



Redeemable Bonds and Orange Groves

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Redeemable Bonds and Orange Groves

Notes by M. Gaffney, 5\95

This model combines the annuity case and the PIPO case in one equation. This gives you, when you master it, a more comprehensive analytical power and grasp than either case taken alone.

What do bonds and groves have in common? The model applies literally to redeemable bonds (defined below). However, the math framework of it applies as well to speculative real estate, like a Riverside orange grove. This explains the heading. It is to let you know this framework applies to much more than the work of a narrow clique of Nicholas Leasons and other backroom money-changers and arbitrageurs shuffling papers. It applies to any investment that couples a cash flow with a resale value.

Literal "resale" isn't even necessary: a re-use value will do. For example, you might plan to rent out a vacation home for several years, and then retire there. The re-use value is what you reckon you would have to pay for a newly-bought home at that time. You might plan to use a lot by your store for parking over several years, then later build on it. You can think of other examples.

Most bonds are "redeemable." This simply means they are repayable debts: the bond pays cash interest (called the "coupon") over a finite life, and then you get back a lump sum at the end, usually in units of \$1,000. As a rule, that is also the original par value paid in, although not always. In either case the bond has a current market price, above or below \$1,000, that fluctuates daily, depending mainly on current interest rates available on alternative investments. Once the bond is issued and traded, the current price is cut loose from the original cost, and depends entirely on discounting future expectations.

Notation:

F = Future Redemption value after n years.

P = Price, or Present Value of bond.

a = annual cash flow from bond, the "coupon."¹

¹Bond coupons are usually paid more often than once a year, but here we use annual periods, for simplicity.

I. Valuation

$$P = a \cdot \frac{1-I^{-n}}{i} + FI^{-n} \quad (1)$$

Eqn. (1) combines the annuity case and the PIPO case in one equation. As I said, this gives you more power and grasp than either case taken alone. The Force will be with you. But you must concentrate, young Skywalker, if you want to become a Jedi Master!

II. Rate of Return on Bond, or "Yield-to-Maturity" (YTM)

This rate is also called the "yield to maturity" (YTM). It combines two yields: the "current yield," a/P ; and the bond appreciation from now to year n . You find it by solving Eqn. (1) for i .

$$i = a \cdot \frac{1-I^{-n}}{P-FI^{-n}} \quad (2)$$

Review the section on IRR, and Eqn. (5,C), in the paper "Basic Math of Finance," pp. 19 ff. Equation (2) here, like Eqn. (5,C) there, is solved using Newton's method of systematic trial and error. This Eqn. (2) is marginally more complex than the old Eqn. (5,C), but if your calculator has to labor another nano-second, so what?

YTM's on redeemable bonds are published daily in the Wall St. Journal. Lots of people are interested. Many papers just publish what are called the "current yields," meaning just $i = a/P$. Anyone who invests based purely on current yield is cruising for a losing. He may often luck out, because for many bonds the appreciation element is minor; but for others it is major, and then you need Eqn. (2).

If $P = F$, the equation collapses into the current yield, $i = a/P$. Run through the algebra as an exercise - be sure you understand how this works. It's not hard, and it's fun to see all the hard stuff cancel out and disappear. This is the same as the old Eqn. (2) from the paper on "Basic Math of Finance," p.11. Getting back exactly what you put in is the same as having a perpetual income stream, the premise of old Eqn. (2).

Ask yourself these questions - and, of course, answer them!

What happens when $F > P$? Why?

What happens when $F < P$? Why?

If $a = 0$, Eqn. (2) looks like big trouble, but you can step around the trouble by going back to Eqn. (1). The first term disappears, and Eqn. (1) collapses into the old familiar PIPO discount formula from the paper "Basic Math of Finance," Eqn. (1,B), p.2. You then solve for i as in Eqn. (1,C), p.4.

III. *Carter's Cut, or How to Lose Money on Junk Bonds*

It is tempting to simplify by decomposing Eqn. (2) into two parts, finding ROR separately, and adding. Don't: it's wrong, and in some cases very wrong.

A dear old friend of mine, a Mr. Carter,² tried to simplify Eqn. (2) by cutting it in two. His procedure looks reasonable, but isn't. It lost him a lot of money -- and he never even knew it! He reasoned that the YTM is the sum of a/P + appreciation, so why not just figure each one separately and add them together? Here is what Carter figured.

$$i = a/P + [F/P]^{1/n} - 1 \quad (3)$$

Carter's Cut just adds together two simple Equations from the paper on "Basic Math of Finance," p.4. He had a good intuition. It works out pretty well if F/P is close to one. Otherwise, however, it leads you by the hand down the primrose path to poverty. If Eqn. (2) were a robot like Hollywood's Johnny #5, it would be pleading "No Disassemble! No Disassemble #2!".

It led Carter to invest heavily in "deep-discount" bonds -- bonds with high F/P ratios. He would compare his figures from Eqn. (3) with those published in the *Wall Street Journal*, which uses Eqn. (2), and notice that his figures were higher than theirs, for the discount bonds. (They were lower for bonds selling at a premium.) He was a man of independent mind and action, who insisted on figuring things out for himself: a true contrarian he was, and usually right. He despised complexity because so much is humbug. He liked to reduce problems to simple elements. He

²No relation to UCR's Professor Susan Carter.

had once worked for Thomas A. Edison, and said that's how "the old man" worked.

