

Capital and 'Reswitching'
By Dr. Mason Gaffney

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Notes by M. Gaffney, Re Capital and "Reswitching."

For discussion and dialogue, Monday P.M., 3:30, January 29, 1979.

The thesis here is that the reswitching theorists have produced apparent paradoxes and anomalies about the effect of interest rates on capital intensity by erroneously identifying economic life of capital with carcass life of physical objects of capital.

These notes undertake to give an economic definition to economic life (or "period of production"), develop tools for measuring it, and for measuring factor proportions and capital intensity. Using these tools we can then evaluate the examples of reswitching paradoxes.

I. Intro. Pooling and the period of production.

An inventory of capital is analogous to a tub of water with an inlet and an outlet. How long does the average molecule of water stay in the pool, as water flows in and out, and inflow = outflow?

If daily flow = stock in pool, then daily flow could just replace each original molecule, daily, so the period in storage = one day. Double the flow and the period = $\frac{1}{2}$ day, etc. In general, this leads to:

$$\text{Period} = \frac{\text{Stock}}{\text{Flow}} \tag{1}$$

(1) is a recognized principle of fluid mechanics, and I postulate it as self-evident, demonstrable, and known.

An allied concept is Turnover, the reciprocal of Period. When the period is $\frac{1}{2}$ day, the water changes twice a day, etc.

$$\text{Turnover} = \frac{\text{Flow}}{\text{Stock}} \tag{2}$$

Water is fungible and miscible. It is pooled, and the molecules are not labelled or dated. Some may tarry in stagnant backwaters or eddies, while others rush through a main stream. We are calculating the mean period for the whole, averaging together the fast and the slow, which we cannot observe individually.

Common examples of turnover in economics are:

-- Money. $$\text{Velocity (turnover)} = \frac{\text{Flow of spending}}{\text{Stock of money}}$$

-- Inventories. $$\text{Turnover} = \frac{\text{Sales throughput}}{\text{Stock}}$$

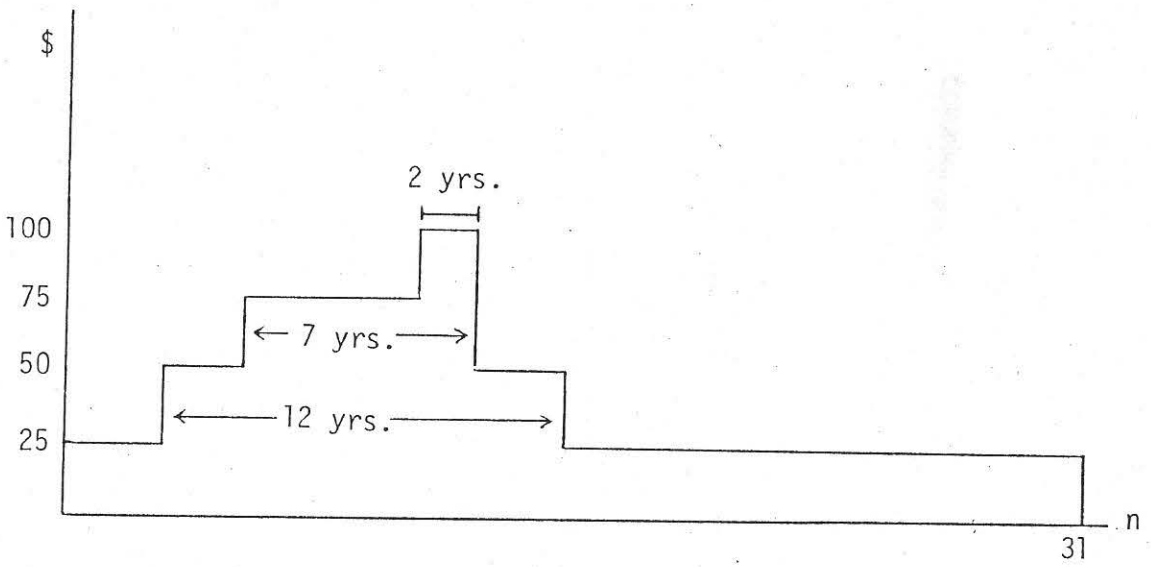
Those are simple examples. Here we will apply the concept to durable capital whose "molecules" are not so obviously fungible as those of water, cash, or inventories; and which complicate things by changing in value as we hold them in stock.

