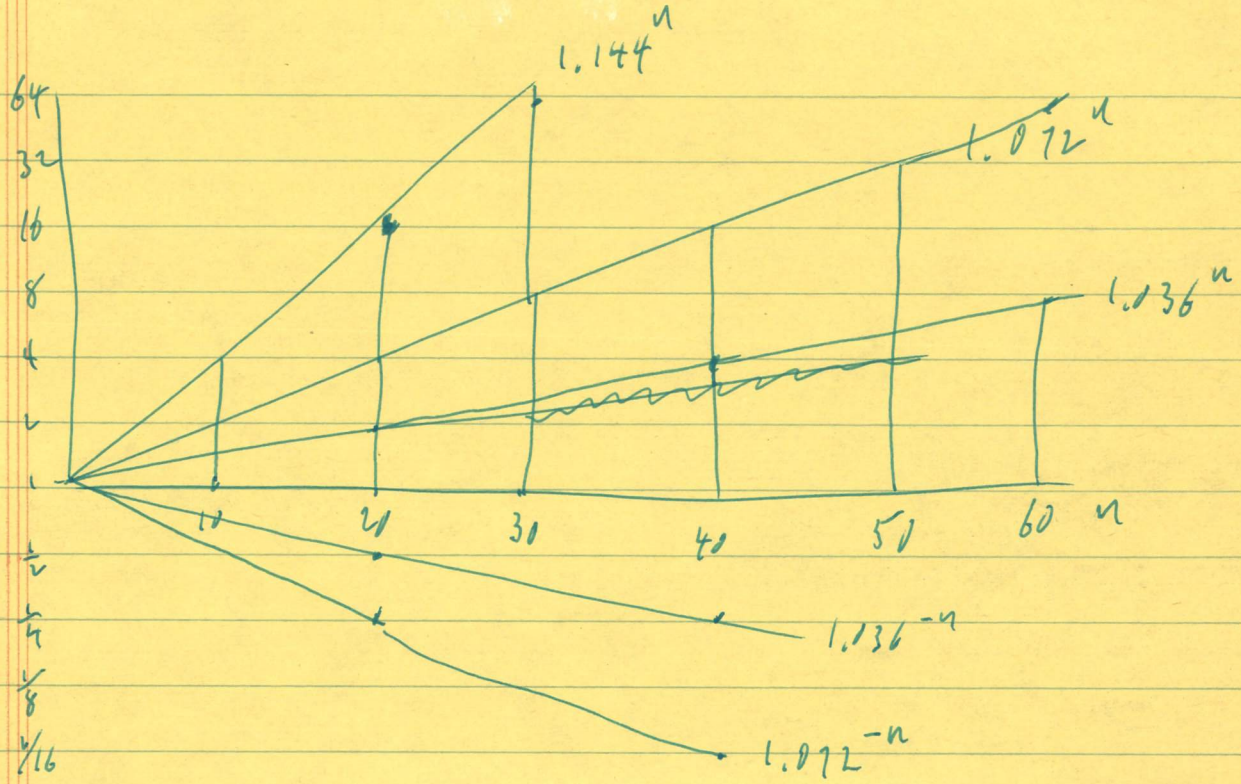
162 USED NOTES S94


1.144^n
1.072^n
1.036^n
64
32
16
8
4
2
1
1/2
1/4
1/8
1/16
10
20
30
40
50
60
n
1.036^-n
1.072^-n

"a 20% annual rate of increase" means

$$\left(\frac{F}{P}\right)^{\frac{1}{n}} - 1 = .20$$

c) Absolute values of P + F don't matter; only their ratio. (They must be commensurable.)

d) If $P = 1$, $\frac{F}{P} = F$

e) Solve a few:

$$5^{\frac{1}{20}} - 1 = 8.38\%$$
$$5^{\frac{1}{40}} - 1 = 4.11\%$$
$$5^{\frac{1}{80}} - 1 = 2.03\%$$

f) Note, $ni = k \cong 1.63 \cong \ln 5$

g) When need we do this, & why?
You must meet a target ROR; when P + F + n are fixed. (unlike Probs. 1, 8, 12, etc.)

h) Solve Probs. { Time 0 = end of year 0 / Time 1 = End of " 1 / Time n = " " " n } = nth year

2, c. p. 1

$$(1.5)^{\frac{1}{1}} - 1 = 1.5 - 1 = .5$$

$$\left(\frac{46}{83}\right)^{\frac{1}{15}} - 1 = .961 - 1 = -.039$$

$$\left(\frac{1563}{128}\right)^{\frac{1}{50}} = 1.0513 - 1 = .0513$$

h) Solve probs (cont.)

