

Ec. 162, Metals.

E. 701

2-14-66

I. Loose ends from 2-7

A. l.g.

1. It is a fallacy to prefer a low i.g., for same reason is a fallacy to prefer max. a.p. — it overlooks cost of fixed input, just because it is implicit.

How about fact that it is "fixed?" That is also a difference.

(Importance of spelling out subtle distinctions, here exemplified. ~~It~~ Is essence of theory — get to essence of things)
Alternative is confusion.
(E.g. of "farm" programs, turn out to be landowner's programs)

Essential idea is that one needs a cushion to absorb uncertainty. Can one get this w/o underusing land? Yes, sell some, use the money as a reserve, for trouble.

What we do is instinctively to identify landowner & risk-bearer. But they may be different parties. They are certainly different functions.

~~Fixity of costs is not of the essence here.~~ If I can supply labor in variable amounts w/o explicit cost, then that is my credit cushion; & if I pay a fixed cash rent for land, it is no cushion.

The labor is not a cushion in it's capacity as labor, either. It

works w/o pay, thus lives on other reserves, or capital. Absorbing risk is the fn. of capital

Skip { Irreversibility of commitment also involved, ex post. But ex ante, nothing is so committed, & that is no reason for underinsuring the fixed factor.

So, to protect vs. risk, essential point is to have a cushion. That means either an equity, or a line of credit, or outside assets, or a family ready to absorb long reverses and still work hard. all of this means having capital on tap.

An Ace in the Hole.

The risk protection under low i.g. is illusory. One is protected vs. visible, explicit losses, by making continual implicit losses. Land is a cushion only bcs. available w/o visible cost.

Neglect o.c.

If lo i.g. landowner was renting land & supplying family labor, he'd be max'g explicit risk by lo i.g., which we cd. show by drawing curves reverse, w. labor fixed & land vble.

(Another fn. of theory - as well as drawing clear distinctions, develop mental agility to transpose, see things differently, etc.)

If a firm wants to min. credit risk, shd. define owned inputs as fixed and hired ones as vble. then min. i.g.

This is only desirable if credit, ^{& other} mkt's. are imperfect - otherwise, can trade off firm's surplus asset for its short input, which is best. E.g. put some of surplus family labor to hire, to get money to buy more land. Or sell

surplus land to finance bldgs. & equipment. Thus keep balance, so you can min. credit risk w/o unbalancing factor proper.

2. Price theory partly underemphasizes this factor of credit rationing. Everyday bus. thinking ~~is~~ overemphasizes, neglects opp. cost, & unqul. anal. So each has something to gain from the other.

$$3. \text{ i.g.} = \frac{1}{\text{a.p.}} = \frac{\text{cost (vble.)}}{\text{Benefit}}$$

4. i.g. is a different concept of "intensity," different from instinctive ^{geographer's} idea of ~~input~~ ⁱⁿput/acre. It bears a closer relation to non-land input / ~~land input~~, a more ec. concept of intensity. But it's more measurable; & num. & denom. are readily commensurable. L.v. is implicit in low ratio.

$$\text{i.g. is } \frac{\text{non-land input}}{\text{land} + \text{non-land input}}$$

Resembles & differs from geog. intensity. Both intensities rise as you intensify a given land use.

But they differ in movg from unqul. to superior land. Geog. int. rises; i.g. falls. Indeed, i.g. = 1 means unqul. land. Which may be either hi or low in output/acre.

Wheat & grazing are lo i.g. — showing that land not unqul.

5. Uses of i.g., other than analyzing risk.

(i) Analyze supply response. (Distrib. fm. paper.)

a. Raise price (a) bring in hi i.g. land, big leverage effects.

(b) Intensify all land, same reason

Stabilizing price has same effect. Same as a big capl. xfr. to producers — fr. of absorbing shocks is performed by govt.

ii) Spot underused land, & ineffic. masquerading as opposite

Lo i.g., probly means that. Not that all shd. be the same.

Identify inqul. land. Wheat in e-n. Wn.; grazing in western states; both lo i.g., superior land — in strong hands. Big cushion — for lower price, higher taxes.

Cukes in Cal., other hand, gross \$800/a, cost \$600/a., are nearer inqul. (altho the land is not inqul. — has other uses.)

Big, rich farms. & indl. corpus. Not efficient.

Lo i.g. lets them look good. Careless defs. of effie. or productivity, arrogat's land rent to credit of inqul. mislead us — w. many implicns. for public policy.

2-19

Cobb-Douglas

1. Land of zero value is not land of zero output.

$$l.a. = \frac{\text{Non-land cost}}{\text{Output.}}$$

2. Rent

3. Cobb-D and elast. of produ.

$$y = k a^{\alpha} b^{\beta}$$

$$\log y = \log k + \alpha \log a + \beta \log b$$

$$\alpha + \beta = 1$$

$$\text{Elast. of produ} = \frac{\% \Delta y}{\% \Delta a} = \frac{\text{MP of } a}{\text{AP of } a} =$$

If we ~~double~~ double a , then $\log a$ rises by one unit.
So output is now $\log y + \log a$ rises by α .

$$\frac{d \log y}{d a} = 0 + \alpha \frac{d \log a}{d a}$$

$$\frac{\frac{dy}{da}}{y} = \alpha \frac{\frac{da}{da}}{a} = \frac{\alpha}{a}$$

$$\frac{\frac{dy}{da}}{\frac{y}{a}} = \alpha$$

$$\frac{\text{MP}}{\text{AP}} = \alpha$$

Ec. 701 or 921

Wed., 4-8-64

1. $AP_a \neq AP_b$
2. Review symmetry. (and relevance of Euler formula)
3. Residual imputation & direct

Traditional treatment of land as residuum

Dissatisfaction w. separate or non-symmetrical treatment

Appropriateness depends on context. For individual firm, it can expand land, too, so symmetry needed.