Those are admirable qualities in principle, which I cherish in students who show them. Ralph Waldo Emerson wrote a wonderful essay on *Self-Reliance* we should all read, full of familiar quotations. The only problem was, in this case Carter was wrong.

He was dead sure, though, and thought the *WSJ* and the whole financial community were missing his point, and were misstating the true YTM. Try as I might, I could never make a dent in his conviction. He was sure he was beating the market. He died (R.I.P.) thinking he did it; he'd have left his widow more if he had thought it out again.

I'll say this for my good old friend, he had self-confidence: never occurred to him maybe they were right, and he was wrong. I couldn't talk him out of it. But in fact, Eqn. (3) is wrong. It overstates the true yield on discount bonds. The reason, in words, is that it credits the bond with a current yield of a/P_0 for every one of n years. That is fine if P remains constant over the life of the bond, but with discount bonds, it doesn't. P rises every year, to reach its terminal level of F . So:

the current yield on a discounted bond (high F/P) is less than a/P in every year after the first; and,

the current yield on a premium bond (low F/P) is more than a/P in every year after the first.

If the last three sentences make your head swim, let's try a *reductio ad absurdum*. A quick and dirty test of any proposition is how it behaves with extreme values. Try Carter's Cut when $F = 0$, and therefore, of course, $F/P = 0$. This *reductio* is not, actually, absurd in the sense of being unusual, but only in being extreme. Lots of investments have no salvage value. Most capital, in fact, just wears out and becomes junk. This is the case we first met in the paper, "Basic Math of Finance," p.20, Eqn. (5,C).

Evaluate Eqn. (3) with $F = 0$, and $a/P = .10$. You get $i = -.90$, regardless of the value of n . This has to be wrong. Common sense tells you that when $n=10$, you will have recovered your entire P without interest, so $i = 0$. This is not so hot, but a sight better than $i = -.90$. It also tells you that as n rises above 10, and gets very large, your true return $\rightarrow .10$. That's

simple enough: if an asset lasts forever, and yields 10% forever, it's true return is 10%. Eqn. (3), though, says it is still $-.90$.

More generally, we can make a table with one value for n , and many values of F/P . In column 1 put F/P ; in 2, put YTM according to Carter's Eqn. (3); in column 3, put YTM according to the *WSJ*, using Eqn. (2). You'll see the difference.

Working out Eqn. (2) is no picnic, unless you have a calculator that does it for you (many business calculators do, and some of you have them). Anyway, I've worked out an example. See Table 1. I've left a blank row at the bottom for you to fill in if you like, choosing whatever value of F/P interests you.

Table 1: Yield to Maturity, Carter vs. the *Wall Street Journal*
Based on $a/P=.08$, and $n=10$.

F/P	Carter	Wall Street Journal	% Error in Carter
0.0	-.92	-.002	459.0
0.5	.013	.037	-64.9
0.7	.045	.057	-21.1
0.9	.070	.073	-4.1
1.0	.080	.080	0.0
1.1	.090	.087	3.4
1.3	.107	.099	8.1
1.5	.121	.110	10.0
2.0	.152	.133	14.3
3.0	.196	.170	15.3

Notice that Carter overstates the yield on deep discount bonds (high F/P); and enormously understates the yield on premium bonds (low F/P), especially perpetual bonds ($F = 0$). (Don't let the positive value in the upper right hand corner throw you. The lack of a minus sign before 459.0 in the upper right hand corner occurs because the *WSJ* figure is negative - it indicates that Carter overstates the negative number, not that he overstates the yield.)

Tables 2 and 3 give values of YTM for many more values of n .

Table 2: Yield to Maturity, when $a/P = .08$

n--->	3	6	10	15	20	30	40
F/P							
0	-.476	-.176	-.002	.023	.050	.069	.076
.5	-.106	.004	.037	.058	.067	.075	.078
.7	.001	.034	.057	.068	.073	.077	.079
.9	.049	.066	.073	.076	.078	.079	.080
1.0	.08	.08	.08	.08	.08	.08	.08
1.1	.110	.093	.087	.084	.082	.081	.080 ⁺
1.3	.165	.117	.099	.090	.086	.083	.081
1.5	.215	.139	.110	.096	.090	.084	.082
2.0	.325	.185	.133	.109	.098	.088	.084
3.0	.501	.255	.170	.130	.111	.094	.086

Table 3: FALSE YTM Using Carter's Cut

n--->	3	6	10	15	20	30	40
F/P							
0.0	-.92	-.92	-.92	-.92	-.92	-.92	-.92
.5	-.126	-.029	.013	.035	.046	.057	.063
.7	-.032	.022	.045	.057	.062	.068	.071
.9	.045	.063	.070	.073	.075	.076	.077
1.0	.080	.080	.080	.080	.080	.080	.080
1.1	.112	.096	.090	.086	.085	.083	.082
1.3	.171	.125	.107	.098	.093	.089	.087
1.5	.225	.150	.121	.107	.100	.094	.090
2.0	.340	.202	.152	.127	.115	.103	.097
3.0	.522	.281	.196	.156	.136	.117	.108

Table 3 FIGURES ARE IN "SHADOW" TYPE. This is to help you stay aware of the difference in the two Tables, when you compare them.