4. p. 2

a)

$$2^{\frac{1}{3}} - 1 = .260$$

b) $2^{\frac{1}{36}} - 1 = .0194 < \frac{.26}{12} = .0217$

c) $1.0217^{36} = 2.163 > 2$

$1.0217^{12} = 1.294 > 1.26$

$1.26^{\frac{1}{12}} = 1.0194$

6. p. 2

$$25^{\frac{1}{50}} - 1 = .0665$$

11. $P_0 = \frac{F}{I^n}$; $P_1 = \frac{F}{(I+\Delta i)^n}$

$$\frac{P_0}{P_1} = \frac{F I^{-n}}{F (I+\Delta i)^{-n}} = \frac{(I+\Delta i)^n}{I^n} = \left(\frac{I+\Delta i}{I}\right)^n$$

$$= \left(1 + \frac{\Delta i}{I}\right)^n \cong (1+\Delta i)^n$$

$$\therefore P_1 = P_0 (1+\Delta i)^{-n}$$

13.

$3^{\frac{1}{3}} = 1.442$

$3^{\frac{1}{10}} = 1.116$

$3^{\frac{1}{50}} = 1.022$

3. Return to compounding factor

a. Sensitivity to Δi increases with ~~f~~ futurity (n).

$$\frac{(I+\Delta i)^n}{I^n} = \left(\frac{I+\Delta i}{I}\right)^n = \left(1+\frac{\Delta i}{I}\right)^n \simeq (1+\Delta i)^n$$

E.g. $\frac{1.06^{50}}{1.05^{50}} = \frac{18.42}{11.47} = 1.606$

$$\left(\frac{1.06}{1.05}\right)^{50} = 1.0095^{50} = 1.606$$

$1.01^{50} = 1.645$

b. Effect of Δi is almost independent of I_0.

c. Doubling i has more effect when you start from a higher base.

$\frac{1.072}{1.036} \simeq 1.036$; $\frac{1.144}{1.072} \simeq 1.072$

4. Logs.

$$F = I^n$$

$$\ln F = n \ln I \cong ni$$

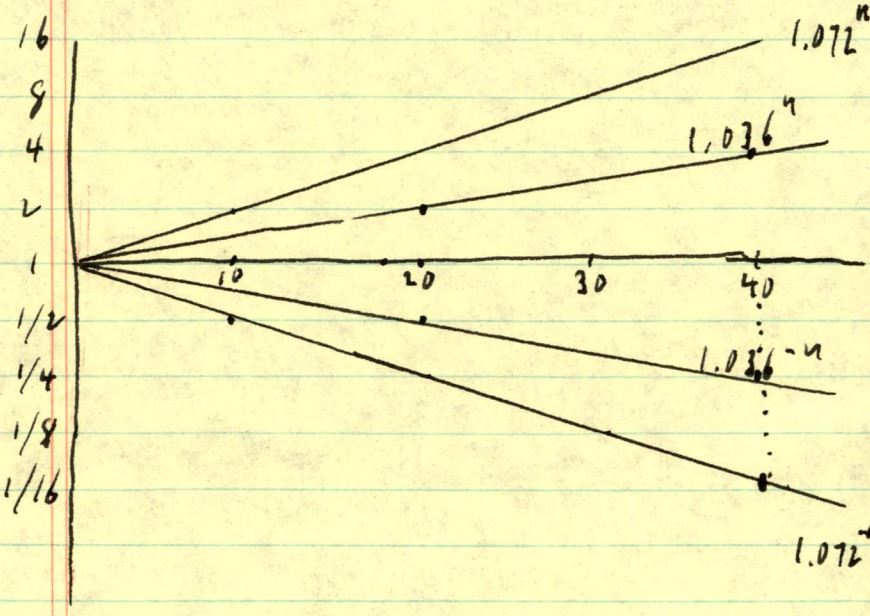
$$\frac{d \ln F}{di} = i \qquad \text{Deriv. of log} = \frac{F'}{F}$$

We already knew that

$$\frac{d \ln F}{di} = n$$

$$\therefore \frac{\frac{dF}{di}}{F} = n$$

$$\frac{dF}{di} = nF$$


16
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1.072^n
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1.036^{-n}
1.072^{-n}