Crude demonstration: take away land, other inputs have alt. uses, so resid. is ΔP due to land. But that isn't really MP from small unit. It is NP, from a large Δ : ANP, when divide NP by Δ land.

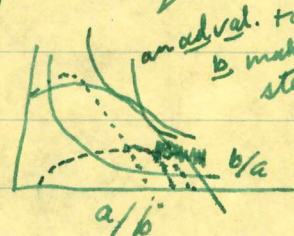
Paying land its MP means from small Δ . Euler only holds when $MP = ANP$. That requires const. returns.

Best demonstration, lens graph.

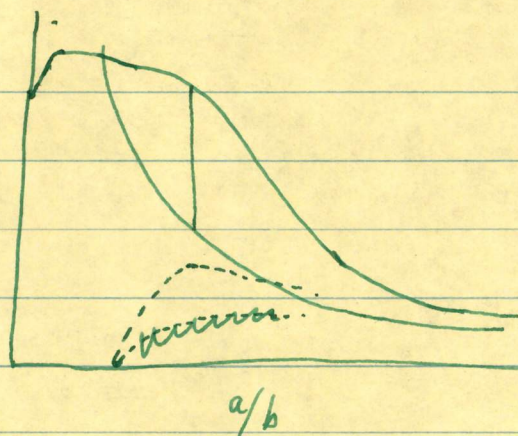
3 1/2. Note $MP = MNP$ anyplace.

4. Best to treat profit as residual b/c of time-lag. [Is most closely allied to interest.] Land, view'g processes over time, is the least residual, the most disserviceable or salvageable. i.e. it has an o.c. constantly.

an ad. val. tax on Δ makes acc. curve steeper



~~But~~ ANP is flat-topped, so can vary intensity greatly w/o big change in ANP. But that is deceptive. Big changes in MNP. Should drive to equal intensity, where mgnt. adjustments are easy. ~~But where they cannot,~~



A fish ~~curve~~ ^{graph}, rather than a lens.

Top curve is not an arc, but a "hump-and-tail"

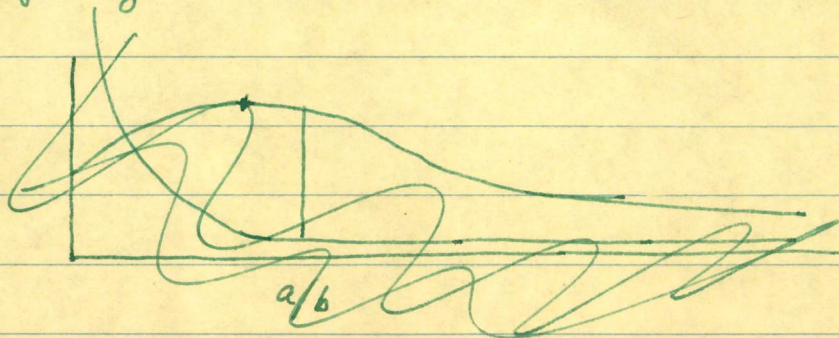
Worst fallacy: Max. $\frac{GR}{CC}$, or min. I.Q.

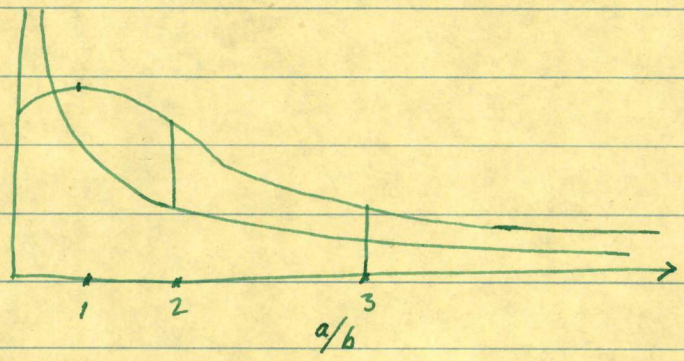
Next bad fallacy: regard equal ANPs as equivalent

Bring in under risk, dynamics, as related to prodn. eqs.

Role in cycle: pressure on land causes move to max ANP, (when before that was often sacrificed to min. risk by min. I.Q.) Hence apts.; hi hotels & office bldgs; intensive farming; always find extraordinary capacity to ΔGR

How do we know shape of AGR ? It's just an ordinary AP curve, with b fixed. At first it goes up, despite falling b/a , bec. in stage I, Δg returns to scarce limitational input. Then it falls bec. b/a falls. Then it tapers out bec. b/a stops falling much. It's unlikely ever to fall below b/a — that wd. entail getting into Stage III.





- 1 - max QGR
- 2 - max ANR
- 3 - max $\frac{GR}{CC}$

Milw. study

can have large differences of intensity or factor proportions.

The moral: differences of intensity or factor proportions are a sure sign of imperfect markets. Other argument is that low density gives ANP just as hi. But watch incl. curve!

5. Concept of i.g. $\equiv \frac{GP}{CC} \frac{CC}{GP}$

$$NP \equiv GP - CC$$

? is symmetrical, but we will apply to land.

✓ Moral: If 2 NPs are =, but I.Q.s not =, are on 2 diff. sides of opt. & MRR are diff.
So I.Q. is indirect index to incl. p.t. rev.

✓ Can have big change of I.Q. with small change of ANP
Note fallacy that = ANPs mean equal economy of resource use. Equal I.Q. is proper index.

✓ Any tax or other policy that ACC will drive you far to right. Has bigger impact on hi I.Q. Likewise tax on GP

? Any pressure to economize on a (land) drives you far to left. Develops excess capacity: agric; apts.

(On graph, pushes up a GR curve) Or does it just range over which error goes undetected, allows greater extremes of range of intensity?)