Now you've seen the Tables, go back to the equations and be sure you understand what is happening. If you really want to understand what finance is all about -- really understand, deep inside -- you want to roll it over in your mind like that, until something comes together inside your own head, in your own way, that makes sense to you.³

Getting back to Carter, and repeating, Eqn. (3) overstates the YTM on discounted bonds; and understates the YTM on bonds selling "at a premium," i.e. with $P > F$. So it leads you into bad deals with discount bonds (aka junk bonds). Mike Milken made a bundle, (and got a jail term), selling to people with Carter's approach to math.

It leads you away from good deals with premium bonds. Worse, it leads you away from productive investments in capital that yield only "ordinary income."⁴ This means income from providing people with real services. It makes anything that wears out, as all human works do, look bad. It makes you look for so-called "capital gains" instead. Mere cash flow without any prospect of resale value seems to have no value at all. Speculative gains are what look good.⁵

It might seem that no real managerial issues hang on what kinds of bonds people buy, but they do. The kinds of real investing borrowers do depends a lot on what kinds of bonds they can sell.

³Most people never have time for that, or never think they have. So they follow trends and authorities, never beating the crowd. If you want to spend your life as a wage-slave, be like that. But if you can make that time, and break through the mental blocks that hold most people down, who knows, maybe you will figure out how to beat the market. Better yet, you may contribute something to improve human understanding, the most gratifying achievement of all.

⁴Congress, the IRS, and tax economists reveal an ominous bias by using this term. It betrays their disdain and contempt for what "ordinary" useful folk do, and get taxed for. The other kinds of income, much of it unearned or predatory, are called "capital gains," and get favorable p.r. and tax breaks.

⁵Ironically, this is the opposite of Carter's moral creed - and he was a very moral person. His error in math led him away from his own ideals.

If a lot of bond buyers make Carter's mistake it leads borrowers to finance more things that yield deferred cash, as opposed to fast cash. That slows down the turnover of real capital.

Turning from bonds to direct investments, a lot of direct investments resemble redeemable bonds in this way: they show a mediocre current yield, followed by a "pot of gold at the end of the rainbow," a hoped-for resale to some rich investor after a few years. The classic and obvious example is speculative real estate in growth areas. (See the paper on "Quantum Leaps and Ecotones.") There is a strong tendency to overvalue that future pot of gold, especially when the herd psychology is running like a buffalo stampede. A lot of bad investing results. Carter's artless error is just one example of a large genus.

Investments that are bad for the investor are also bad for society, especially if the capital gets frozen or destroyed. As Winston Churchill said, "Capitalism is not a social evil because investors make profits, but because they fail to."⁶ So this is more than a selfish individual concern. People who misallocate land and capital are hurting everyone.

Tax policy is biased in favor of discount bonds, too. The "capital gain" on bonds, and many real investments, is partly exempt from income tax. Presidential Candidates Bush (R), Clinton (D) and Tsongas (D) (March, 1982) all wanted to exempt even more of it, back to the way it was before the reform of 1986. Capital gains are the source of most campaign contributions, so some politicians sacrifice many other issues to show favor to capital gains.

In Bush's case, he deemed it so important that in 1990 he broke his magic pledge, "read my lips, no new taxes" in order to secure another break for capital gains. (This was a deal with Democrats in the House: Bush accepted a tax hike to swap for a break for capital gains.) Can you believe that anything as theatrical as who wins the next election hangs on something else as dull and abstract as Eqn. 1? Believe it! You'll read and hear volumes on this subject between now and November. The public rhetoric on both sides, naturally, will be pitched to the Yahoo level. Behind the scenes, though, calculating accountants will be

⁶He might have qualified the point, but he was making a speech, and that would have spoiled the epigram. He did have a point, a very good one. He could have gone on to discriminate between profits from production and others from predation. Some of his other speeches reveal that's what he meant.

studying these equations, rubbing their hands, and studying the pathway to the bank.

ADDENDUM, 2 May, 1995. I left the above paragraph just as written in Spring, 1992. As we now know, Bush did lose that election of 1992, and this was a big reason why. I'd say, "I told you so," but a certain radio-TV talk show host has already taken that line.

IV. *Land and Capital*

The Redeemable Bond model is a way of bringing the far future into evaluating any investment, instead of limiting ourselves to Generation #1. We have seen elsewhere another method of doing that, named for Faustmann. See the papers "Finding Optimal Life of Capital Affixed to Land," "Maximizing Value in Perpetuity," and "When to Cut a Tree." See also Appendix 2, hereto.

How does the Redeemable Bond model relate to Faustmann? In Faustmann we find the value of future generations endogenously, making them clones of Generation #1. The Redeemable Bond method takes the value of future Generations as given exogenously, expressed in Future Resale Value (F). It takes all future cycles after #1 and expresses their value at one point, the end of Generation #1. This is the model to use at ecotones, where land is rising in expectation of a different future use.

