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4. Ripening; opt. Life; math. models

A LAND SPECULATION MODEL: THE ROLE OF THE PROPERTY TAX AS A CONSTRAINT TO URBAN SPRAWL

by

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A LAND SPECULATION MODEL: THE ROLE OF THE PROPERTY TAX AS A CONSTRAINT TO URBAN SPRAWL[†]

Roy W. Bahl*

1. INTRODUCTION

Much academic attention has been devoted recently to describing and analyzing the causes, nature, and economic implications of urban sprawl (Gaffney [4] and Harvey and Clark [6].) More specifically, most research in this area has focused on the costs and causes of non-compact forms of urban development. Perhaps the most costly, and certainly the most often attacked form of sprawl, is the leapfrog development — a subdivision separated from the periphery of the urbanized area by some amount of land which is either completely vacant or unused for urban purposes. Abstracting from motives which are irrational in an economic sense, and from topographical considerations, a major cause of this inefficient development is speculation in land values which, for purposes of this paper, will be defined as occurring if land is withheld from the market for economic reasons by a rational (a profit maximizing) individual.

The specific objectives of this analysis are twofold: (a) to develop a relatively uncomplicated land speculation model from which the optimum time period (from the landowner's viewpoint) for withholding a parcel of land from the market may be deduced (given that this land is both desired and suitable for urban purposes), and (b) to assess the relative effects of the property tax as a measure by which local fiscal and physical planners may discourage a leapfrog pattern of urban development resulting from urban land speculation. No attempt is made to test this model empirically. Rather, the general purpose of this paper is to apply the tools of economic theory to the urban land speculation problem and thereby develop an analytical framework in which the alternative constraints to this cause of urban sprawl may be more thoughtfully examined. Given the model, certain hypotheses are offered regarding the relative effectiveness of the property tax as one of these alternatives.

The development of a model to explain the actions of an urban land speculator necessitates a statement of temporal rather than static equilibrium conditions. This problem is analagous to the classic examples of how long should the tree be permitted to grow or how many years should the wine be permitted to age. In each of these two examples an additional year of waiting brings an increase in the quality (and therefore in the value) of the product, but it also means a higher

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opportunity cost to the holder. An early statement of marginal conditions for the optimal length of investment (attributed to W.S. Jevons) is that a "ripening" asset should be held until the discounted rate of increase in value is equated with the opportunity cost (per cent return).¹ If one identifies the costs and returns associated with speculating in urban land, it should be possible to develop a similar set of optimality conditions.

More specifically, the model developed here is an extension of Gaffney's technique for analyzing the effects of time indivisibility.² The central problem in Gaffney's paper is to define the conditions under which a rational individual will continue to add to his land holdings. Applying an equimarginal analysis, Gaffney develops what he terms a "dynamic equilibrium equation" which shows that an individual will add units of land to his holdings until the rate of return plus the expected future rate of return *on the last unit* just yields him his interest rate. A simple transformation of Gaffney's equilibrium from rates of return to stock terms permits a restatement of his conditions: the landowner will continue to add units until the marginal revenue productivity of the land in the present period plus the discounted marginal revenue productivity expected for all future periods is exactly equal to the return which he could earn on the value of the land at this interest rate. Given this statement of equilibrium conditions, it may be shown that the length of time the landowner will speculate may be partially controlled by the local government if values of certain of the parameters in the system (notably the expected value of marginal revenue productivity) can be effected by public policy. Within this general framework the potential effectiveness of the property tax in shaping land use patterns may be examined.

The model is presented in a discrete form primarily for purposes of simplicity. The optimality conditions are developed in terms of the ratio of the difference between the rate of return on the land and an opportunity cost for the current period, to the discounted difference between these rates for the aggregate of all future periods. These conditions can be much more easily explained when discrete time periods are considered. In formulating the model this way, it is implied that the landowner faces the sell or hold decision at the beginning of each period and makes his decision on the basis of what he expects in terms of net benefits for the coming period, and his evaluation of aggregated, discounted net benefits in all future periods.

2. A LAND SPECULATION MODEL

In order to develop a model to explain the optimum length of time to hold a parcel of land, assume the value of the land to be increasing at a decreasing rate, i.e., $dV/dt > 0$ and $d^2V/dt^2 < 0$, where dV/dt represents the rate of increase in market value. The traditional incremental reasoning of economic theory suggests that, with perfect knowledge, the speculator will hold the land an additional period only if the increase in market value is great enough to compensate him for the additional cost incurred during that period. Abstracting from risk considerations,

¹ For a brief state of the Jevons argument, see Baumol [2, pp. 427-443].

² Gaffney [5, p. 467] defines time-indivisibility as a "... tying of the present to the future such that he who could use a durable good cannot easily buy its services from week to week or year to year as he needs them, the way he buys his labor and raw materials, but must buy in one piece a claim to its services over its full span of years."

these costs include the property tax and the opportunity cost, i.e., the amount which could have been earned by investing an amount equivalent to the current market value of the land in the best alternative investment.³

Symbolically, this condition may be expressed as:

$$(1) \quad R_t = P_t + iV_{t-1},$$

where:

R_t = increase in the value of the land during the t^{th} year,

P_t = amount of the property tax for the t^{th} year,

i = rate of return which could be earned on the best possible alternative investment, and

V_{t-1} = market value of the land at the beginning of the t^{th} year.

However, since property taxes are fully deductible for federal income tax purposes, the landowner does not consider the entire amount P_t an incremental cost. Specifically only an amount $(1-s)P_t$ is borne by the speculator, where s represents his marginal personal income tax rate.⁴ Thus, from (1) it may be seen that the optimal time for selling the land occurs when:

$$(1a) \quad [R_t - (1-s)P_t] / V_{t-1} = i$$

The left side of Equation (1a) (hereafter referred to as r) is the net rate of return to the landowner, or alternatively, may be thought of as his indifferent rate of interest, i.e., that market rate of return at which he will be indifferent between holding the land for an additional period or selling now and reinvesting the return. Conversely, if i is expected to exceed r in the t^{th} period, the landowner will be acting rationally if he sells.

This relatively simple statement of optimality conditions from the viewpoint of the speculator is based on two assumptions. First, the landowner must be certain that he may re-evaluate his position at the beginning of each time period. Thus, his expectations for future growth in the value of the land beyond the first year are disregarded. Second, the value of the parameters of the system are estimated with certainty (though it would be possible to assign probabilities to both sides of the equation).

A second and more realistic case may be developed in which the landholder feels that he must consider not only his position in the next period, but also his expected returns and costs for the following periods. Under these conditions, the

³For the balance of this analysis, the problem of capital gains taxation will be ignored since both the increase in the value of the land, R_t , and the return from an alternative investment, I_t , are assumed to be subject to the same rate. A margin of error is introduced by this omission, however, as the payment of the tax on capital gains from the land is shifted forward to some future period, whereas if the land were sold and the proceeds reinvested in an alternative investment, a capital gains tax on the land would have to be paid at the time of the sale. This omission may be corrected for by subtracting from the cost side of the equilibrium statement a factor $G(1+i)^t - G$ where G represents the amount of taxes paid if the land were sold in period t . It will be assumed that the magnitude of this error is not sufficiently large to build the capital gains factor into the model.

⁴For example, an individual in the 52 per cent income tax bracket may shift 52 per cent of his property tax bill forward via the deduction from federally taxable income.

optimality condition in (1a) must be amended to reflect: (a) the expectations of the landowner for future increases in the value of the land, (b) the estimate of the landowner of the expected future return from an alternative investment, and (c) the marginal cost of time. Applying these modifications, the equilibrium statement⁵ becomes:

$$(2) \quad R_t + \sum_{j=t+1}^n \frac{R_j - (1-s)P_j}{(1+i)^j} = (1-s)P_t + iV_{t-1} - \sum_{j=t+1}^n \frac{I_j}{(1+i)^j}$$

where⁶

R_j = the dollar increment in the value of the land in the j^{th} year, and

I_j = the dollar return which could be earned in the j^{th} year by selling the land and reinvesting the proceeds in the best alternative investment.