6. Scale & intensity

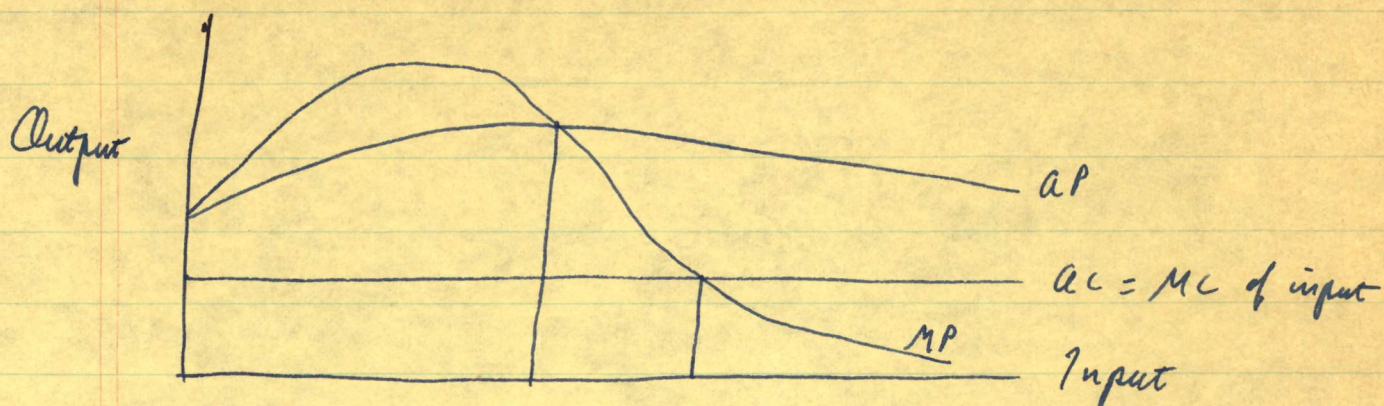
Some inputs are less divisible than others. Require certain scale to get best proportions.

Other scale advantages are priv., not social.

- Credit rating
- Bulk purchase
- Capture own externalities
- Overcome bilateral mpy. in imperfect mkt
- Overcome inheritance of control of business.

What is ec. scale depends on what society does for you.
U. F. in Lat. Am.
Quality control -- if govt. doesn't.
Etc.

Where govt. comes in conflict w. priv. enterprise is likely to
be that shadow zone of natural monopolies, etc. Need some govt.
there to make compen. work.



$$\frac{AP}{AC} = \frac{\frac{P}{I}}{\frac{C}{I}} = \frac{P}{C}$$

If AC is const., max AP is also max. AP/AC

If AC of input varies, max AP is not same.

GB holds AC const. by measuring input in \$.

101

- (-

Mon., 2-12-68

~~4th~~ Week 2

Review, and pick up

1. Loose ends, re meaning of mgnl. product

a. Assumes adaptability of complements in form, not in quantity.

Thus $MP = MNP$

b. ~~the~~ MP refers to the total effect on the output of the ^{whole operating} unit being analyzed and not just the effect ~~on~~ physically associated w. the mgnl unit.

Thus, presupposes a firm, of finite size, "indivisible" below a certain level.

The 11th acre. (Note is mgnl. in location as well as quantity. Is not homogeneous to the operating unit, [a better term than "firm".] gets further from center, a basic fact re scale ecs.)

MP depends on tenure instns., ~~and~~ ^{bes.} that determines what is internalized and what is a spillover.

Open range, no tenure, complete externalization. $MC = AP$.
This is worse, maybe, than setting $\rightarrow$ zero price on variable input; MP can be negative.

(This wd. not be ^{so} bad if all land had no tenure, ^{as in nature, or primitive societies.} It is

✓ the contrast that is bad -- unused prt. parks, over used public parks.

A general sort of rule -- unequal treatment is bad.

But Beware misuse of the rule. It is only unequal treatment of equal things that is bad. Equal treatment of unequal things is equally bad -- and equally common. [E.g. uniform area pricing; or gen. prop. tax.]

c. Dim. rets. to human inputs implies rent to others.

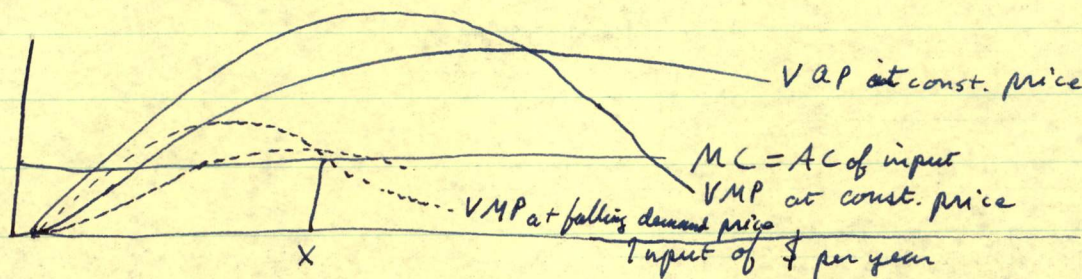
Show rent on graph.

When rent = 0, $MP = AP$. Land then is, & must be, treated as a free good. (Capl. & labor hardly ever are.)

Intensive & extensive mgnl. In city, stratify, & "extensive"

mgn. is on top of each lot.

Ag. rets. implies neg. rent. (if product sold at price low enough
so ~~MPP~~ VMP



SMP vs. LMP (In SMP, no time to adapt.)

9-30-85 Ec, 162

Ass.

Ch. 1 Scope

Ch. 2 Optzn.

2 Mid & Final.

1st. Mid. Mon. 10-21

Cover, ^{Optzn.} Midn., Cost, Regn., ~~Cap. Budgeting~~
(2d Midt. cover Cap. budgeting)

Prereg: Calc., Ec. 100A

Buy: 2 books + hand calc.

Attendance. Quiz every Fri. Problems: key words.