Unlike Faustmann, it cops out on how that future value is determined. It just takes it from that great outside force, "the market," which many modern economists treat as the new infallible source of spiritual, moral, and material guidance. However, that doesn't mean we must cop out. If we want, we can value the future use ourselves. The Redeemable Bond method just says that the job is done outside the analysis at hand.

Another difference is that Faustmann applies to PIPO investments, not DCF investments. However, that is only superficial: Faustmann can be adapted to DCF investments. See the paper "Finding Optimal Life of Capital Affixed to Land."

The analysis of most direct investing (i.e. actually making or owning real things, not just paper) involves the same math as Eqn. (1), which is why I started you out with it. Most direct investments consist of capital occupying and preempting space. When the capital is terminated, the space is released for another

use. The space, therefore, is analogous to the Redemption Value, F, of the redeemable bond. The notation for space is T (Terra).

The other difference is that the original investment, P, is separated into two elements, K and T, for capital and land.

$$T + K = a \cdot \frac{1-I^{-n}}{i} + TI^{-n} \quad (4)$$

Why distinguish T and K? Notice that T appears twice in Eqn. (4), and K only once. This is because K is depreciable, it wears out and has no resale value. T does not depreciate, but often appreciates.⁷ That is why T appears twice in Eqn. (4). You put it in at the start, like K, but you also get it back at the end, unlike K. You recover K in the cash flow, bit by bit, year by year.

Market forces work this out because buyers and sellers know the land outlasts the capital, and they price it accordingly,⁸ so the part of the product imputed to land is just enough to cover rent, with nothing left over to recover principal. K, on the other hand, has to be priced low enough so its cash flow covers both recovery of principal, and interest on the unrecovered balance each year. We worked that out in the paper "Basic Math of Finance," pp. 14-16.

⁷Land has infinite life, except for exhaustible resources, which are not treated here. They are a special case, not quite like other land, and not quite like stores of man-made capital. See the paper on Reserve, an excerpt from *Extractive Resources and Taxation*, 1967.

⁸This is on the premise that the market works reasonably well, and forecasts correctly. In practice land is sometimes underpriced: then the buyer gets a free ride. More often it is overpriced, and the buyer gets in trouble. These dynamics are important, but are not dealt with here in the brief ten weeks available. Those interested in pursuing this matter see me for references.

A. Valuation

1. Value of Land (T)

Solve Eqn. (4) for T:

$$T = \left[-K + a \frac{1-I^n}{i} \right] \cdot \frac{1}{1-I^n} \quad (5)^9$$

The expression in brackets is the DCF of Generation One (G_1), net of capital cost, K. The ratio to its right is the Land Value Multiplier (LVM). Here are some LVM values, to give you a feel for it, when $i=.05$.

Table 4: Land Value Multipliers when $i=.05$.

n:	1	2	4	8	16	32	64
LVM:	21.0	10.8	5.6	3.1	1.8	1.3	1.05

For example, when $n=16$, generations following the first one add 80% to the value of generation one.

What we have here is similar to Faustmann in its DCF form, as developed in the paper, "When to Cut a Tree." There is also a skeletal demonstration of Faustmann in Appendix 2 to the present paper. A big difference is that Faustmann applies to PIPO, and Eqn. (5) applies to flows.

A second difference is that Faustmann, gives us a way to solve for optimal life, n . Here, in Eqn. (5), the terminal value of n is just assumed. " a " goes along at a level value and suddenly self-destructs. This (the part in brackets) is therefore often called the "One-hoss Shay" model.¹⁰ It's also called the "light-bulb" model, for the same reason.

⁹Yes, I know, this can be simplified a bit. That comes later.

¹⁰The One-hoss Shay is from a classic poem, *The Deacon's Masterpiece*, by Oliver Wendell Holmes, about a "Wonderful One-hoss Shay" that lasted 100 years and a day, when it abruptly fell all apart. With such a mind for irony and economics, no wonder the poet's son and namesake became Chief Justice of the U.S. Supreme Court. It was something like Cinderella's coach at midnight (except the coach turned back into a pumpkin).

Eqn. (5) can be adjusted (with added complexity) to show "a" declining, as is often true as capital ages. I have done so in a separate set of notes. See the paper, "Declining Cash Flow Models."

2. Value of Capital (K)

If you know T, and want to find K, solve Eqn. (5) for K.

$$K = a \cdot \frac{1-I^{-n}}{i} - T(1-I^{-n}) = (a/i - T) \cdot (1 - I^{-n}) \quad (6)$$

Eqn. (6) gives you the value of K net of land. It is the method to use once a building has been built. First you value the land from Eqn. (5), using values of a and K derived from the highest and best future, prospective use. Then plug that value of T into Eqn. (6) to value the standing, existing structure. This is the standard way that valuers separate land value from building value. It is called the "building-residual" method, because T is taken as given, and K (the building) gets what is left over.

Eqn. (6) has several uses:

One, valuing K tells you when the time has come to tear down the old building. It's pretty simple: when $K = 0$, the time has come. $K = 0$ when $a/i = T$.¹¹ a is the cash flow from the current use (land-cum-building). T is the DCF derived from foreseeing the best future use. When $T > a/i$ it is past time to rebuild.