Rearranging the terms in Equation (2) and dividing through by V_{t-1} yields:

$$(2a) \quad \frac{R_t - (1-s)P_t}{V_{t-1}} - i = \frac{\sum_{j=t+1}^n I_j / (1+i)^j}{V_{t-1}} - \frac{\sum_{j=t+1}^n [R_j - (1-s)P_j] / (1+i)^j}{V_{t-1}}$$

or

$$(2b) \quad r - i = i^* - r^*$$

where:

$r - i$ = difference between the net rate of return on the land and the landowner's interest rate (in year t), and

$i^* - r^*$ = difference between the discounted expected percent return on an alternative investment, and on the land (over n years).⁷

Therefore, if the equilibrium condition is stated in terms of actual and expected per cent returns, one concludes that the speculator will hold the land for the marginal period only if the difference between present net rates of return exceeds the difference between discounted expected per cent returns.

⁵ However, if $d^2I/dt^2 < d^2R/dt^2$ (where dI/dt is the average rate of return associated with the alternative investment) an equilibrium will not exist. For purposes of this paper, it will be assumed that second order conditions are satisfied.

⁶ The convention in Equations (2) and (2a) is adopted of denoting the discount factor of the land as $\hat{i}$ and that of an alternative investment as i . For convenience, it will be assumed that the alternative investment involves no risk. Thus, i is purely a cost of capital while $\hat{i}$ also includes a risk factor.

⁷ It should be emphasized that r^* and i^* are not annual rates of return, but ratios of discounted expected flows to original outlay and that they are calculated from period $(t+1)$ forward to some period n . For example, $r^* = (R_{t+1} + R_{t+2} + \dots + R_n) / V_{t-1}$. The number of periods, n , is defined as the speculator's time horizon.

To use a numerical example, assume a speculator is certain of a return of ten per cent (net of property taxes) if he holds a parcel of land for the coming period, and further, he is aware that if he holds the land, he must forego a return in that period of six per cent on an alternative investment. After choosing an appropriate time horizon (n periods), the speculator estimates a discounted net return on the land of 25 per cent and a comparable rate for the alternative of 27 per cent.⁸ Since $(r-i)/(i^*-r^*) > 1$, the land should be held for the t^{th} period. The landholder may then reevaluate his position at the beginning of each succeeding period by the same criterion. It is implicitly assumed in this modified statement of the optimality condition that the landowner sees his position at the beginning of each period as one of selling now or holding the land until time period n (the number of years in his time horizon).

3. POLICY IMPLICATIONS OF THE MODEL

The above analysis may be generalized in the following functional relationship:

$$(3) \quad t_o = f[i, i^*, r, r^*, s, n, g, P]$$

where:

- t_o = optimum time period for holding land,
- r = net rate of return on the land,
- r^* = discounted expected per cent return on the land,
- i = the individual's interest rate,
- i^* = discounted expected per cent return on an alternative investment,
- s = marginal personal income tax rate of the landowner,
- n = number of years in the speculator's time horizon,
- g = a subjective discount factor reflecting risk, and
- P = the property tax.

If the effect of changes in each variable in the system on the optimum time period for holding the land (t_o) could be identified, it would be a simple matter to evaluate the relative effectiveness of alternative policies which are followed by fiscal and physical planners in attempting to constrain sprawl.

The cost of capital, or opportunity cost (i), varies among individuals with their ability and capacity to invest and the quantity and quality of their opportunities. While Gaffney's i represents "... the rate of interest at which the individual discounts future values," the discount factor for land in the present paper includes a risk premium, i.e., $\hat{i} = i + g$, thus, R_j will be discounted at a higher rate than I_j . The length of the speculators time horizon (n) will depend in part on his forecast of the intensity and regularity of the demand for urban land. For example, if the landowner suspects that a current flurry of building activity will leave the community "overbuilt" in a couple of years, his time horizon will be shorter than if he expects population growth to be sufficient to support the higher level of building activity for a longer period of time. Given the length of the speculator's time horizon, an increase in either i or g will shorten the optimal period for holding the land. Likewise, an increase in the property tax or a reduction in r or r^* will accomplish the same result. In summary, there would appear to be

⁸ See preceding footnote.

two ways in which the optimal number of time periods for speculation might be shortened by action of the local government: (a) an increase in the risk factor, i.e., an increase in the difference between i and $\hat{i}$, (b) an increase in property taxes. Since these factors might be influenced either directly or indirectly by government policy, the discussion below is a qualitative examination of the potential for using each to constrain land speculation.

Clawson's [3] suggestion that "... local real estate taxes could be made into a conscious instrument to implement (land use) plans," must be tempered by the fact that only a portion of an increment in property taxes – specifically, $(1-s)P_t/P_t -$ is eventually borne by the speculator. Even if this ability to shift the tax forward did not exist, it is doubtful that the property tax could be used to discourage speculation in land at the periphery of the urbanized area.

In the above analysis it was shown that *ceteris paribus* the smaller the difference $R_t/V_t - (1-s)P_t/V_t (r-i)$, the less apt the landowner would be to hold the land for another period. Consequently, to the extent (a) P_t/V_t is small relative to R_t/V_t , (b) $\Delta P_t/\Delta V_t < P_t/V_t$, and (c) the potential for increasing P_t/V_t is small, the property tax will be an ineffective constraint to urban sprawl. Since aggregate data on the property tax treatment in the urban fringe are not available, it is not possible to examine for all urban areas in the United States the size and movement in the ratio P/V for these areas through time. However, the growth in market value of building sites at the margin of urban development has been phenomenal in recent years, increasing by over 160 per cent between 1950 and 1962. (See Maisel [7].) This contrasts with a rise of less than 30 per cent in the general price level. Given that market values of these potential building sites in the urban fringe are rising dramatically and that assessment practices in most localities are such that these sites are generally taxed at (agricultural) use value which is usually a small fraction of market values;⁹ it would seem safe to assume the ratio P/V for this type of land to be small (less than one per cent);¹⁰ and declining through time.

The levels of and temporal change in property tax burdens on owners of sites at the periphery of the urbanized area may be illustrated by the results of a case study of five plots of land which were "leapfrogged" at the urban fringe of the Lexington, Kentucky SMSA. (See Bahl [1].) The data in Table 1 show the comparisons between 1956 and 1962 of market values, effective assessment ratios, and effective property tax rates for each of the five sites.

These comparisons show the size of the ratio P/V to be quite small – less than one-half of one per cent in all five cases – and lower in 1962 in each case. The effective assessment ratios shown in Table 1 together with the actual assessment ratios shown in footnote *b* of the table reveal the size of the ratio of present use valuation to potential use valuation also to be quite small and suggests the degree to which property taxes might be increased if the land were assessed at its highest use. For example, if the effective assessment ratio for plot 1 were raised by a multiple of seven (to 28.4 per cent in 1962), the resulting increment in

⁹ Maisel [7, p. 58] notes that at the fringes of suburban development, land for building will sell at ten times or more its purely agricultural value.

¹⁰ The average effective rate of taxation on all taxable property in the United States estimated at less than 1.5 per cent. See Netzer [8, pp. 102-103].

TABLE 1: A Comparison of 1956 and 1962 Market Values, Effective Assessment Ratios and Effective Property Tax Rates:^a
For Five Plots of Land on the Periphery of the Lexington, Kentucky SMSA

Plot Number	Increment in Market Value 1956-1962 (Thousands of dollars)	Ratio of Assessed to Market Value (Per Cent)			Ratio of Property Taxes to Market Value (Per Cent)		
		1956 ^b	1962 ^b	Change	1956	1962	Change
1	201	5.2	3.8	-1.4	0.12	0.11	-0.01
2	10	2.5	1.9	-0.6	0.06	0.05	-0.01
3	120	5.6	4.1	-1.5	0.16	0.12	-0.04
4	7	8.0	5.8	-2.2	0.18	0.17	-0.01
5	150	5.0	3.7	-1.3	0.12	0.10	-0.02

Source: Bahl [1, Chapter 3].