Hard work. If that doesn't fit in w. ^{life-style or} yr plans, do
us both a favor & take a diff. course.

People who pass will know something.

Note that in cases like ~~this~~, there, we begin with a Ratio of 1/1, express or implied, and then modify it by adding or subtracting % changes. Several ~~the~~ people feel at sea because they wonder what ^{the} is the firm's ~~base~~ ~~bottom~~ we started from? + ~~where are we going?~~

Remember ^{how} we early on developed ~~at~~ the compound interest formula. Begin with \$1, see how it grows. If we can do it for 1, we can do it for any number, by multiplying the result by that number. 1 is the base, and i is a % change in the base. Then:

$$F_1 = P_0 (1 + i) \quad (20)$$

Now we are carrying this ^{a step} further, to cover cases where the Bottom of the ratio also changes. We have already done some of this before, when we ~~discuss~~ found the ~~of~~ PV of growing cash flows. Then we said:

$$P_0 = \frac{C_1}{1+i} + \frac{P_1}{1+i}$$

(11)

$$\begin{aligned}
 p_0 &= \frac{a_0}{1+i} \left[1 + \frac{1+g}{1+i} + \left(\frac{1+g}{1+i} \right)^2 + \dots + \left(\frac{1+g}{1+i} \right)^{n-1} \right] \\
 &= \frac{a_0}{1+i} \left(\frac{1 - \left(\frac{1+g}{1+i} \right)^n}{1 - \frac{1+g}{1+i}} \right) = a_0 \frac{1 - \left(\frac{1+g}{1+i} \right)^n}{i - g} \quad (21)
 \end{aligned}$$

At that time we were not focusing our consciousness on Ratios as such, but just following the flow of the compounding logic. Now we are integrating our experience with financial math, with the ~~rat~~ Quotient Rule of differential calculus, to handle Ratios generally.

Let's try some more problems.

Continuation

12

The money value of your stamp collection grows at 15% / year, but the value of money drops at 11% / year. What happens to the real value? Grows by 4% / year.

usent
p. 1A

~~Cash flow from the farm left to you by old Gramps in Kansas rises at 8% / year.~~
and the remainder to your children.

~~Old Gramps in Ks. leaves you a lease on his farm, The lease runs 15 years; the cash rent, ^{begins at \$10,000 +} rises by a yearly increment of 8%.~~ ~~What is the PV of the last year's payment?~~
 ~~$(1 + .08 - i)^{-15}$ a, where a, is the 1st price.~~

10,000 x

~~Note that in cases like this we begin with a Ratio of 1/1, (express or implied), and then modify it by adding & subtracting % changes.~~

~~The "E of production" is the % Δ output caused by a given % Δ input. If the E of production is .90 ~~when~~ for land, what happens to output and yields when you add 20 acres (half of a back 40) to a 320 acre farm? Output rises by 90% of $\frac{1}{16}$, or 5.63%. Yields ~~fall by~~ change by $\frac{1}{16} (.90 - 1)$ or -.63%. (Yield means output/acre.) That is, output is up by 90% of $\frac{1}{16}$, and ~~land is~~ acres are up by $\frac{1}{16}$, so the Ratio of output/acre is down by 10% of $\frac{1}{16}$.~~

That + the next problem imply require that 1A
you understand how a drop in the value of money (over)

On ~~unit~~ to inflation, a city lot grows in
nominal dollar value by 6% / yr., while ^{also} yielding
an annual income.

The stock of National Natural ^{TV} ~~gas~~ ^{box} rises at
6% annually, while the CPI rises at 11% / yr.
What happens to the real value R after 8 years?
Down to $(1 - .05)^8$; (or by $1 - (1 - .05)^8$) of the
initial real value. (Return to p. 1)

$$\frac{1 - .4}{1 - .4} = 1$$

The value of whiskey grows ^{beginning w. year 3:} at the ff.
rates in a sequence of years: .25, .19, .15, .12,
.10, .08, .065. At which rate is the which
year is its PV a max? Use a discount rate
of 9%. In year 7, when it grows at 10%.
Because PV is a Ratio whose Bottom grows by i
each year. In year 8, PV ~~drops~~ ^{changes} by $\pm (.08 - .09)$
or -1%.

is measured. There is a Consumer Price Index (CPI) which begins at ~~100%~~ ^{100%} in the Base Year,

~~(which keeps adjusting forward, periodically).~~

When prices rise by ~~$\frac{d}{100}$~~ ^{$\frac{d}{100}$} , the Index goes to ~~$1.0 + \frac{d}{100}$~~ ^{$100 + d$} . (meaning it.)

If When, e.g., $d = 100\%$, $CPI = 200\%$. That says a Base Year \$, ~~$(\$_B)$~~ ^{$(\$_B)$} has the same value as 2 Current Year \$, ~~$(\$_C)$~~ ^{$(\$_C)$} .

If When, e.g., $d = 5\%$, $CPI = \frac{105}{100} = 1.05$. Prices are up 5%; a dollar buys 5% less.

~~$\$B = 2 \C~~

$$\underline{\$C = \frac{\$B}{2} = \frac{\$B}{CPI}}$$

The value of the dollar, ~~is the goods/\$~~ ^{is} the reciprocal of the price of goods, i.e. $\$/\text{goods}$.

To find the value of goods in ~~current~~ constant or base-year \$, we ^{would} multiply the current price of a good $(\$/\text{good})$ by the current value of money $(\text{goods}/\$)$;

or, what is the same thing, divide the current price ^{of a good} by the current price of goods in general $(\$/\text{goods})$.

If I were king I would have chosen the first way; but the custom now is the second way, divide by an index of $\$/\text{goods}$, called the CPI. We will follow the custom ~~this time~~.