To do that you must first tear down and haul away. Remember, though, the idea is to rebuild. If you just tear down and haul away you've only destroyed value. Obvious, yes; but look around the city and you'll see it is not obvious to all owners. They lose a lot by taking the down-time of land so lightly; and so does our whole society. Once they put their assets into an airplane, ship, or power plant, they sweat buckets to minimize its downtime. The traditional American attitude toward land is more cavalier. It's an attitude that should change, but is mainly changing for the worse.

Two, valuing K this way is theoretically meaningful because a structure, once built, has little or no salvage value. I.e.,

¹¹ $K = 0$ also when $n = 0$, but that is redundant because it is $a/i = T$ that makes $n = 0$.

it has no "opportunity cost" separate from its present form and site. The site, on the other hand, never loses its opportunity cost. You see them tearing down old buildings all the time, to salvage the land beneath them. "They ain't making any more of it," so such salvage is basically the only source of land for the future, as it has been for the past.

Three, land and building may have separate owners. In such cases, land income takes legal priority over other income.

Four, buildings are depreciable for income tax purposes; land is not. You have to separate them to state your income correctly. Also, improvement assessments levied to pay for public works are levied against land, not buildings.

Five, you need fire insurance for the building value, not the land value. (You need title insurance for the land.)

B. Rate of Return (ROR) as Affected by Capital Depreciation and Land Appreciation¹²

Eqn. (4) shows how the parts are tied together. Like all equations it can be solved for any element, so you can reason from what you know to what you want to find out. Let's solve it for the ROR, or i .

To adapt Eqn. (2) to the case of direct investments we break P into two parts, T (land) and K (building, or other depreciable capital). Then we break T itself into land price at two different times. T at time zero is Z ; and T at the future time n is F . I do this to avoid extensive use of subscripts, which clutter things up. Z is T_0 , and F is T_n . $\Delta T = F - Z$.

$$ROR = a \cdot \frac{(1-I^{-n})}{(Z+K) - FI^{-n}} \quad (7)$$

¹²For an extended analysis of this case, showing the effects of different income tax provisions, see the paper on Reserve, "Land Gains, Fast Write-off, and Incentives to Build." My students in Public Finance master this material without swooning; so can you.

1. How Z depresses ROR

You put land into the beginning of the investment cycle, and recover it unchanged at the end. It is like a catalyst in a chemical reaction, participative but untouched. This does not mean its presence is free. Occupying a time-slot in space costs you. The cost is the money tied up in land over n years. The effect is to lower the ROR. Table 4 gives some examples.

Table 4: ROR on Cash Flow Followed by Sale of Land, with Stable Land Price. (Parameters: $n=10$; $a = .15$; $K = 1$; $F = Z$)

Z	Z+K	ROR (%)
0.0	1.0	8.14
.25	1.25	5.90
.5	1.5	4.60
1.0	2.0	3.17
4.0	5.0	1.10

2. Rent or Buy?

The need to carry land at a high value over a normal investment cycle lowers the ROR: lowers it a lot. It lowers ROR by adding to the base of total assets on which a is the return. This reduction is only partly compensated for by recovering land after n years -- 10 years in Table 4.

You might avoid this by renting the land. In that case, however, you add annual rent as an expense, deducted before arriving at cash flow, a . You might do better or worse, depending on particulars. It depends on whether land is overpriced relative to its rent, something that varies with circumstances. Your financial success depends in part on your judgment call, whether to rent or buy. Your judgment call in turn depends in part on your technical ability to reckon ROR correctly.

But wait, there may be another plus for buying. Maybe the land will rise during your time-slot, or "period of ownership," as the tax folks say. That would improve your ROR. Let's see how much.

3. Rising Land Value

We need some math to overcome the complexities, but let's begin with a simple case that almost explains itself.

a) Simple example: ΔT offsets depreciation

Here, the appreciation of land value, ΔT , is just enough to offset the depreciation of the building or other capital (K) that yields a cash flow for n years. Your salvage value thus equals your initial investment, so your ROR is simply the cash flow divided by the initial point-input of $Z+K$. Look at Eqn. (7) and see how much cancels out when $F = Z+K$. You are left with:

$$\text{ROR} = a/(Z+K) \quad (8)$$

It is as though K didn't wear out at all, but lasted forever, like land. Eqn. (7) becomes like the case of a redeemable bond you can buy for its redemption value of \$1,000. This makes $\text{YTM} = \text{current yield} = a/P$. (See the 3rd paragraph after Eqn. (2), p.2.) In the bond market such a condition is not unusual, and would in fact seem "normal" and expected. In the case of a direct investment it is a coincidence caused when $\Delta T = K$.

It's amazing, though, how many owners in a rising market come to expect this, and forget that capital (K) wears out. They fail to provide for replacing it, assuming that ΔT will take care of that, and more besides. They consume ΔT by letting it substitute for replacing their K as it depreciates. I have listened to some who have come to regard this as not only natural, but as some kind of natural right, or entitlement. They feel outraged and cheated when it doesn't work out that way. The human mind plays funny tricks - you need to know about these quirks as well as the math.