II. The average period of economic life when the stock varies.

Our pool of water may rise and fall. We can still compare flow with a mean value of stock, and find a period and frequency.

We can also transpose this technique to the typical investment cycle of large units of durable capital -- buildings, machines, timber, roads, lines, etc. We invest capital in one or several large pulses, and recover it in one or many later flows of cash and services. The volume of unrecovered capital in the corpus rises and falls, from zero back down to zero again.

So we simply match up each input with an output in some systematic way, and calculate a mean period. Figure 1 shows this schematically.



The carcass-life here is 31 years, but the mean economic life of the $100 capital which is invested at maximum value is much less. If $100 were invested for 31 years there would be 3100 "dollar-years" of capital tied up. But only $25 is in for the full 31-year term (bottom row of graph). The next row up is in for 12 years only, and so on. Summing up:

$	Years	=	$ - yrs.
25	31		775
25	12		300
25	7		175
25	2		50
			1300

(It is often more convenient to add the columns instead of the rows, when our data give us the value of unrecovered capital each year. In either case we get the area under the curve, the number of dollar-years. Cash flow analysis, (below, V), gives us the column values.)

The mean period of the $100 of capital which we used at the peak is the same as though we had used $100 for 13 years, i.e. 1300 dollar-years. More generally:

$$\text{Mean Period, } P = \frac{\text{Sum of dollar-years}}{\text{Sum of Investments}} \qquad (3)$$

We can also arrive there by figuring the 1300 dollar years is the same as holding a constant stock of $\frac{\$1300}{31}$ or $41.94 for 31 years. Throughput is $100, so turnover is $\frac{100}{41.94}$ or 2.38 in 31 years, which is $\frac{2.38}{31}$ or .07677 per year, which is $\frac{1}{13}$, the reciprocal of the mean period. Generally,

$$\text{Turnover} = \frac{\text{Throughput per year}}{\text{Mean stock}} = \frac{1}{p} \qquad (4)$$

III. Capital, Interest and Turnover.

So far we have omitted the yield and the income of capital per se. It was provided free, and there was no increase during storage. Now we deal with those matters.

To simplify, assume the land in use is marginal and free; and supplies and materials are negligible and cheap. So we are just dealing with labor producing capital which is then recovered over time, with interest.

What is capital? Anything of value produced by man (in conjunction with land and capital under his management) and not yet fully consumed. This also means the original costs are not fully recovered, as a rule. [Owing to imperfect foresight this rule is seldom met exactly, and the treatment of changes of data in mid-cycle needs addressing in a complete system, but here we skip over the question.]

Who supplies capital?

a) Whoever consumes less than his income creates capital currently. This includes workers who build up pension funds (if no one raids them).

b) Whoever devotes existing stocks to pay current factors to produce new ones. These stocks include part of the service flow of non-expendable capital like buildings -- i.e. one may devote the amortization funds from an apartment building to finance a new building, or anything else. These amortization flows represent recovery of existing capital, not income or new capital.

Capital is productive -- it yields an increase with time. Timber and livestock are obvious concrete examples, where the corpus literally grows. Other capital, non-living, also yields an increase, and has an existence separate from that of the labor that started it. We can discuss this if anyone thinks examples are needed.

We know that capital is productive because a given labor input produces more when conjoined with more capital.