2 ~~the~~

The "E of production" is the % Δ output from a given % Δ input. Thus, add half of a back forty to a ^{half-}section of Illinois prairie soil — (that's 20 added to 320, for you city folks) — that's a 1/16 increase of land input. Get a ~~2.8~~ 2.8% increase of output, and:

$$E = \frac{\cancel{44.8} \cdot .028}{1/16} = 44.8\%$$

(22)
(20)

What does (22) mean for yields? ("Yield" means gross output/acre.) They change by $.028 - 1/16 = -.03450$. That is, they fall by 3.45%.

Here's a tonguepic, you could put on the back burner for a while. If you know E, you can predict the effect on yields of adding acreage. Multiply E by the % Δ acres to get the % Δ output. Then subtract the % Δ acres from the % Δ output to get the % Δ yields.

$$y = p - i = Ei - i = i(E - 1)$$

23
(21)

where ^{b.c. means} ~~y~~ is % change; Y = Yield; P = Product (used in lieu of Output, to avoid confusing "0" with Zero); and I = Input. (We shall return to E -- I know you find it bewildering at this point.)

Continuation

The ^{Quotient} Rule of (19) is useful in maximizing present values. The PV of a single or "point" ~~of~~ FV is a ^{Ratio} ~~function~~ whose Bottoms grows at a rate i each year. If the Top grows with time, the ^{Ratio} ~~Quotient~~ is a max when $t = b$. (This is precise, because calculus is exactly true when $t = b$; that's why calc. rules are good for maximizing.)

Example. The value of whiskey aging in barrels of the highest grade White Oak, resting quietly in a cool limestone cave in ~~Kentucky~~ ^{Tennessee}, grows at the ff. rates in the series of years beginning with #3: .25, .19, .15, .12, .10, .08, .065. In which year is its PV a max? Using a discount rate of 9%, PV is max toward the end of year #7, because in year #8 it falls by 1% (.08 - .09). You can also say that PV grows in the sequence .16, .10, .06, .03, ~~and~~ .01, -.01, and -.025.

Here's a tricky one for you. ~~There is~~ White Oak barrels are not cheap. White Oak of "whiskey-barrel grade" takes 50 years or so to age, ^{in the barrel} a lot longer than whiskey. In Europe, used barrels are recycled several times. Not so here - by law they cannot be. A boon for gardeners ^(over) and much more so for producers of White Oak, of course, (for whose benefit the law was passed in ~~this free market~~ ^{our highly political} economy.)

to whom the old barrels are uncoiled, ^{unwound in half,} for planters;

of the numerator -- i.e. the Bottom
of the Top -- if you get my meaning,

But what is the effect on the date of max PV of whiskey? ~~But~~ In Europe, add the value of the barrel to the denominator -- (Bottom, ~~bottom + land~~ ^{I prefer to say} ~~long~~ words) -- and reduce its % growth rate. Yes, Va, adding a constant to a variable dilutes the variable and ~~lowers~~ ^{lowers} the % growth[†] of the whole. [‡] So this factor moves Europeans to age their whiskey a bit less, to salvage the barrels for recycling.

By the same logic, White Oak growing on valuable land will be harvested sooner than it would if growing at the same rate on cheap ~~or~~ free land. Explain the logic -- why are used whiskey barrels like the forest floor in this economic analogy? In either case, you are adding a constant to a growing value, reducing the % growth rate, and thereby advancing the date of ripeness (or "financial maturity").

^{commonly,}
† Subtracting a constant, ^{commonly,} fortifies the variable and raises the % growth -- an effect known universally in business as "leverage", because of the multiple effect on % growth. The analogy with mechanical leverage is strained, as we will see, (over)

and Archimedes would turn over in his grave
to hear such ^{usage} ~~usage~~, ~~except he anticipated this~~
~~problem by inventing Archimedes' screw to~~
~~turn for him.~~

B. The Quotient Rule with Finite Changes.

As in (16), let $R = T/B$. Then, using our convention that l.c. letters are % changes:

$$R(1+r) = \frac{T(1+t)}{B(1+b)} \quad (24) \quad (\text{cancel})$$

Dividing by (16):

$$1+r = \frac{1+t}{1+b}$$

$$r = \frac{1+t}{1+b} - 1 = \frac{1+t-1-b}{1+b} = \frac{t-b}{1+b} \quad (24)$$

$$r = \frac{t-b}{1+b} \quad (25) \quad (\text{cancel})$$

and t is not close to b ,

(25) is the Quotient Rule modified for finite changes. If b is large, you'll have to pay attention, obviously. Thus, if input doubles and output triples, you cannot say that yields rise by ~~200% - 100%~~ double. Rather, $r = \frac{2-1}{1+1} = \frac{1}{2}$, meaning yields rise by 50%.

Example. Al Avocado is hit with higher water rates so he cuts his irrigating by $\frac{1}{3}$. By ~~more~~ irrigating more conservatively, he keeps ~~his~~ output constant. What happens to output / acre-foot of water?

~~Up by $\frac{0 + .33}{1 + .33} = \frac{.33}{1.33} = \frac{0 + \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$~~

Up by $\frac{0 - (-\frac{1}{3})}{1 + (-\frac{1}{3})} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$. Magic!

To confirm, assume output is 100, while water use drops from 1 unit to $\frac{2}{3}$.
 Note that $\frac{100}{\frac{2}{3}} = \frac{300}{2} = 150$, a
 50% rise of R caused by a $\frac{1}{3}$ decline in B.

(What a lovely thought: if everyone used $\frac{1}{3}$ less water, we could support 50% more people! }
 If 50% less, 100% more people, etc. No wonder people preach
 get excited about conservation.)