^aAssessed values and property taxes from the Office of the Fayette County (Kentucky) tax commissioner. Market value data is from a special appraisal report prepared by the Kentucky State Department of Revenue. Though assessed by the local government at (agricultural) use value, the appraisal of the land was in terms of residential use.

^bThe Fayette County median assessment ratio was 31 per cent in 1956 and 28 per cent in 1962.

property taxes would not bring the ratio P/V up to one per cent of market value. For this particular case the ratio P/V is less than one half of one per cent. $\Delta P/\Delta V < P/V$, and a 600 per cent increase in the effective assessment ratio would not increase the ratio P/V to one per cent; therefore, the property tax would appear in this instance to be an ineffective deterrent to land speculation. Moreover, since neither the assessment practices nor the spectacular growth in market values described above are unique to the Lexington SMSA, one might offer the hypothesis that manipulation of the property tax is in general not an effective method of discouraging speculation in urban land.

Apart from the manipulation of property tax, local planners may shorten t_o by increasing the risk associated with holding the land. The speculator must of necessity attach a higher risk premium in a city in which planners have a history of utilizing low density zoning and other subdivision control measures, and where there is substantial pressure from local residents for the preservation of open space within the metropolitan area. The size of the risk premium could also be affected if local authorities had a history of periodic reassessment of urban land on a market value rather than a present use basis. On the other hand, Clawson [3] notes that the drastic shortening of the suburban planning horizon and the ensuing irregularity of the intensity of demand for urban land has increased the uncertainty for any land user. At any rate, all of these effects on r^* could be of a "moral suasion" nature, and are a function of interactions between market forces and the degree to which the local authorities have supported a responsible planning program in the past. Given the limitations of existing data, it is not possible to construct an estimate of the elasticity, $(\partial_t / \partial_i)(i/t_o)$, by which to quantitatively evaluate the relative dampening effect of a higher risk factor. However, one might hypothesize, on the basis of the relatively small effect of changes in the property tax on t_o , that a one per cent increase in the subjective risk factor may well be relatively more effective in reducing the optimum speculation time than a one per cent increment in the property tax.

4. CONCLUSIONS AND LIMITATIONS

If local government policies of constraint are to be directed toward the speculator, it appears that the effect must come either directly through an increase in the property tax or indirectly through an increase in the degree of uncertainty associated with holding the land.¹¹ The evaluation of the relative effect of these alternatives in shortening the optimum time period would necessarily involve a comparison of the property tax and discount factor elasticities. From Equation (2a) and a knowledge of the general size of the ratio P/V as discussed above, a strong case could be made that the potential success of the property tax as a measure to discourage land speculation is quite limited. Even in the simple model where expectations are not relevant (See Equation 1a), increases in the property tax would not appear to be a strong deterrent to urban sprawl. Consider a hypothetical case of a speculator in a 50 per cent federal income bracket, holding a piece of land which has a market value of \$500,000 and is increasing in value at

¹¹ Other potential measures which the local government could undertake, e.g., direct prohibition of subdividing the land, are excluded from consideration here since the objective of this paper is an analysis of potential nondirectly coercive constraint measures.

at a rate of two per cent per year. In the first place, the assessed value will be only a small fraction of market value, and in addition, 52 per cent of any increment in property taxes will be shifted on via the federal tax deduction. For example, if the effective assessment ratio is ten per cent and the property tax rate is two per cent, total property taxes will be \$1000 of which \$520 may be shifted forward via the tax deduction. Consequently, $\Delta R = \$10,000$ and $\Delta P = \$420$. Therefore, the increased cost (tax) to the speculator would appear to be a relatively small proportion of the increase in the value of the land.

The model developed in the present paper is subject to a number of reservations, the most severe of which relate to the apparent difficulties in empirically testing the relationships. From the subjective nature of many of the variables, an empirical test of the model would, if at all possible, be extremely crude and would necessitate data derived from some type of sample survey. Therefore, the relative size of the elasticities is posited only on subjective grounds.

A second shortcoming of this model is the inability to explain non-economic motives for withholding 'ripe' land from the urban market, e.g., personal idiosyncracies, such as the desire to own land or to keep a parcel of land in the family, or the existence of some legal impediment to the sale of the land.

The purpose of this paper is to present a general framework by which to at least partially explain the actions of a land speculator, and to analyze the relative effectiveness of the property tax as a constraint to urban sprawl. It is shown that real estate taxes may have little effect on the sell or hold decision of the speculator, and that the role they play as a measure to implement land use planning may be only minor. Consequently, though the speculator may be the active agent of sprawl, effective urban development plans may have to be implemented through restrictive policies aimed at the developer or prospective homeowners. Actions such as the levy of additional charges for public services on residents of leapfrog developments (which may affect the speculator by increasing his subjective risk factor and/or dampening the rate of growth of value of land) may offer more effective constraints to a sprawling pattern of urban growth than policies of raising property assessments to more realistic levels.

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$$S = \frac{V(L)}{1 - (1+i)^{-L}}$$

The denom. Δ s w. L .

$\therefore$ its being there lowers opt. L that max's S

Qn: How is opt. L sensitive to i ?

Method: See how i affects % growth of denom. wrt L .

If $\Delta i \rightarrow$ higher % growth of denom., it shortens opt. L .

$$\frac{d}{dL} [1 - (1+i)^{-L}] = 0 - \ln(1+i) (1+i)^{-L} (-1) = p(1+i)^{-L}$$

$$\% \text{ growth wrt } L = \frac{p(1+i)^{-L}}{1 - (1+i)^{-L}} = \frac{p}{(1+i)^L - 1}$$

As i gets larger, this ratio gets smaller. That is, higher i reduces % growth of denom. So higher i lengthens opt. life.

This seems odd until you consider that higher i means lower future rents. (Also, it only shows effect of denom., which reflects future rents. It does not show how higher i affects (over)

It now follows that a $\frac{1}{2}$ factor, g , showing higher PV of each succeeding generation as fu. of L will also shorten life (bes. it acts like lower i)

? growth of numerator, which is a PV of 1st gen. How does it? See p. 4.

$$\frac{d}{di} \frac{i}{(1+i)^L - 1} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{[(1+i)^L - 1] \cdot 1 - i L (1+i)^{L-1}}{(1+i)^{2L} - 2(1+i)^L + 1}$$

Is this derivative positive?

Denom. is +, bec. $[(1+i)^L - 1]$ is +, and its squared anyway.

Num. is + if

$$(1+i)^L - 1 > i L (1+i)^{L-1}$$

But $L \cdot i = .72$ when $(1+i)^L = 2$

So $2 - 1 > .72 \cdot 2$

$1 > 1.4$ no

$$(1+i)^L [1 - i \cdot L (1+i)^{L-1}] > 1$$

$$(1+i)^{L-1} [(1+i) - i L] > 1$$

$$(1+i)^{L-1} [1 - i(L-1)] > 1$$

Again, no, using $L \cdot i = .72$ when $(1+i)^L = 2$

So derivative is negative. Confirming previous page.

Finally, use tabular example
 $n = 20$

i	Tabulated $\frac{(1+i)^n - 1}{i}$	(Reciprocal) $\frac{i}{(1+i)^n - 1}$
.01	22.0	.0454
.02	24.3	.0411
.03	26.9	.0372
.04	29.8	.0336
.05	33.1	.0302
.06	36.8	.0272
.07	41.0	.0244
.08	45.8	.0218

$\frac{(1+i)^n - 1}{i}$ is the amount of an annuity. Obviously a positive fn. of i . So its recip. is a neg. fn. of i .

How does higher i affect % growth of PV of 1st gen. w.r.t. L ?

It must reduce it, by reducing relative weight ~~of~~ of later years.

Thus it reduces % growth of num., at any given value of L .

Thus tends to shorten L .