What is "capital input"? It is the <u>use</u> of capital over time (as opposed to impairment of substance), just as the labor input is labor-time and the land input is land-time. Capital, like income, consists of many concrete things which are conventionally aggregated and measured in money, with all the problems that may entail (which are not treated here). Capital has the added quality of turning over regularly, and changing concrete forms, so quantification poses fascinating speculations which bemuse and divert philosophers. But we always have an investment cycle where current factors are hired and paid off by an investor who finances the wait between investment and recovery, and commands a payment for this.*

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*The payment is rival to wages, but also to the rent of land when rent is paid in advance, or is reduced by the payments made to command capital. I mention the landowners because it is often thought that interest is extracted from labor alone, due to political power or class relations. But landowners, of high political power, find interest cutting into their rent, too. They put up with this because they need the capital

We here focus on the investment cycle, where payments to hired factors, and receipts from sales are both measured in money. We short-circuit, without prejudging other aspects of the "capital controversy."

Because capital has the job of financing current inputs, including the use of capital itself sunk previously, there is a plowback of unrealized income into capital stock each year that capital remains sunk and unrecovered. This cumulates over time until recovered, with a compounding effect, as follows (Fig. 2).

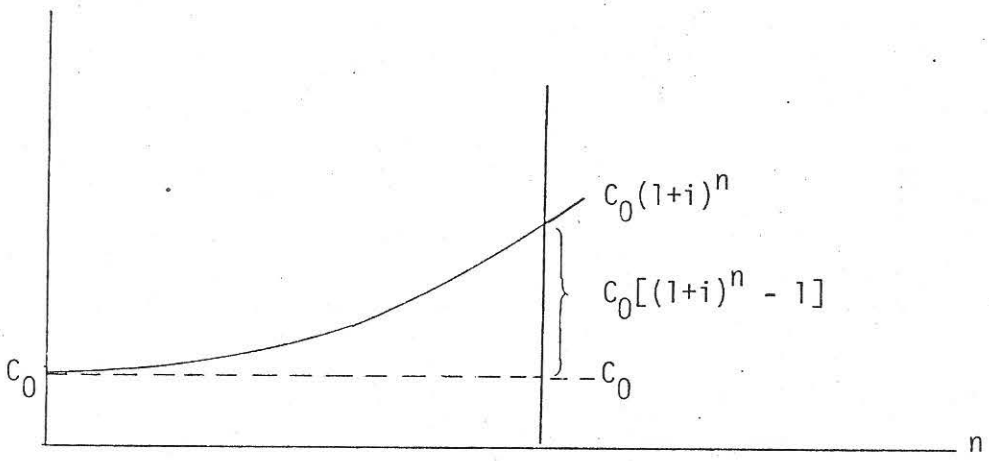


Figure 2

C_0 is the initial labor cost, assumed to be one-shot only. But there follows a capital input each year that capital is unrecovered. The payment is sunk but unrecovered which adds to the capital tied up in service, so the following year there is interest on this, too. (If you have trouble with implicit payments, just picture capital represented by cumulative debt where the investment is 100% financed from outside.)

The investment is liquidated in year n. The labor input remains fixed at C_0. The capital input has grown to $C_0[(1+i)^n - 1]$. This is less than the total stock, which is the whole bundle, $C_0(1+i)^n$.

Now we can redraw Fig. 1, showing capital input added to labor input. In Fig. 3, the vertical risers are successive labor inputs. The intervening curves of exponential growth are smoothed curves showing capital inputs.