Insur p. 6A

Example: The Canadian \$ drops 20% against the U.S. \$, while the yen rises 32%. What happens to the yen in Canadian \$? Rises by
 $\frac{.32 + .20}{1 - .20} = \frac{.52}{.80} = 65\%$. (To confirm, $\frac{1.32}{.80} = 1.65$)

What happens to the U.S. \$ in Canadian \$?
 Rises by $\frac{0 + .20}{1 - .20} = 25\%$. This one was caused a

Settler's Ditch,

Ad's water comes from Crayfish Creek, 15 miles away, through a leaky old canal. 40% of Withdrawals (W) are lost in conveyance, so ~~to~~ Deliveries (D) = .60 W . We can define Transmission Efficiency (E) ~~as~~ as D/W . But they neglect cleaning, ~~and~~ Settler's Ditch becomes choked with "phreatophytes" — water-loving plants which transpire & waste water. Line losses rise by 50% — i.e. to 60% right? — so E falls by $1/3$, to 40%. (Much of that is noise. Hold ~~fast~~ ^{fast} to the signal: E falls by $1/3$.)

Ad has a firm right to a fixed D . By how much must W rise — by $1/3$? Not so simple.

$$W = \frac{D}{E}$$

(26)

$$w = \frac{0 - - \frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

Efficiency falls by $1/3$, so Withdrawals must rise by $\frac{1}{2}$. Very interesting. So does inefficiency waste resources? You'd better believe it!

Is it just a coincidence that the 50% rise in required withdrawals equals the 50% rise of line losses? Yes, that's all, a coincidence. But it's interesting to note that ~~the~~ Efficiency is the complement of ~~the~~ line losses:

$$E = 1 - L \quad (27)$$

where L is the % of ^{input (I)} ~~withdrawals~~ lost in a process.

$$E(1+e) = 1 - L(1+l)$$

$$1+e = \frac{1 - L(1+l)}{1-L} = \frac{1-L-Ll}{1-L} = 1 - \frac{Ll}{1-L}$$

$$e = -l \frac{L}{1-L} \quad (28)$$

Thus, if L is already 20%, $\frac{L}{1-L}$ is 25%, and a 5% ΔL ($l = .05$) means a $1\frac{1}{4}$ % loss in efficiency.

Another approach is to substitute (27) in (24).
Then

$$W = \frac{D}{1-L} \quad (29)$$

In (29), L appears directly as a % decline in the denominator, and we apply (25) directly.

$$w = \frac{0 - - L}{1 - L}$$

Thus, ^(as compared with no losses) a 75% loss means Withdrawals must rise by:

$$w = \frac{3/4}{1/4} = 3 = 300\%$$

The world is full of things like like losses: which take a % "off the top" of things like income taxes; share rentals; royalties; commissions; losses from heat, friction + windage; spoilage; vacancies; etc. In all such cases, apply (26), (28), or (29). ~~and carry on.~~

Remember that (25) may be applied backwards, like most equations. That is, we may know x , and be seeking $\underline{t}$ or $\underline{b}$. Then we just solve for $\underline{t}$ or $\underline{b}$ and carry on. It is even easier to solve (16) first for T or B , then apply the product rule for $T = BR$; or the Quotient Rule of (25) for $B = \frac{T}{R}$, where

B + R have simply been switched. Using (26), here is an example of each.

Water withdrawals at the head of pioneer ditch drop by 10%; transmission Efficiency rises by 20%. What happens to Deliveries? $V D = W E$, so use the product rule. Up 8%.

~~But~~

$$d = w + e + we = -.10 + .20 - .02 = .08$$

$$(.90 \times 1.20 = 1.08).$$

Withdrawals rise by 8%; Deliveries rise by 22%. What has happened to Efficiency? Up by 12.96%.

$$e = \frac{.22 - .08}{1 + .08} = \frac{.14}{1.08} = .1296$$

Problems can get more interesting as we add more detail. Thus, line losses are always some function of distance, ~~usually to a power near 2. above one, after near 2.~~ Say Joan has a right to D units of water, no matter how far out Pioneer ditch she locates. How much more W ~~water~~ does she require by moving from a

a 2-mile point out to a 6-mile point?
 Losses $\approx W \cdot 0.06 M^{.80}$, (where M = distance in miles).

$$\cancel{D = W(1 - .06 M^2)}$$

$$W = \frac{D}{1 - .06 M^{.8}}$$

The ~~safest~~ safest way to handle this one is
 in 2 stages. At 2-mile point,

(l.c.) $\rightarrow W = \frac{\cancel{0} + .24}{1 - .24} = \frac{.24}{.76} = \frac{0 + .06 \cdot 2^{.8}}{1 - .06 \cdot 2^{.8}} = \frac{.1045}{.8955} = .1167$

at 6-mile point,

$$W = \frac{0 + .06 \cdot 6^{.8}}{1 - .06 \cdot 6^{.8}} = \frac{.2516}{.7484} = .3362$$

$$\frac{.3362}{.1167} - 1 = .1965$$

Joan requires 19.65% more W because of her move. ⁶

A true loss formula would be based on D
 rather than W ; and M would be raised to a power greater
 than 1. And we require a better formula for losses. This one

than one. Let's try that.

$$W - .06 M^{1.2} D = D$$

$$W = D(1 + .06 M^{1.2})$$

This turns out to be simpler.

(l.v.) $\rightarrow W_2 = .06 \times 2^{1.2} = .1378$

$$W_6 = .06 \times 6^{1.2} = .5151$$

$$\frac{1.5151}{1.1378} - 1 = .3316$$

Joan requires 33.16% more water because of her move. The higher increase is due to ~~over~~ a more realistic formula which bases losses on deliveries, not W . Too bad it contains no Ratio, but it would if we held W constant and ~~then~~ found the effect of M on D . But enough for now; let's move on.

