this would be "reaching".

Most people will assume debt-financing, and reinvestment via retirement of debt.

3. Overlooking ~~[illegible]~~ replacements after the first

If people think about future replacement at all, they look no further than the first one. You are right to look further.

Let's compare the present values (PV) of the first replacement and an infinite chain of replacements. (infra). It strengthens your case.

4. Choosing a high i.r. & ignoring inflation

Tab. 1 shows the case for longevity is stronger at $i = .03$ than $i = .10$.

~~Using a "real" i.r. ad~~

With debt financing the debtor gains from

inflation. The $\sum_{i=1}^{n}(k_i)$ ~~$\sum_{i=1}^{n}(k_i)$~~ do not yield enough to replace the trunk, but