When i rises, this is probably, almost certainly, the paramount effect.

But when we are considering our present problem, it is not really i that changes. That is just a surrogate. It is really " g ", subtracted from i , which is being analyzed. And this applies only to denom., not to numerator.

So effect of g is to shorten life of each generation.

This implies a marginal condition at each renewal where o.c. of dr. is higher. And so it must be higher at present renewal point, as well.

Can I derive this as a marg. solution, showing neeq. marg. condition? That wd. be very useful.

Another problem is: I'm assuming equal life for each gen. in perp. Maybe they get shorter?

$$S = PV_1 + \frac{PV_2}{(1+i)^{L_1}} + \frac{PV_3}{(1+i)^{L_1 \cdot L_2}} - \dots$$

$$= PV_1 + (1+i)^{-L_1} (PV_2 + PV_3 (1+i)^{-L_2} + PV_4 (1+i)^{-L_2 \cdot L_3} - \dots)$$

$$= PV_1 + (1+i)^{-L_1} [PV_2 + (1+i)^{-L_2} [PV_3 + (1+i)^{-L_3} [PV_4 + \dots]]$$

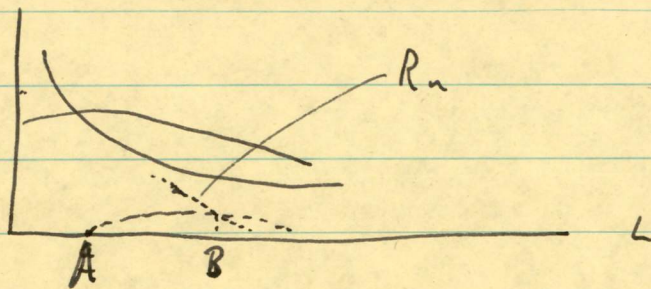
Maybe could systematize & compress that.

Why wd. lives get shorter? Res. o.c. of future keeps rising. On other hand, you probly raise C_0 each time, compensating so, to avoid great conjecture & variation. I submit simple model in which every gen. has same L , even tho they are not assumed identical, as in tree case.

Check my tree book - didn't I handle this?

$R_L = S_L \cdot i$ is condition of death of S_1 .

$R_0 = S_0 \cdot i$ birth



Another model is the squeezed interim use. Here we assume a 2d gen. Ch_2 of much higher rent.

Interim use can be squeezed as short as A , and still yield rent > 0 .

In comparing it w. Defender, & finding equil. yr. to begin, o.c. of Ch_1 is probably $R_n > a = \frac{[pv(R-c)]i}{1 - (1+i)^{-L}}$

What is R_n ? Not s.r. cash flow, but ~~some~~ l.r. Δ from extending L . I.e., from gross R_n , deduct negy $\Delta \left[C \frac{i}{1 - (1+i)^{-L}} \right]$

This rises bcs. C_0 & L both rise.

$$S = PV \left[1 + (1+i)^{-L} (1+g) + (1+i)^{-2L} (1+g)^2 \dots \right]$$

$$= PV \frac{1}{1 - \frac{1+g}{(1+i)^L}} = PV \frac{(1+i)^L}{1+g} \frac{1}{\frac{1+g}{(1+i)^L} - 1} \frac{1+g}{(1+i)^L}$$

$$S = \frac{1}{(1+i)} + \frac{1+g}{(1+i)^2} \dots = \frac{1}{1+i} \cdot \frac{1}{1 - \frac{1+g}{1+i}} = \frac{1}{i-g}$$

$$S = \frac{1}{(1+i)^L} + \frac{1+g}{(1+i)^{2L}} \dots = \frac{1}{(1+i)^L} \frac{1}{1 - \frac{1+g}{(1+i)^L}}$$

$$= \frac{1}{(1+i)^L - 1 - g}$$

$$S = \frac{1}{(1+i)^L} + \cancel{\left[\frac{1+g}{(1+i)^L} \right]} \frac{(1+g)^L}{(1+i)^{2L}} \dots$$

$$= \frac{1}{(1+i)^L} \frac{1}{1 - \frac{(1+g)^L}{(1+i)^L}} = \frac{1}{(1+i)^L - (1+g)^L}$$

(Anomaly: In this one, g seems to reduce % growth of denom. w.r.t L — hence to lengthen opt. L .)

Probably due to assum. of not beginning for L yrs., but check it out.)

Wahsan

$$S = \frac{1}{i} + \frac{g}{i(1+i)} + \frac{g(1+g)}{i(1+i)^2} \dots$$

$$= \frac{1}{i} + \frac{g}{i} \frac{1}{1+i} \frac{1}{1 - \frac{1+g}{1+i}} = \frac{1}{i} + \frac{g}{i} \frac{1}{i-g}$$

$$= \frac{i-g+g}{i(i-g)} = \frac{1}{i-g}$$

$$S = a \frac{1 - (1+i)^{-L}}{i} + a(1+g)^L (1+i)^{-L} \frac{1 - (1+i)^{-L}}{i} \dots$$

$$= a \frac{1 - (1+i)^{-L}}{i} \left[1 + \frac{(1+g)^L}{(1+i)^L} + \frac{(1+g)^{2L}}{(1+i)^{2L}} \dots \right]$$

$$= a \frac{1 - (1+i)^{-L}}{i} \frac{1}{1 - \frac{(1+g)^L}{(1+i)^L}} = \frac{a}{i} \frac{1 - (1+i)^{-L}}{1 - (1+i)^{-L} (1+g)^L}$$

$$\frac{\left[\frac{[?+1]}{[?+1]} - 1 \right]}{1} \cdot \frac{?}{[?+1] - 1} = \left[\frac{[?+1] - [?+1]}{1 - [?+1]} \right] \frac{?}{1} =$$

$$\left[\frac{[?+1] - [?+1]}{1 - [?+1]} + 1 \right] \frac{?}{1} = \left[\frac{\left[\frac{[?+1]}{[?+1]} - 1 \right]}{1} \cdot \frac{[?+1]}{1 - [?+1]} + 1 \right] \frac{?}{1} =$$

$$\left[\dots \frac{[?+1]}{[?+1] - [?+1]} + \frac{[?+1]}{[?+1] - [?+1]} + \frac{[?+1]}{1 - [?+1]} + 1 \right] \frac{?}{1} = S$$

$$\dots \frac{[?+1]}{1 - [?+1]} + \frac{?}{1} = S$$

Then

$$\frac{?}{1 - [?+1]} = \text{Let } PV =$$

$$\dots + \frac{[?+1] - 1}{[?+1] - [?+1]} \frac{[?+1]}{PV} +$$

$$\frac{[?+1] - 1}{[?+1] - [?+1]} \frac{[?+1]}{PV} +$$

$$\frac{[?+1] - 1}{[?+1] - [?+1]} \frac{[?+1]}{PV} +$$

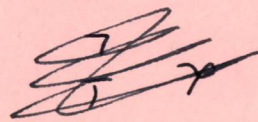
$$\frac{[?+1] - 1}{[?+1] - [?+1]} \frac{[?+1]}{PV} + \frac{[?+1] - 1}{[?+1] - [?+1]} \frac{[?+1]}{PV} = S$$

$$1 - 10 +$$

$$\frac{[(b+1)(b-1) - 1] \left[\frac{b+1}{1+1} \right] - 1}{(b+1)(1+1)(b-1) + \left[\frac{b+1}{1+1} \right] - 1} =$$

$$\frac{(b+1)(1-)(1+1) \left[\frac{b+1}{1+1} \right] - 1}{[(b-1) - (1-0) \left[1 - \left[\frac{b+1}{1+1} \right] \right]} = \frac{\frac{b_p}{n p} x - \frac{b_p}{n p} x}{x p} = \frac{b_p}{x p}$$

$$X \equiv$$



$$\frac{1 - \left[\frac{b+1}{1+1} \right]}{b-1 - \frac{b}{1+1}} =$$

$$\frac{\left[\frac{1+1}{b+1} \right] - 1}{\left[\frac{1+1}{b+1} \right] - 1} = \frac{\left[\frac{1+1}{b+1} \right] - 1}{\left[\frac{1+1}{b+1} \right] - 1} = \frac{\left[\frac{1+1}{b+1} \right] - 1}{\left[\frac{1+1}{b+1} \right] - 1}$$

Ripening.