Here's an easy one, to regain your bearings. The equity capital of Gump's Gisunos rises by 50%; profits rise by 20%. What happens to return on equity? (Cover)

Change ~~list~~ by $\frac{.20 - .50}{1 + .50} = \frac{-.30}{1.5} = -.20$

By the way, a profit "rate" is a Ratio, with a Bottom. ~~Half the~~ 90% of the people just see the Top & ignore the Bottom -- as though they thought "rate" just meant "per year". The other 10% become potential corporate raiders. They see fat old firms with low real profit rates masquerading as high rates by undervaluing the fat in undervalued and misused assets, ~~and overvaluing the fat in the managers' heads & tails.~~

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lot of confusion; ~~Our~~ Theirs drops 20% against ours, while ours rises 25% against theirs! If the C\$ is worth 80¢, then the U.S. \$ is worth C\$ $\frac{1}{.80} = \text{C\$} 1.25$. (over)

~~Problem~~ True Story. In 8/76 the Gaffneys sold their home in Victoria, B.C. for ~~\$108,200~~ ~~C\$~~ ~~means Canadian~~. They transferred \$C 108k

the funds to U.S. \$ when ~~the \$C = \$1.03~~ ~~was~~ ~~One week~~ later the \$C started dropping like a stone. $\text{\$C } 1.00 = \1.03 U.S. They ~~bought~~ ^{put the proceeds into} a house in Riverside.

Within ~~2~~ ^{their} years the \$C dropped to 81¢ U.S., and ^{their} house in

Riverside rose by 65%. How did the Gaffneys make

out in \$C? Their worth rose by $\frac{.65 + .22}{1 - .22} \cdot 1.03$

$$= \frac{.8636}{.7864} = \frac{1.098}{109.8\%}$$

(F) Without lifting a finger! And do they feel ^{unlucky?} ~~miserable~~?

~~I do not think they do!~~ In fact, poor old dad

reads the foreign exchange quotes ^{whenever he needs} ~~just~~ for an upper, and prefers to measure his assets in \$C. ~~but~~ They do not feel

(F) Check. $\frac{1.65}{.81} \neq \frac{108 \cdot 1.03 \cdot 1.65}{108 \cdot .81} - 1 = 109.8\%$. They (over)

Here we see that reciprocals do not behave as simply as we smoothly assumed when we were talking CPI. You have to watch them when % changes are large, and use $(\frac{25}{100})$.
 E.g. the CPI rises by 100%; what happens to the value of the dollar? Down by 100%?
 Hardly! T is no. of dollars, no change; B is CPI, and $b = 1$.

$$r = \frac{0-1}{1+1} = -50\%$$

gained 3% on the exchange and 65% on the price rise, and the U.S. \$ became worth 1/.81 \$C,
 or ~~\$1.235~~ C\$ 1.235

~~unlucky~~ ^{sorry} for the chauvinistic Canadians who drove them out because they wouldn't have "that American person" ^{in a political job} ~~serving as a govt. adviser. post.~~ They do feel ~~unlucky~~ ^{unlucky} for the rising generation of Americans who ~~will~~ find housing priced so high, including their own 6 kids. So we are more unlucky than lucky, & so are you. And anchoring - but the point (one)

Accuracy of approximations to quotient rule

We have seen that the change in a Ratio is about $= 1 + \epsilon - b$; and this is handy for calculating PV when we ~~have to~~ subtract the inflation rate from the interest rate to find the real rate, etc. How great is the error? ~~Before~~ "Who gives a hoot?", you scream. Well, ^{here is motivation,} ~~if~~ ^{where} you're transferring \$108 k between countries, ^{for example,} believe me, you give a hoot about accuracy to several places. And if you don't the bankers do, and they will clip you plenty. ~~So carry on, here we go.~~

Again, when you're raising a Ratio to the 60th power, as we do in discounting, a small error can turn into a big one. Try it on your calculator. OK? So carry on, here we go.

now is how to handle reciprocals.

Here's a tax case. Disposable Income (D) is Gross Income (G) less Tax (T). Let τ = tax rate.

$$D = G(1 - \tau) \quad \text{because taxes are based on } G.$$

But wouldn't it be better if taxes were based on D ?
Why should you pay tax on money you never get?
Then,

$$D = G - \tau D$$

$$D(1 + \tau) = G$$

$$D = \frac{G}{1 + \tau}$$

For low values of τ , macht's Wicht. But try $\tau = 100\%$.

Sounds strange, but the property tax is actually reckoned that way. It is based on market value, which is capitalized from after-tax income. We should be so lucky! Ask me about this one.

Let a ratio, R_0 , change so that $R_1 = R_0 \frac{1+t}{1+b}$.

$$R_1 = R_0 \frac{1+t}{1+b} \quad (26) \text{ ~~(24)~~}$$

What kind of error do we introduce if, to save time, we say "Oh, what the Heck!"

$$R_1 \approx R_0 (1+t-b) \quad (27) \text{ ~~(25)~~}$$

The error as a % of the correct error is:

$$\text{Error \%} = \frac{R_0 (1+t-b)}{R_0 \frac{1+t}{1+b}} = \frac{1+t-b + b \frac{1+t}{1+b} + b(t-b)}{1+t}$$

$$= 1 + \frac{b(t-b)}{1+t} \quad (28)$$

The "Error Factor" is the excess over 1. It is obviously 0 when $b=0$, and when $b=t$. How bad does it get? If your calculator is programmable, write the program and generate all the answers you want. Here are a few.

Table 1: % error from approximating change in Ratio by calculus rule. Data in %, to 2 digits

$b \backslash t$	2	4	8	16	32	64
2	0	.04	.11	.24	.45	.76
4	-.08	0	.15	.41	.85	1.55
8	-.41	-.31	0	.55	1.45	2.71
16	-2.2	-1.8	-1.2	0	1.9	4.7
32	-9.4	-8.6	-7.1	-4.4	0	6.2
64	-39.4	-37.	-33.	-26	-16	0

For large values of b , the error becomes unacceptably large for most purposes. E.g. when b is 64% and t is 2%, the estimate is 39% below the true Ratio, or 61% of the true Ratio:

$$\frac{1 + .02 - .64}{1.02} = \frac{.38}{.622} = .61 = 1 - .39$$

On the other hand, when $b = 2\%$ and $t = 4\%$, who's to worry about an error of .04%?