Coincidence or not, that simple finding in Eqn. (8) is a handy landmark to keep in view as you sail in these strange new seas. You need a stable pole star for orientation. It is not in human nature to trust mathematical formulae until we have worked with them long enough to develop understanding and confidence. Nor is it easy to keep in mind highly generalized formulae, nor to adapt them quickly to changing particulars, even though they are designed for such adaptation. But here is something you can understand and accept readily. Use it as a rule of thumb for situations that approximate it. Use it as a benchmark to check the credibility of your answers for situations that are obviously better or worse.

b) Example where $F = 2Z$

Eqn. (8) makes it clear that you get a higher ROR when Z is low -- i.e. you buy in cheap. If you are really lucky you could start from a base with Z at zero. Then, $ROR = a/K$. Life is more than fair to those who get in on the ground floor. That is why it is less than fair to newcomers. One may do even better when ΔT is more than K, obviously, and we will be seeing how much better. However, mark this well: the enjoyment of ΔT by one investor makes it tougher on the next one.

The gratuitous gain enjoyed by Groundfloor O'Grady is an added burden to his successor Newcomer Nellie who had the bad business judgment to be born 30 years later. O'Grady's juicy resale value, F, is Nellie's initial land cost, Z. Nellie gets on where O'Grady got off. O'Grady got a free ride, but it's Nellie who pays. In a rising market, each generation holds the next one up for ransom. Even though the elevator may keep going up, Nellie's ROR is to be based on a higher ante -- a number, remember, that you divide by in figuring ROR. The higher Z will offset much of, all, or more than the gain, (varying with particulars).

So the same ΔT that gladdens the hearts and raises the animal spirits of one generation serves to depress those of the next. Here we have the makings of a business cycle. This point is not pursued further here, but is worth a lot of thought by people venturing into the future world of finance.

There are two opposing forces at work here on the ROR. The burden of carrying Z lowers the ROR; the promise of ΔT acts to raise ROR.

Table 5 illustrates the relative weight of forces acting on the ROR. It is an exercise in Sensitivity Analysis. It's a bit subtle, so prepare to slow down and study it a while, or you'll miss its points.

In the Table, T doubles over an investment cycle of 10 years. The higher the base (Z), the higher is the absolute value of ΔT . But there is a loss in the ROR, caused by having a higher base, as illustrated in Table (4). In Table 5 the loss is partly offset by the gain from ΔT , resulting in a slight drop in ROR as Z and ΔT are both higher. (ΔT is tied to Z, the initial value of T, because of the assumption that T doubles.)

Row 6 (ROR_1) shows what the ROR would be if $\Delta T = 0$, and therefore $F = Z$.

Read the "Morals" under the Table to help interpret the findings.

Table 5: ROR When T Doubles in 10 Years, Various Values of Z & F
Parameters: $n = 10$; $K = 1$; $a = .15$; $F = 2Z$ (except in Row 6)

		1	2	3	4	5	6
1	$Z =$	0	.25	.50	1.0	2.0	5.0
2	$Z + K =$	1.0	1.25	1.50	2.0	3.0	6.0
3	$F =$	0	.5	1.0	2.0	4.0	10.0
4	$\frac{a}{(Z+K)}$	.15	.12	.10	.075	.05	.025
5	$\frac{Z}{Z+K}$	0.0	.20	.33	.50	.67	.83
6 ($F=Z$)	ROR_1 (%)	8.14	5.90	4.60	3.17	1.95	0.90
7	$\frac{2Z}{(Z+K)}$	0	.40	.67	1.00	1.33	1.67
8 ($F=2Z$)	ROR_2 (%)	8.14	7.82	7.66	7.50	7.37	7.27
9	$\frac{ROR_2 - ROR_1}{ROR_1}$	1.00	1.32	1.67	2.37	3.78	8.08

Morals:

i. Row 6 is there to show the effect of Z on ROR when $F = Z$, so $\Delta T = 0$. Row 5 is the ratio of future value to present value used to find ROR_1 in Row 6. Moving from left to right, higher values of Z are posited - but remember, the value of T remains constant here, equal to Z, throughout each time-slot being analyzed. The higher they get, the lower goes ROR_1 , even though F remains equal to Z. It's the higher level of Z that depresses ROR_1 .

ii. Row 8 shows how a positive ΔT acts to compensate, and offset the tendency of T to depress ROR. In Row 8, ROR_2 holds nearly steady, moving from left to right. (If we showed the effect of tripling T, ROR would rise slightly instead of falling slightly as it does in Row 8.)

iii. Row 9 is the ratio of Row 8 to Row 6. It shows how much the ROR is boosted up over the actual ROR on K by the factor of ΔT . The boost is, as you might expect, greater where Row 1 is greater. Row 1 is Z, and since $K=1$ it is also the

ratio to K. In a rising land market, investors will favor land-using investments, relative to what they should be doing.

iv. The effect of ΔT is to make ROR insensitive to the amount of land (Z) being used with a given amount of capital (K). This insensitivity is shown in Row 8. It reflects the fact that land appreciation offsets the cost of carrying land over time, making land very cheap to hold during the course of the investment cycle.

c) Example where T rises by various multiples from given base

To round out our understanding of this matter, I now add a third dimension to the table, letting T rise by multiples other than 2. (2 is the fixed multiplier in Table 5). Accordingly, Table 6 shows the effect on ROR of having T rise by multiples from .5 to 12, over 15 years.