~~4-2~~

Ex. Opp. cost of rite is $S \cdot i$

$S \cdot i = a + \Delta S$, where ΔS is gain from approaching $Ch_2 - \infty$ a year sooner, & a is rent of Ch_2 . (Prove that when $L > 1$) (Obvious. S is const, except for approaching Ch_2)

Defender rev. is $a_0 + \Delta S'$, where $\Delta S'$ is Δ from getting Ch_1 a year later.

So net effect hinges on whether $\Delta S'$ or ΔS is larger.
(i.e. net change caused by departing from simple rent & looking to future.)

$R_L =$

Omit

Another way to view this may be: If Ch_1 to be replaced by equal Ch_2 , o.c. of $yr. = a$. But if $Ch_2 > Ch_1$, $R_L > a$ in last yr. of Ch_1 , even tho a made smaller by this. ~~R_L~~

But this reduced a , (a') is not full o.c. of time.

R_L might be. ~~R_L~~ But it must be discounted;

and later generations added in. That is probly how we get to ΔS .

✓ g. growth model. $S = PV_1 + \frac{2 PV_1}{1 - (1+i)^{-L_2}} (1+i)^{-L_1}$ (Like Lutze, p. 110)

This adds more to S & hence $S \cdot i$.

Alt. model. $S = PV \left[1 + \frac{(1+g)^{\frac{L}{2}}}{(1+i)^L} + \frac{(1+g)^{2\frac{L}{2}}}{(1+i)^{2L}} \dots \right] = PV \frac{1}{1 - \frac{(1+g)^{\frac{L}{2}}}{(1+i)^L}}$

This damps effect of L on ~~$(1+g)^L$~~ growth factor, a compromise between $(1+g)^n$ and $(1+g)^{Ln}$

✓ Might Cite Jorgensen on how cycles begin anywhere, so shorter L means earlier renewal.

MS ~~$\equiv$ Moving Site Value.~~

there are cases where waiting will allow future gens. to yield higher rents in perp., not bcs. of later R_2 -date, but bcs. something irreversible is done like subdividing. If by wait'g a few yrs. can make smaller lots, get higher rents in 'perp.', or at least 2-3 generations. Avoid suckers; preemptive lower uses — anything that binds future generations.

f. [Upper limit on waiting, 10-15 yrs.

If we defer P_2 that long, might as well build P_1 as interim use, ~~glean~~ rent of early yrs., & defer P_2 a little longer b/c P_1 is there.]

where S is S if you bld. today.

(f.) Handle, w/ usual method, by defining o.c. of site as $S \cdot i$ instead of $PV_1 \cdot i$. $S = PV_1 + \sum_{t=1}^{\infty} PV_t \approx PV_1 \div (1 - (1+i)^{-L})$

The premium to cost is $i \cdot \sum_{t=1}^{\infty} PV_t$. ~~at smallest, if assume no higher rent than P_1 , is~~ $\frac{PV_1}{1 - (1+i)^{-L}} - PV_1$

Mark. rent.
 $S \cdot i = \text{rent} - \text{cost of future}$

$$= PV \left[\frac{1}{(1+i)^L - 1} \right] \approx PV \cdot \frac{1}{L} \text{ if } (1+i)^L = 4.$$

Too large to ignore.

(Not so unfamiliar -- is just $\frac{a}{i}$) (says o.c. is a , i.e. rent of last year)

(g.) And if $Ch. 2-\infty$ ~~are~~ yield higher rent, is larger ^{than $\frac{a}{i}$} . And they do - that is implicit in the very assum. that made us consider deferral in primo. Cost is a of 1st yr. + int. on PV of extra future rents, deferred. Same up or compress as $i \cdot S$. (over)

(h.) $\bar{F}$. Gain from waiting is ΔMS . A 1% , too, may gain from $Ch. 2-\infty$ if higher rents, b/c. it defers each of them a yr.

This may under selected assms. nearly offset the higher o.c.

But the neg. assum. is ~~not likely~~ -- ~~is~~ that future rent Δs be geometric, ^{etc.} More likely, as realized rent gets larger -- this adds to cost of waiting -- absolute Δs from wait's a year decline as scale effs. are achieved, and $\%$ growth of demand declines.

See p. 68 for Method.

However
(over)

Outline, 3

h, summary.

Cost of waiting is int. on PV of P_1 + PV of $P_2 - \infty$

Gain " " is growth in "

Net change made by considering $P_2 - \infty$ is:

- (1. They shorten L of ~~SA~~ SAC_2 , reduce gain of wait.)
2. Added int. cost vs. added growth of S_1 .

Added int. cost is $i \cdot PV_{2-\infty}$

(over) Added growth equals that only if $PV_{2-\infty}$ ^{"differential"} grows by int. rate or faster. (And we ignore shorter L of SAC_2)

$PV_{2-\infty}$ "differential" is added value resulting from deferring P_1 .

~~It~~ This is not large. ^{implausibility evident from accepting it -- it results (over)} SAC_1 cd. be extended another yr or so to take advantage of any spectacular gains then.

And there isn't likely to be high gains then as now, b/c LAC flattens; % of demand declines.

And higher future gains will always to some degree refuse to be deferred. Thus raise costs of squeezed interim use.

i. Relate to rent, our orig. criterion. $S-i > \text{level}_{\text{rent}} \text{ of } P_1$.

Justifies liberties we have taken, advancing R-date into Defenders later years.

$S-i$ is rent plus Δ of PV by advancing future inc.?

✓ If $\frac{\Delta MV_1}{MV_1} = i$, then $\frac{\Delta MV_1 + \Delta MV_2}{MV_1 + MV_2} < i$ whenever $\frac{\Delta MV_2}{MV_2} < i$

~~in endless deferral. Wd. never reach a year in which it was not better to wait another. The solution of Schekunzade. p. 72~~

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THE OPTIMAL TIMING OF
URBAN LAND DEVELOPMENT

By
DONALD C. SHOUP

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THE OPTIMAL TIMING OF URBAN LAND DEVELOPMENT

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While economists have devoted much attention to the problem of allocating urban land among competing uses, much less attention has been given to the problem of the *timing* of the development of land from rural to urban use or from a lower to a higher urban use. Such development and redevelopment of land continually take place in and around urban areas, and usually considerable land is temporarily held idle or in a low-valued use, awaiting development to a higher use. Land awaiting new development has typically been said to be "ripening" for a higher use.¹ The purpose of this paper is to go more deeply into the question of the optimal timing of the development of land from a lower to a higher use and to clarify some of the important considerations that a private investor or a government should take into account when making land use decisions.

Some observations on the role that speculators play in bringing about optimal development timing decisions in the real estate market also emerge as a by-product of the analysis.

THE STATIC CASE

In a static situation, the criterion for the profit maximizing allocation of land among different uses is that value of marginal product in all uses should be equal. If the marginal product of land in one use is higher than in another, land should be shifted from the lower into the higher valued use, up to the point where the two marginal products are equal. Since land yields its service through time, this statement must be taken in a present value sense; that is, the present discounted value of the marginal product of land in all uses should be equal.²

The value of the marginal product of an urban site is determined by the production function into which it enters and by the demand for the services of improved property in its location. Given the expectations of the future demand for the

The author would like to thank Mason Gaffney and C. Peter Rydell for valuable comments.

¹ Richard T. Ely and George S. Wehrwein, *Land Economics* (New York: The Macmillan Company, 1940), pp. 148-150.