Notice that the error is much more sensitive

to $\underline{b}$ than to $\underline{t}$. ~~Thus it~~ It would be helpful if we dealt with small values of $\underline{b}$, especially. But in discounting, $\underline{b}$ is normally larger than $\underline{t}$. Therefore we are driven to another easy approximation which is more accurate than $(\frac{27}{27})$ when $\underline{b} > \underline{t}$. This is the one we have used before, it will continue to use in this course, for discounting:

$$R_1 \approx R_0 \frac{1}{1 + \underline{b} - \underline{t}} \quad (29)$$

Table 2 shows why.

Table 2: Ratios calculated precisely, and by 2 methods of estimate

$\underline{t}$	$\underline{b}$	$\frac{1 + \underline{t}}{1 + \underline{b}}$	$1 + \underline{t} - \underline{b}$	$\frac{1}{1 + \underline{b} - \underline{t}}$
.02	.04	.9808	.9800	.9804
.04	.08	.9630	.9600	.9615
.08	.16	.9310	.9200	.9259
.16	.32	.8788	.8400	.8621
.32	.64	.8049	.6800	.7576

Use of the reciprocal form of estimate, the right column, cuts the error by half or more. You can show that

(29) is always a better estimate, so long as $b > t$.
 Just apply the method of (28), and compare results.
 But I'll leave that to whosoever wishes to explore
 it, and move on.

How bad is the error in discounting? Take
 the 2d row of Table 2 and ~~compare~~ ^{compare} .9630⁵⁰ with
 .9615⁵⁰. Your discount coefficient becomes $7\frac{1}{2}\%$ too
 low, or 92.5% of the true figure. In this
 imperfect world, that's not bad, not bad at all.

Another benefit of using (29) is that it fits so nicely into the PV formula, where the Bottom turns out to be exactly $b - t$: — only we have called it $i - g$, where i is the $(1+i)$ is the Bottom of the discount formula, (so i is b), and $(1+g)$ is the Top (so g is t).

How come the Bottom turned out to be $i - g$ instead of $\frac{i - g}{1+i}$? Because when we derived the formula⁽¹⁹⁾, to keep it simpler, we made the first term $\frac{1}{1+i}$ instead of $\frac{1+g}{1+i}$. That let us multiply the Bottom by $(1+i)$, and so remove the $(1+i)$ from under $(i - g)$. This ^{raises the Bottom, and} ~~makes the PV lower, but~~ the PV.

$1+$ compensates for that bias if we ~~use~~ make the Top a little higher. The effect of using $\frac{1}{1+i-g}$ in lieu of $\frac{1+g}{1+i}$ is exactly that. $1+$ lowers the second term, which is subtracted from one, and raises

the Top.

So it leaves us with a nice simple rule that is ^{pretty} accurate, ~~enough~~. ~~It~~ because of ~~some~~ the errors in it are ~~sure~~ compensating, as well as very small.

Before you use the PV formula, subtract any growth rates, ^{g} from ~~the~~ i . Add any decay rates. You can also, by the way, add the property tax rate, (τ), (because it is based on value, like interest.)

Then, just grind out your present value formulas, using $(i - g + d + \tau)$ in lieu of i , and

there est compris.

so, when & how can we use the calc. Quotient Rule?

1. To know the direction of change in a Ratio - up, down or sideways, (if $a=b$, there's no change.)

2. When a & b are small.

3. When $a \approx b$

4. When small errors are tolerable

5. When you're not going to raise the Ratio to a high power.

6. When you're rusty or hurried or flustered and trying to get your bearings to get started.

7. When you're marooned on a desert island & your calculator batteries run down.

Complements, & Ratios.

When a fixed resource is shared between two parties, their [%] shares add up to one, which means they are complements. Thus a football field is shared between two teams, Red & Blue. When B gains 4 yards, or 4% of the turf, R loses an equal amount, and an equal 4% of the field.

But if B gains 4 yards from its 4-yard line, its % gain is 100%; while R has lost $\frac{4}{100-4}$ or 4.17%.

In absolute terms this is a "zero-sum game" — what B gains, R loses, and the cynic is right, ТАК НАСТАЕТ. Again, in terms of % of the whole field, B gains 4% and R loses 4%. But in terms of what they had before, B gains much more than R loses, and ТИСТААЕТ.

Example. 85% of the water used by man in Calif. goes into farming; ~~the rest~~ ^{the rest} goes to cities. If ~~from~~ 5% of farm use is transferred to urban use, how much do cities gain? ~~$\frac{.05 \times .85}{1 - .85} = 39.22\%$~~

$$\frac{.05 \times .85}{1 - .85} = 28.33\%$$

In general, the Ratio of the % gain of the Gainer to the % loss of the ~~loser~~ Donor ~~is~~ can be formalized.

Let D = initial % held by Donor

Let x = % of D transferred

$$\frac{\% \text{ gain}}{\% \text{ loss}} = \frac{\frac{x D}{1-D}}{x} = \frac{D}{1-D}$$

Example. In a labor-intensive crop like strawberries, payroll cost is 91% of all costs. Wage rates drop by 2%; what happens to the cash flow of ~~employ~~ growers? Rises by $.02 \frac{.91}{1-.91} = 20.22\%$. (No wonder they fight Cesar Chavez!)

Example. The proceeds of a tree farm are divided between the Planters and the Investor. The Investor's share of the harvest value is:

$$I = \frac{(1+r)^n - 1}{(1+r)^n} = 1 - (1+r)^{-n}$$

The Planter's share is initially $\frac{1}{4}$; but ^{the unit} falls by 20%.

✓ Failure is the opportunity to begin
again more intelligently. Henry Ford