Table 6: ROR when T rises by various multiples over 15 years
Parameters: $n = 15$; $K = 2$; $a = .20$; $Z = 1$; $Z+K = 3$

F/Z	ROR on pure Z	ROR on Z+K	ROR_1/ROR_2
(1)	(2)	(3)	(4)=(2)/(3)
.5	-4.52	1.77	-2.55
1.0	0.00	3.11	0.00
2.0	4.73	5.14	0.92
2.25	5.56	5.56	1.00
3.0	7.60	6.67	1.14
6.0	12.69	9.85	1.29
12.0	18.02	13.69	1.32

Morals from Table 6:

i. Column (2) shows the ROR on bare land (Z), assuming there is no K at all, and no a . At the higher values of F (above 2.25) the investor does better not to use the land at all, even though this entails idling the land and foregoing the positive net cash flow it could yield. This strange effect results because at discount rates higher than 5.56% the present value of the positive cash flow is negative. Putting it another way, the

ROR on K alone, at $Z = 0$, is only 5.56%, so if pure land speculation yields more, the individual chooses to buy more land instead of building on what she already owns.

ii. The combined ROR in column (3) is dominated by F, the resale value of land. With Z assumed equal to one, F is the multiple of F/Z . The investor will pay close attention to the likely future of the value of land under the capital she has built.

The condition that bare land speculation yields more than productive building is a disequilibrium condition, i.e. one that tends to work itself out in the perfect markets premised these days by ordinary theorists. As they put it, arbitrage anticipates all future windfalls. That just means that the Groundfloor O'Gradys of this world keep raising the entrance fees for the Newcomer Nellies. However, real life is likely to be a series of journeys between equilibrium conditions which elude us like the rainbow's end, the will o' the wisp, and the Holy Grail. So don't think this "disequilibrium condition" never happens. It is all around us all the time, and has been since Columbus landed and introduced the Old World concept of fee simple land tenure. Still, let's ask how equilibrium is approached.

Faced with high expected returns from land speculation, the investor seeks to buy more land rather than build on what he has. This hikes the ratio of land to capital in his operation, and this, in turn, raises the marginal return expected from adding capital.

When many people buy more land their bidding raises the price. This reduces $\Delta T/T$ and also reduces a/T . The combination of those two effects with the first one finally may produce a new equilibrium where the positive cash flow from using land is not neglected. It's amazing, though, how long it can take: 1492-1995 is 503 years, and we're still disequilibrated. So don't hold your breath waiting for equilibrium.

Those are the basic tools. The applications and variations are endless. If the tools seem hard to learn, remember poor Mr. Carter. You can learn the cheap way from me, or the expensive way from your creditors. There are lots of folks out there eager to help you learn by taking your money.

May The Force be with you. Keep away from The Dark Side.

Appendix 1. Alternative Expression for ROR

If T is exogenous (fixed, given), solve for i . Again, this is not pure recreation: you may want to maximize i .

$$i = \frac{a}{T + K \times LVM} \quad (1)$$

You must be getting tired of having me find all these expressions for you: I'll let you figure out how I got from (4) to (6). As you do, I hope you'll appreciate how nice and simple (6) is compared to the hash we might have made.

Here are some LVM values, repeated from the text, to give you a feel for it, when $i=.05$.

<u>Table 4: Land Value Multipliers when $i=.05$.</u>							
n:	1	2	4	8	16	32	64
LVM:	21.0	10.8	5.6	3.1	1.8	1.3	1.05

For example, when $n=16$, generations following the first one add 80% to the value of generation one.

Appendix 2: PIPO Model of Land and Capital (Faustmann)

Note. Owing to time pressure, these notes are only in unfinished form. Students wanting to pursue this, and needing guidance, see me. Equations are numbered independently from Appendix 1, and the main text. Notation follows:

K = Capital outlay at time zero, (e.g. for planting trees)
 H = Harvest value of product (e.g. timber) resulting from K
 Z = Price of land at time Zero
 F = Price of land at time n , if different from Z

We begin with constant land value, at level Z. In case 3 we consider rises of land value ($F > Z$).

1. Compounding

$$K + Z = [H + Z]I^n \quad (1)$$

You put in Z at the beginning of the cycle, and take it out at the end. It is a kind of "dead weight", but it changes the necessary relationships among H, K, I, and n.

2. Discounting

$$K + Z = [H + Z]I^{-n} \quad (2)$$

Solving for K, if Z is given:

$$K = HI^{-n} - Z[1 - I^{-n}] \quad (3)$$

Eqn. (3) says that the discounted product of a current outlay, K, that produces a future return, H, is less than simple discounting would have it. Besides discounting in the familiar way, we have to subtract something for using a time-slot of space.

It helps to understand that time-slot valuation by noting that $Z = a/i$, where a is the annual rent of the land. Now the second term is seen to be the same as $(a \cdot DCFF)$: that is, it is the present value of the rents of the space used in the PIPO production process (e.g. growing timber).

3. Rate of Return

$$i = \left[\frac{H + F}{K + Z} \right]^{1/n} - 1 \quad (4)$$

(4) shows how the presence of the deadweight cost of Z acts to reduce i , the ROR, so long as $F = Z$. The higher the value of Z, the closer the ratio in (4) approaches one.