² The marginal social product of land may differ considerably from the private marginal product if there are externalities associated with its use. To some extent these externalities are taken into account by zoning and other planning regulations. To the extent that these externalities are not accounted for by the planning framework, the optimal timing conditions presented here will differ from socially optimal conditions, for the present analysis is concerned with optimal timing from the point of view of the investor.

services of improved urban property, a production function will relate the inputs of capital and land to the output services of real estate in value terms:

$Q = f(K, L)$, where K = capital and L = land. Although the service of real estate will be a flow through time, Q can be interpreted as the present value of this flow.

If there are diminishing returns to capital within the relevant range, the profit maximizing outlay on improvements to the site will occur where the marginal revenue product of an increment in capital investment in construction declines to the point where it equals the cost of the increment in capital investment in construction. The classic example of calculating the marginal revenue product of improvements to land is the determination of the optimal economic height of a skyscraper. As building height increases, structural elements, mechanical equipment, and elevators increase as a proportion of rental space, causing the price per square foot of rental space to increase. As the price per square foot increases with height, there comes a point where the cost of additional space is just equal to the discounted value of its future rental returns.³ Though the example is most obvious in relation to building height, the method also applies to any sort of capital expenditure for improvements to land.⁴

Both the average and marginal revenue products of capital as a function of the capital input (K) to a fixed site are illustrated in Figure 1. In this figure, the average and marginal revenue products are calculated net of operating and maintenance expenses, and the optimum input of capital is $\bar{K}$. The resulting value of the improved site would be $OBC\bar{K}$. Subtracting the cost of construction, $OAD\bar{K}$, from the total property value will give the value of the bare site, $ABCD$, as a residual. This value formation process is implied by the real estate term "highest and best use," which refers to the form of development that would justify the highest payment for the site, whether as ground rent or as purchase price. If there is sufficient competition among potential users of the site, its market price should equal its value in the highest and best use, provided the optimal time for development is the current time.⁵

THE DYNAMIC CASE

But in a changing community, the optimal time for development may not be the current time. Population growth and redistribution, growing income, technological change, etc., will continually alter the demand for the services of real estate in specific locations. Thus, the APK and MPK curves will be shifting through

³ Alfred Marshall called this point the "margin of building." *Principles of Economics* (8th ed.; London: Macmillan Ltd., 1961), p. 371.

⁴ Alternatively stated, the profit maximizing outlay on improvements occurs where the marginal rate of return on capital investment in construction equals the interest rate.

⁵ This description of the static case follows, for instance, that of Ralph Turvey, *The Economics of Real Property* (London: George Allen & Unwin Ltd., 1957).

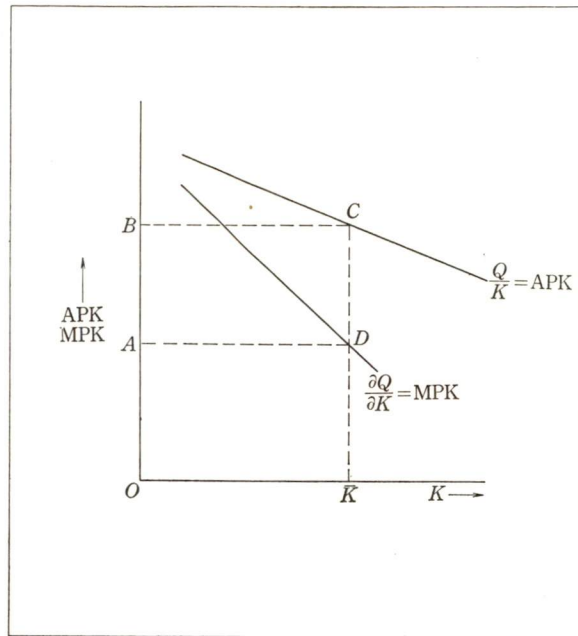


FIGURE 1

time, and the highest and best use of a particular site will consequently change.

In a growing community, there should be an increasing demand for the services of real estate, and this increased demand will be reflected in the demand for vacant sites by an upward shift of the APK and MPK curves of Figure 1. In addition to the effect of general community growth (or even in the absence of it), in particular areas the construction of new highways or other public or private investments may increase the demand for the services of real estate. Thus, as population and income grow, and as new infrastructure is created, increasing demand will justify more intensive use of the land, with larger construction outlays on the remaining vacant sites being called for. When the APK and MPK curves are gradually shifting upward as time passes, what is the optimum construction outlay, and when should the land be developed? The optimality criterion remains the same—that the present value of the land's marginal product be maximized. The problem is that, once the land is developed, the construction outlay usually freezes the land into a particular use and fixes for a long time the services that the property will render. For most uses, building codes, building technology, zoning regulations, and consumer tastes result in the construction of very durable structures. Thus, the development process usually involves the commitment of both land and capital to a long-lived combination, especially when high rise construction is involved. Further, quite aside from the great durability and immobility of most improvements, development

tends to freeze land into a particular type of use merely by splitting the land into separate parcels; once subdivided into a large number of separate ownerships, land is difficult to reassemble for subsequent higher reuse. And the bundle of ownership rights to land can be subdivided even further, as between tenant and owner or between owner and lender, all of whose interests may conflict in regard to decisions about redeveloping improved land. These factors introduce an aspect of putty-to-clay irreversibility in allocation that can only be overcome by demolition or conversion, usually at great cost.⁶

The conclusion to be drawn from these observations on the rising demand for the services of improved property and the durability of improvements to land is that the appropriate form of development of a vacant site changes through time and that delaying the development of a site to a later date may result in a different (usually more capital-intensive) form of development, justifying a higher payment for the land. Thus, in the development timing decision, the value of a lower

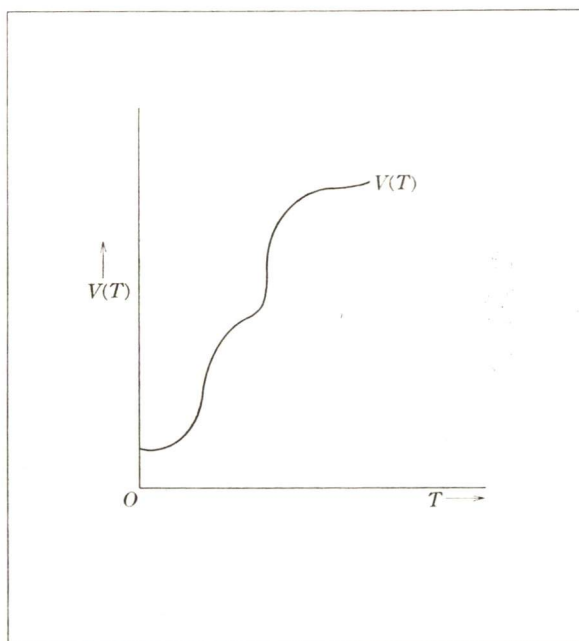


FIGURE 2

⁶ Some buildings are even built with the intention of being potentially permanent. As an example, when the Singer Building in New York, once the tallest building in the world and still the tallest building ever to be demolished, was torn down recently, it was reported that the architect who designed the building stated that it was "designed to last as long as the pyramids." And as one example of the way that buildings face obsolescence in response to changed consumer tastes, the Singer Building had a central vacuum system with outlets in each executive office for cleaning silk hats.

intensity development justifying a lower payment for land now must be compared to a higher intensity development justifying a higher payment for the land at a later date.

To find the development date and capital intensity that maximizes the present value of the land, let $V(T)$ stand for the development value of the land at any future time T if it is developed in its highest and best use at time T . That is, $V(T)$ is given by the area $ABCD$ in Figure 1, reflecting the optimal capital intensity if development were to take place at time T . But this development value changes through time because the APK and MPK curves are altered by the increasing demand for the services of improved property. For land on the fringe of an urban area, the path of $V(T)$ might take the shape of the curve in Figure 2. As population grows, previously rural land is demanded for urban uses, and $V(T)$ rises. The completion of highways, sewers, or other lumpy public or private facilities might cause a rapid or even discontinuous rise at some points; anything that affects the demand for the services of real estate in the vicinity of the site will affect $V(T)$. Naturally, unfavorable circumstances, such as the location nearby of a nuisance land use, may cause the development value of some land to decline, for the process is not one-way.

