The PIPO Building Block: 1, Compounding
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THE PIPO BUILDING BLOCK: 1, Compounding

Comparing present and future Point Values: the "PIPO" case

A value at a single point in time is a "Point Value," aka a "lump-sum" value, "balloon payment," "single payment value" (spv), salvage value, redemption value, liquidation value, resale value, etc. It is standard, but wrong, practice to call it a "fund" (as opposed to "flow") value, but many point values are not really funds, e.g. land values, and common stock values, as discussed later.

Two such single Point Values together form a "PIPO" pair. PIPO is the acronym for "Point-input, point-output."

Notation:

F = Future Point Value at a time, n, meaning the end of the nth year. If there is any doubt about what year is meant, I add a subscript to F. To avoid clutter, I only do it when needed.

P = Present Point Value at time 0, meaning the start of the first year.

i = rate of interest, annual. (You can use periods other than years, too, and adjust n and i accordingly. Months are commonly used in normal business practice today; for simplicity, I use years in these notes.)

I = 1+i. This seemingly redundant term is used to cut down the clutter in equations which make repeated use of (1+i).

n = number of periods, usually years, between time zero and the time that F is evaluated. n can be a whole number or a fraction.

1. Compounding

$$F = PI^n \qquad (1,A)$$

This is called "compounding." I^n is the "Compounding Factor" (CF). Remember that I=1+i. This is the last time I will remind you, and in some later strings of equations you will see "I" suddenly change, without further explanation, to (1+i), where that helps find a solution.

Compounding is the basic building block of all the math that follows. Be sure you have a firm grasp of it. I'll repeat this shortly.

Compounding looks a little mysterious at first, but it all goes back to simple addition. "Money makes money, and the money money makes makes more money," is how smart old Ben Franklin put it. That's about as compact and concise as language gets. After that, we need a little math notation.

You put $1 in the bank at the start of a year; at the end you have 1+i, or I. Next year-end you have $I+iI = Ix(1+i) = I^2$. Next year-end you have $I^2+iI^2 = I^2x(1+i) = I^3$.

It goes on like that, so in general, at the end of any year, n, you have I^n dollars. Again, compounding is the basic building block of all the math that follows. Be sure you have a firm grasp of it.

Firmly to fix your understanding, make a table with three columns. Column 1 is n. Column 2 shows what you have at the start of each year; column 3 at the end. I could do this for you, but you learn by doing, so do.

I habitually do this from top to bottom. You could do it either way, as the spirit moves, but do it my way to facilitate comparing data.

First, do 4-5 rows algebraically. Next, since you now understand what you are doing, do it again numerically, using $.03<i<.15$. Extend column 2 down in ten-year jumps to about n=50. Plot the points on a graph, and join them with a smooth curve. This is a curve of "exponential growth" (since the exponent, n, is the independent variable). It should look like Bob Hope's ski-slope nose, with Bob facing to your left.

The Rule of 72.

The Rule of 72 is used to estimate doubling times. It is a rule of thumb, not 100% accurate, but very useful for two things: a) quick and dirty estimates;[1] b) getting a better feel for and

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[1]The rule is most accurate when i=.08. When i>>.08, the rule gives too high a value for n. When i<.08, the rule starts giving too low a value for n. (So? check that again. What about when

understanding of exponential growth. (Each doubling is a growth of 100%.)

The rule comes from the fact that LN2 = .69. The .69 is raised to .72 to allow for the error in the approximation setting LNI=i.[2]

To estimate the doubling time, divide .72 by the interest rate, expressing the interest rate as a decimal; or divide 72 by the rate expressed as a percentage. Thus, at 3% money doubles in 24 years; at 6% in 12 years; at 9% in 8 years; at 12% in 6 years; and so on.

It is useful to have on the tip of your tongue that money doubles every 10 years at 7.2%. It follows from that that in 60 years each original dollar has doubled 6 times, and grown to $\$2^6 = \$64$. It's enough to make a young person start an early savings program. It only works, however, if you faithfully plow back <u>all</u> the interest, never withdrawing a bit for that new Toyota pickup, wedding bash, or <u>anything</u>.

We will be showing how to solve (1,A) for elements other than F. First, however, we will see how to use semi-log scales to get a firm visual grasp of (1,A).

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i=0?)

[2]In fact, lnI is a shade lower than i. If we used continuous interest, it would be a rule of 69, and would be 100% accurate. However, we use annual interest rates, so it is a rule of 72, and tolerably accurate for economic and managerial purposes, where we are dealing with a narrow range of interest rates, and pretty fuzzy data most of the time anyway.

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THE PIPO BUILDING BLOCK: 2, Solving for any element

Discounting

Sometimes you know F, i and n, and want to find P. Solve (1,A) for P.

$$P = FI^{-n} \qquad (1,B)$$

Eqn. 1,B is called "discounting"; I^{-n} is the discount factor. It is the opposite of compounding. Since compounding is adding interest to P, discounting has to mean subtracting interest from F.

Mathematically, to be sure, it involves division, but this is just a short cut. Logically it is a matter of subtracting: P may be viewed as F net of the interest cost of getting from now to then.[1] Test this by discounting one year at a time, e.g. from F_1 back to P_o. In this case, P=F/(1+i), so P+Pi=F, and P=F-Pi. You have valued P by subtracting Pi from F. In fact, you have done both in one stroke: you had to know what P is in order to subtract Pi from F.

This kind of thing used to cause dizziness and vertigo until they invented math, which makes it all so easy. Trouble is, it makes it so easy you are apt to overlook the wonder of what you are doing, and thus not really to understand it.