If you want to get a quick fix on how rising F affects the ROR, Eqn. (4) is the place to start. If F is expected to rise above Z that offsets part of the deadweight effect and pushes ROR up toward what it would be if Z were zero. If F/Z gets up as high as H/K , then the rise of F completely offsets the deadweight effect, and ROR is the same as if Z were zero.

Now watch carefully. If F/Z gets greater than H/K , that would seem to raise the ROR. But it will never happen. Why? Because then investors would just buy more Z, rather than put any K on land they already have, and sit back and maximize their ROR without using the land at all. They would become pure land speculators, since the ROR on just holding land is higher than on using land.

That in turn will push up Z until the ROR on holding land falls. This gets tricky, however, because:

a) A rise of Z may generate expectations of equal or greater rises of F. Brokers do their best to make this happen: they are often telling you to "buy it today because it went up yesterday." They have been likened to the mythical Gooney Bird, which flies backwards so it can see where it's been.

b) A higher value for Z will lower the ROR on using land, as in Eqn. (4), which tends to turn investors back towards pure land speculation again.

We cannot begin to unravel all the possible scenarios here, but it would pay one to look at some history of speculative bubbles, which gives us an idea of the range of documented human behavior in the past. See me for references on this topic.

4. Period of time.

Beginning with (4), solve for I^n and take the Ln of both sides:

$$n \ln I = \ln[H+F] - \ln[K+Z]$$

$$n = \frac{\ln[H+F] - \ln[K+Z]}{\ln I} \quad (5)$$

The effect of Z (so long as $F = Z$) is to bring the two logarithms on the right side closer together, thus lowering the n required to achieve a given ROR from given values of H/K . "Lowering" the n required might sound as though it's lowering a hurdle, but *au contraire*, it's raising the required speed of turnover. You have to turn the investment over faster to compensate for the lower % gain on each turnover.

5. Value of Z

$$Z = \frac{-KI^n + H}{I^n - 1} \quad (6)$$

Eqn. (6) is named the Faustmann Formula, after a German forester who discovered it in 1849. I refer to it simply as "Faustmann." Faustmann is premised on there being many generations of investment and recovery after the first one. In fact, that is how Faustmann first derived Eqn. (6), which we have reached by a different route.

6. The PV form of Faustmann.

It is generally useful to put Faustmann in the PV form. Z is already a PV, and this change won't affect Z . But we multiply both top and bottom of Faustmann by the discount factor, I^{-n} . This makes Faustmann more directly comparable with other valuation equations, which commonly reduce values explicitly to time zero.

$$Z = \frac{-K + HI^{-n}}{1 - I^{-n}} \quad (7)$$

The top of (7) is the PV of the first generation of land use. The bottom is a divisor that converts the PV of generation one into a land value derived from all future generations. Rather than divide, it is more convenient to take the reciprocal and multiply, calling it the "Land Value Multiplier", or LVM. For values, see Appendix 1.

7. Value and Rent

Multiplying Z by i we get annual rent. We need a notation for rent: $b = \text{annual rent}$.¹³

Note that in the top of (7) all values are present values. Multiplying by i , the rest of it becomes the IPF, so we have:

$$b = [-K + HI^{-n}] \cdot IPF \quad (8)$$

What (8) does is to take the irregular stream of investments and revenues that we actually get, and convert them to a level annual amount, b , which is the annual value of land, or rent.

8. Optimal life, and replacement timing.

A by-product of Faustmann is determining the optimal value of n , which is the length of the cycle of investment and recovery. Optimal n is the value of n that maximizes Z .

The condition for maximizing Z is that the log derivative of the numerator equal that of the denominator. That means, recall, that their % rates of change are equal, so that the ratio doesn't change. Let's see where that leads us. Let $H' = dH/dn$. Then:

$$\begin{aligned} \frac{H' - KiI^n}{H - KI^n} &= \frac{iI^n}{I^n - 1} = \frac{i}{1 - I^{-n}} \\ H'[1 - I^{-n}] + Ki &= Hi \\ \frac{H'}{H - K} &= \frac{i}{1 - I^{-n}} = IPF \end{aligned} \quad (9)$$

(9) says that you harvest a tree when the % growth rate of the harvest value, net of restocking costs, is a little higher than the interest rate, i .

That "little higher" can be shown to be equal to the annual value of Z . See the paper, "When to Cut a Tree," pp. 3-4, Eqns. (6,7). So what (9) is saying is that a growing tree has to grow fast enough to pay interest on its own value, plus the ground rent, or else you harvest it.

¹³From German *Bodenrente*. German nouns begin with upper case, but lower case is used here to hew to my convention that flows are denoted by lower case letters, and upper case letters are reserved for point values.

Therefore, optimal harvest time comes when:

$$H' = H_i + b \quad (10)$$

(10) says that a growing tree or other capital has to grow in value fast enough to pay interest on its own value, and pay rent on the land it preempts from alternative uses. The left side is the marginal product of time, and the right side is the marginal cost of time.

"b", of course, = Z_i : they are generally interchangeable. So you can find optimal n by maximizing annual ground rent, b , just as you can by maximizing ground value, Z .