Given the curve of $V(T)$, the method of determining the optimal time of development closely resembles Wicksell's famous solution for the optimal period of maturing wine or for the optimal period of growth before harvesting trees.⁷ The maximand is the present value (at time zero) of the development value of the land. If there are no operating expenses and no interim rent receivable for use of the land in the period preceding the time of development, the present value (at time t) of the land, $P(t, T)$, for any future development date, T , is given by the formula

$$P(t, T) = V(T)e^{-r(T-t)}, \quad t < T \quad (1)$$

where r is the (instantaneous) rate of discount applicable in the real estate market.

To maximize the present value with respect to date of development, set

$$\begin{aligned} \frac{\partial P(t, T)}{\partial T} &= \frac{\partial}{\partial T} [V(T)e^{-r(T-t)}] = 0 \\ &= V'(T)e^{-r(T-t)} - rV(T)e^{-r(T-t)} = 0. \end{aligned}$$

Therefore,

$$\frac{V'(T)}{V(T)} = r. \quad (2)$$

Thus, the land should be developed when the rate of increase of the development value, $V(T)$, of the bare site equals the discount rate (this date is, of course,

⁷ Knut Wicksell, *Lectures on Political Economy*, E. Classen, trans. (London: Routledge and Kegan Paul Ltd., 1934), I, pp. 178-181. $V(T)$ as defined here is analogous to the lumber content of a growing tree whose optimal date of harvest is to be determined.

not necessarily easy to identify in practice).⁸ The second order condition must also be fulfilled.⁹ It is that

$$\frac{V'(T)}{V(T)} = \frac{V''(T)}{V'(T)}.$$

If we let T_0 signify the optimal date of development thus determined, the present value of the land at any prior time, t , is

$$P(t, T_0) = V(T_0)e^{-r(T_0-t)}, \quad t < T_0 \quad (3)$$

and the rate of increase in the capitalized value of the land is

$$\frac{P'_t(t, T_0)}{P(t, T_0)} = r, \quad t < T_0. \quad (4)$$

This implies that, with unchanged expectations, we should expect unused land awaiting development to a higher use to appreciate at a rate equal to the interest rate at which future values are capitalized in the real estate market.¹⁰ This result must be modified, however, to take into account the effect of ad valorem property taxes which are (or ought to be) levied on land as a fixed proportion of its market value. Let us assume that the tax is levied each period as a fixed percentage, a , of the market value of the land in that period, and that the development value of the land, $V(T)$, is figured after all taxes. Then equation (1) is altered to

$$P(t, T) = V(T)e^{-r(T-t)} - \int_t^T aP(i, T)e^{-r(i-t)} di. \quad (5)$$

To find the optimal development date in this case, set

$$\begin{aligned} \frac{\partial P(t, T)}{\partial T} &= V'(T)e^{-r(T-t)} - rV(T)e^{-r(T-t)} - aP(T, T)e^{-r(T-t)} \\ &\quad - \int_t^T a \left[\frac{\partial P(i, T)}{\partial T} \right] e^{-r(i-t)} di = 0. \end{aligned} \quad (6)$$

From equation (5), at the optimal time of development the market price of the land must equal its development value

$$P(T, T) = V(T),$$

and also

$$\frac{\partial P(i, T)}{\partial T} = 0 \quad (\text{the condition for optimum } T).$$

⁸ Equations (1) and (2) show that this optimal date of development maximizes the present value of the land for all $t < T$.

⁹ There is also the possibility of nonunique solutions, in which case the global maximum is the relevant one.

¹⁰ The result is, of course, expected. Since appreciation in value is the only return from holding vacant land, the rate of appreciation should equal the rate of return on comparable alternative investments.

Therefore, from equation (6)

$$V'(T) - rV(T) - aV(T) = 0,$$

and

$$\frac{V'(T)}{V(T)} = r + a. \quad (7)$$

Thus, the effect of property taxes on vacant land is to bring development when the rate of increase of the development value, $V'(T)/V(T)$ of the bare site equals the sum of the interest rate and the tax rate. It is interesting that the difference between this optimal development timing condition and the one found previously in the absence of a land tax is one sense in which even a pure site value tax may not be perfectly neutral in its effect on resource allocation, as is frequently claimed.

The present value of the land at any time t prior to development is now

$$P(t, T_0) = V(T_0)e^{-r(T_0-t)} - \int_t^{T_0} aP(i, T_0)e^{-r(i-t)} di. \quad (8)$$

Therefore,

$$\frac{\partial P(t, T_0)}{\partial t} = rV(T_0)e^{-r(T_0-t)} + aP(t, T_0) - r \int_t^{T_0} aP(i, T_0)e^{-r(i-t)} di$$

and the rate of increase in the value of the land is

$$\frac{P'_t(t, T_0)}{P(t, T_0)} = r + a, \quad t < T_0. \quad (9)$$

Thus, the effect of the property tax on vacant land is to *decrease* the value of the land but to *increase* its rate of appreciation.¹¹ This could have been expected, because appreciation, the only return from holding the land, must now cover both the interest cost and lost taxes.

A further complication must also be mentioned. This is that land awaiting development or redevelopment to a higher use may be put to uses that will yield an interim income, either from agriculture, from an existing or remodeled lower-valued urban use, or from an intentionally temporary new improvement to the land that is meant ultimately to be removed to make way for a more permanent improvement (such temporary improvements are called "taxpayers" in real estate terminology). If a temporary improvement is contemplated, its form must be suited to the length of time it is expected to be used until the land is redeveloped. For short periods, the appropriate decision may be to make no temporary improvement at all, but rather to leave the land in its current use, or perhaps no use at all; this would be the case if no form of improvement would permit full amortization over

¹¹ This assumes that the level of property-related government services would be unchanged even in the absence of real estate taxes. If, however, the tax revenues were used to finance services of benefit to the property taxed, the value of the land might not decline at all.

its short lifetime. But as longer holding periods awaiting redevelopment are considered, some forms of short-term improvements will no doubt be feasible; for instance, paving for a parking lot, construction of advertising signs, or even temporary structures. For any given period of temporary use, the profit maximizing developer would choose that form of temporary improvement which would yield the highest present value of interim net rental returns, with the knowledge that the improvement would have to be removed at the end of the period.

Let $A(i, T)$ stand for the interim income of the land in each year, i , for the profit maximizing temporary use beginning during the current period (time 0) and ending in period T , the year of redevelopment of the land to a future higher use.¹²

Then, the present value of the land at time zero, $P(O, T)$ is found as before,

$$P(O, T) = V(T)e^{-rT} - \int_0^T aP(i, T)e^{-ri}di + \int_0^T A(i, T)e^{-ri}di. \quad (10)$$

The optimal date, T_0 , of redevelopment to a higher use is again determined as in equation 6. Set

$$\begin{aligned} \frac{\partial P(O, T)}{\partial T} &= V'(T)e^{-rT} - rV(T)e^{-rT} - aP(T, T)e^{-rT} - \int_0^T a \frac{\partial P(i, T)}{\partial T} e^{-ri}di \\ &+ \int_0^T \frac{\partial A(i, T)}{\partial T} e^{-ri}di + A(T, T)e^{-rT} = 0. \end{aligned}$$

As before,

$$P(T, T) = V(T) \text{ and } \frac{\partial P(i, T)}{\partial T} = 0.$$

Therefore,

$$V'(T) - rV(T) - aV(T) + e^{rT} \int_0^T A'_i(i, T)e^{-ri}di + A(T, T) = 0,$$

and the optimal date for redevelopment occurs when

$$\frac{V'(T)}{V(T)} = r + a - \frac{A(T, T) + e^{rT} \int_0^T A'_i(i, T)e^{-ri}di}{V(T)}. \quad (11)$$

The economic interpretation of equation (11) is that development to a higher use should take place when the rate of change of development value of the land, $V'(T)/V(T)$, is equal to the sum of the interest rate and the tax rate, minus the rate of return in interim use earned by delaying the date of redevelopment one more period.¹³ The rental return from interim use earned by delaying the date of develop-

¹² It will be assumed that the development value of the land, $V(T)$, is independent of the interim use to which the land is put. Any cost of removing interim improvements is deducted from $A(T, T)$.