When F is the deferred product of some other inputs, P is called their "discounted product," meaning their product net of the interest costs of holding the product to maturity. There is also a discounted marginal product. This is a handy way to impute to a current input like labor its worth to the employer. Cf. section on "Imputation."

Discounting is used to put present values on future sums. **It is the basis of valuation of all durable wealth.** However, most durable wealth consists of assets that yield future flow values, or periodic pulses of value, rather than single point values: therefore more complex formulae are needed. You will soon learn them. Remember, though, they are all based on and constructed from this one basic building block.

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[1]There are other time-costs too, like ground rent, dealt with later in (16,A to 16,F).

Make a table with three columns. Column 1 shows n, starting with n=1. Column 2 shows the discount factor. Column 3 is cumulative: add each discount factor to the sum of those before it. Use a high interest rate, about .13 or more, so you won't need to work out a lot of sums to see the shape of the curve.

Plot the points from Column 2. This is a curve of "exponential decay." This time Bob Hope is facing to his left (your right), but it may be Cyrano de Bergerac, or perhaps Michael Doonesbury, because the slope is distinctively different. It starts out slower, and flattens out more.

Shade in the first five columns on your graph. The shaded area is the sum of five discounted values: this sum is the value of an asset that yields five payments in successive years. Save this graph to review when we do the Discounted Cash Flow Factor, coming up soon.

Plot the points from Column 3. This is a "cume curve," (cume for "cumulative"). If your integral calculus is operational, you'll recognize it as the integral of Column 2. If it isn't, no matter, "cume curve" is self-explanatory. It shows how the present value grows with the life or durability of an asset. Note that it grows, but at a decreasing rate. Put this aside for the moment: we use and develop it extensively soon enough.

Rate of Return

Sometimes you know P, F, and n, and want to know i.

$$I = (F/P)^{1/n};$$

Remember that I is short for 1+i.

Therefore,

$$i = (F/P)^{1/n} - 1 \qquad (1,C)$$

"1/n" is the reciprocal or inverse of n. As an exponent it means the nth root. Do not be intimidated: that's why you have a calculator. With this wonder-working little device, you can take the 53d root as easily as you can add and subtract. Practice a little so you can do it smoothly on an exam: you'll need to.

When you find i like this, as the dependent variable, it is called the "rate of return" (ROR).[2] One also sees IRR (internal rate of return); and RRBT and RRAT, for rates of return before taxes and after taxes. In algebra, I generally use r for ROR when it is the dependent variable, as in (1,C).

ROR is more general than (1,C), which is just the simplest form of ROR, when an investment yields its return only in one lump sum. In general whenever you solve a financial equation for i the result is called ROR. Usually the data are more complex than in (1,C).

Period of time

Sometimes you know everything but n. Here we need to use logarithms. Don't panic, it is time you learned how simple that is with a modern calculator in your hand.[3]

"Lnx" means "the natural logarithm of x." It is usually written lower case, but I use Upper Case to make equations easier to read: wouldn't want you confusing ln for 1 x n, for example.

Solving for n:

$$nLnI = LnF - LnP \qquad (1,D)$$

To simplify, set P=1.[4] Then LnP=0, and:

$$n = LnF/LnI \qquad (1,E)$$

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[2]Keynes called this the "marginal efficiency of capital," and it is so designated in many standard macro texts and writings. Wicksell called it the "natural rate of interest," which many standard macro texts and writings also discuss. (Economists have a weakness for using different words to mean the same thing, [and also, alas, sometimes the same word to mean different things]).

[3]I was raised on long log tables that were still too short to give accurate answers without "interpolating," which is an invitation to errors; and on the technical marvel of the slide rule, with which you interpolated by eye, and guessed out to three significant figures on the left side of the rule, and two if you had to work on the right side. It helped to be myopic.

[4]When P is not equal to one, use the ratio of F/P instead of F in (1,E and 1,F). This has the same effect.

Simplifying more, it can be shown that LnI nearly equals i. For a proof, see the section on Napier. Meantime, check it out on your calculator. You'll see that LnI is slightly lower than i. LnI is called the "force of interest." It is the rate that corresponds to i when compounding is continuous. In practice hardly anyone uses continuous compounding in business, except occasionally as a sales gimmick. Monthly compounding has nearly the same quantitative effect as continuous.[5]

So finally we have:

$$n = \text{LnF}/i \qquad (1,F)$$

You can also use (1,F) to solve for i, when you know F/P and n. We already solved for i by taking the nth root of F/P (1,C). Dividing a log by n is another way of taking a root.[6] I mention this only in passing, for the record, for your interest, and for your better understanding. In this course we will use the method of (1,C) to find i, and the method of (1,F) to find n.

"LnF" is still a mental block for many, and understandably so: we don't use them every day. This is where the method of tabulation is useful. Let's just have a look at some numbers, to get an idea where they fall.

Table 1: Values of F, and corresponding logs

F:	1.0	1.5	2.0	3.0	4.0	8.0	16.0	32.0
LnF:	.0	.41	.69	1.10	1.39	2.08	2.77	3.47

Play with the "Ln" button on your hand calculator until you have a feel for logarithms. Don't mess with the "LOG" button, which gives logs to the base 10: we don't use that in this course.

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[5]Continuous compounding is still very useful to simplify derivations, resolve conundrums, and give added insight: we will be using it later.

[6]In the ancient years B.C. (Before Calculators), taking roots was one of the main uses of logs.