¹³ Note that this does *not* say that redevelopment of improved property to a higher urban use should be undertaken as soon as the development value of the site, net of clearance costs, exceeds the value of the existing improved property, as is sometimes stated.

ment one more period (the numerator of the last term) is composed of two parts: (1) the rental in period T and (2) the cumulated value at time T of the increase (if any) in rental returns from interim use associated with a lengthening of the planned lifetime of the interim improvement by one period. The explanation for the second part is that a longer-lived interim use might also be a more capital-intensive use, with higher site rental over the whole interim use interval. For short interim use intervals with no interim improvement, this second term of the numerator would be zero.

The present value of the land at any time, t , prior to development becomes

$$P(t, T_0) = V(T_0)e^{-r(T_0-t)} - \int_t^{T_0} aP(i, T_0)e^{-r(i-t)} di + \int_t^{T_0} A(i, T_0)e^{-r(i-t)} di, \quad (12)$$

and the rate of appreciation is

$$\frac{P'(t, T_0)}{P(t, T_0)} = r + a - \frac{A(t, T_0)}{P(t, T_0)}, \quad t < T_0. \quad (13)$$

That is, the rate of appreciation of land awaiting development to a higher use is the sum of the interest rate and the tax rate, minus the rate of return earned in any interim use in that period. This result follows strictly from the assumption that the value of the land is determined by capitalization of future returns. The curves of $V(T)$ and $P(t, T_0)$ could look something like those shown in Figure 3, and

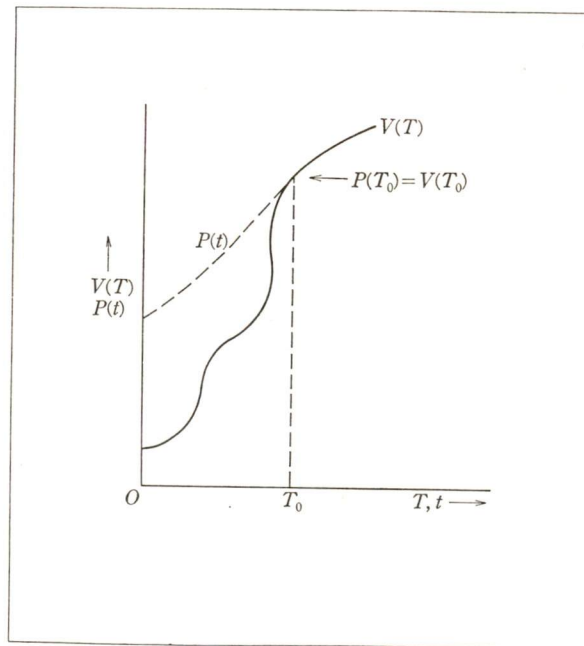


FIGURE 3

the optimal date of development would be at T_0 .¹⁴

If there were perfect competition and perfect information in the real estate market, the market price of the site (at any time prior to T_0) would be $P(t, T_0)$, and the site would appreciate at the rate given by equation (13). The market price of the site can be substantially above its value in whatever current use it may have, and above the value it would have even if it were developed for its current highest and best use (that is, $V(T)$). Such behavior is familiar with land on the fringe of urban areas, where price is determined primarily by future development potential, rather than by any return in current use. The land may be held by farmers, estate owners, speculators or others, but whoever holds it serves the function of preserving the land from premature development when expected increases in demand would justify more capital intensive development at a later date. Unless their actions are based on incorrect forecasts or tend to be destabilizing, speculators can contribute to the efficient working of the real estate market by bearing the risks and costs of holding land during the period of transition between rural and urban use, or between a lower and a higher urban use. However, note that this is essentially a short-run model applying to land in the process of transition from one use to another, and has little application to very long-run or static situations.¹⁵ Also, it is a partial equilibrium model, referring to individual sites in a given (though changing) environment. Finally, the only form of taxation incorporated in this model is the property tax, and a more complete model would have to take into account the effects of other relevant forms of taxation.

UNCERTAINTY AND MARKET IMPERFECTIONS

Obviously, the smooth and efficient operation of the real estate market along the lines described makes extraordinary demands on information about the future. The conventional textbook application of the Wicksell model to determine the optimal date for cutting trees relies on well-established biological growth rates and some assumption about future lumber prices to predict the future time path of $V(T)$. But, in the real estate market, investors will not have certain or uniform expectations concerning the future course of $V(T)$, and so there will be no firm

¹⁴ A recent study found that between 1954 and 1962, vacant land awaiting urbanization in one area of Philadelphia appreciated at a rate of 9.7 per cent per year in real terms when several factors (location, state of land, zoning) were held constant. Real estate taxes reduced this return to 7.7 per cent per year, which the authors felt represented a normal rate of return considering the risk, illiquidity, and transaction costs involved in land investment. F. Gerard Adams, Grace Milgram, Edward W. Green, and Christine Mansfield, "Undeveloped Land Prices During Urbanization: A Micro-Empirical Study Over Time," *Review of Economics and Statistics*, L, No. 2 (May, 1968), pp. 248-258.

¹⁵ Theoretically, with perfect certainty and an infinite time horizon, an investor would plan a sequence of developments for the site. Practically, the typical building lifetime is probably so long in relation to the feasible time horizon that investors would not plan beyond the lifetime of the next improvement.

idea of the optimal date of development, T_0 , or of the development value of the land at that date, $V(T_0)$. At best, investors will have some joint probability distribution for $V(T_0)$ and T_0 which will alter as time passes and more information becomes available about the speed, direction, and character of new urban development. Thus, though the capitalization process theoretically underlies the formation of market price for land awaiting new development, the actual price of land, $P(t)$, cannot be expected to show the smooth upward path that differentiation of the simple present value equation suggests. Particularly, as more information about prospective development patterns becomes known, the price path of land can be expected to shift up or down discontinuously according to how the information affects investors' expectations, and speculative gains or losses will occur.

Since the uncertainty surrounding $V(T_0)$ usually is greater when T_0 is more distant, the discount rate used to arrive at the present value of land awaiting new development may be higher earlier in the period before development, decreasing as T_0 nears. If this is the case, one would expect the land to appreciate more quickly in the earlier stages of the period prior to development, other things the same. Also, uncertainty about the future demand for real estate in an area would lead to a desire for flexibility in improvements. In particular, it may discourage permanency as an aspect of construction in favor of buildings with a lower initial capital outlay and higher maintenance costs so that in case a building is scrapped for redevelopment of the site there will be a smaller capital loss.

Finally, the simplicity of the capitalization process is marred by the many imperfections of the real estate market: land is infrequently transacted, transactions costs are high, there is limited dissemination of transaction prices, land is not homogeneous, and special needs of buyers and sellers enter; all of which will lead to many errors of judgment based on imperfect knowledge of the future.¹⁶ However, the resulting dispersion of market prices about an upward trend is not, in itself, inconsistent with the capitalization process, but is, instead, a reflection of the inevitable uncertainty and the many imperfections of the real estate market.

CONCLUSION

A modified Wicksell model has been used to explain optimal timing decisions concerning the development of land from a lower to a higher urban use. The analysis has shown that, within the context of the assumptions, the optimal date for development or redevelopment of urban land depends on (1) the discount rate applying in the real estate market, (2) the property tax rate, (3) the earnings in any interim use, and (4) the way in which the highest and best use of the land is expected to change in the future. The analysis has also shown that the mere presence of rises in the value of land awaiting development to a higher use should not automatically be interpreted as speculative gains; rather, an increase in the value of such

¹⁶ The earlier mentioned externalities associated with land use are a different sort of imperfection which would lead to nonoptimal allocation even in the absence of other market imperfections.

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