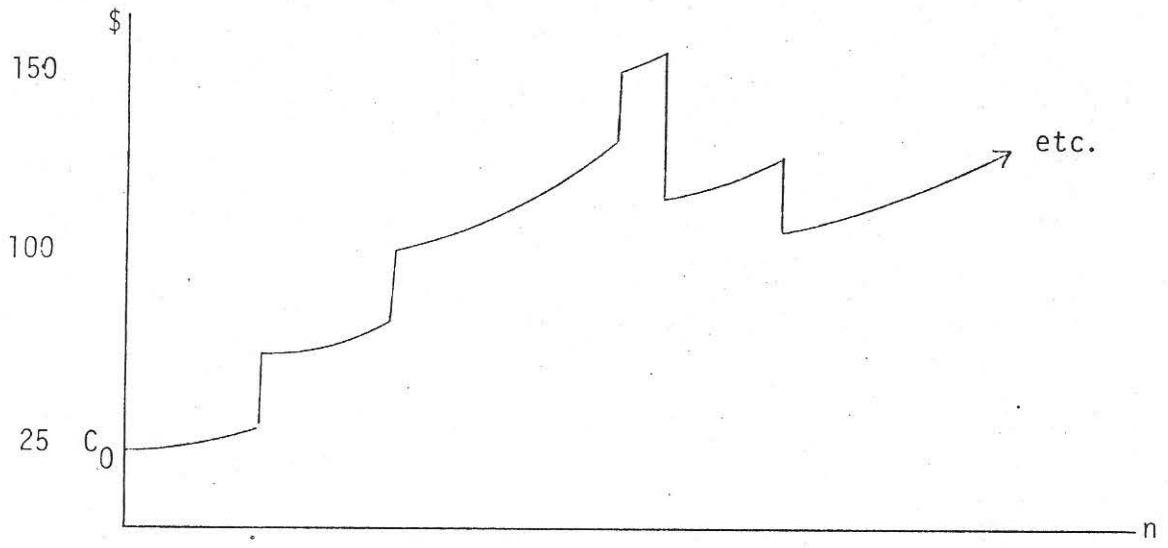


Figure 3

Fig. 3 treats capital inputs the same as labor inputs of the same date. The difference between the discrete jumps showing labor input and the smooth growth curves showing capital input is just for our visual convenience, to distinguish them. It also reflects the fact that a capital input goes on continuously, so long as there is any capital unconsumed and unrecovered.

Where there is only one pulse of labor input, C_0, and one liquidation at n, as with timber, we can formularize the period and frequency of capital readily. Refer to Fig. 2, and relate the next paragraphs to it.

Total flow of throughput and receipts is $C_0(1+i)^n$.

The total number of dollar-years of capital used is the area under the curve; thus:

$$\text{Stock, } S = \sum_{0}^{n} \left[C_0(1+i)^n\right] = C_0 \frac{(1+i)^n - 1}{i} \tag{5}$$

$$\text{Period} = \frac{\text{Stock}}{\text{Flow}} = \frac{1 - (1+i)^{-n}}{i} \tag{6}$$

Note that:

a) (6) shows the period is less than n. This has to be, since only part of the peak capital is in use throughout carcass life (n).

b) (6) is an increasing function of n, but at a decreasing rate. As n becomes large the ratio approaches an asymptotic limit, $\frac{1}{i}$. There is an upper bound on the period of production (but not on capital-intensity, as we will see).

Turnover or frequency, of course, is the reciprocal of period (6):

$$\text{Turnover} = \frac{\text{Flow}}{\text{Stock}} = \frac{i}{1 - (1+i)^{-n}} \tag{7}$$

c) Note last, that C_0 cancels out and does not affect period or frequency. That means that the absolute size of the wage bill is not a factor. It only affects scale, not periods. We will see that it does not affect factor proportions, either.

IV. Capital-intensity and the period of production.

The measure of capital-intensity is the share of the outflow of ripe goods going to capital (at a given i).* Capital-intensity is always an increasing function of n, time.

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*We do not treat the determination of i, or how to interpret changes of i.

Note that we are looking at proportion of factors in investment cycles where the factors are vertically integrated through time. We are comparing the income of capital with the wages of labor who built it, not those who operate it (and produce more goods with their own life cycles and financing costs). We can analyze the whole economy vertically this way. Alternatively, we could take a horizontal cross-section at any fixed date. And we can easily show that longer investment cycles require there be more capital per worker at any fixed date. But beware! What one can not do is wander carelessly back and forth between a vertically integrated framework like the one used here, and a horizontal cross-section. The transition must be carefully executed, lest seeming paradoxes and contradictions be generated by unsystematic, inconsistent procedure.

Hewing to the vertically integrated framework, the share of capital is:

$$\frac{\text{Cumulative Interest}}{\text{Flow of Output}} = \frac{C_0[(1+i)^n - 1]}{C_0(1+i)^n} = 1 - (1+i)^{-n} \qquad (8)$$

As before, C_0 cancels out. The share of capital is simply an increasing function of the investment cycle, n. It equals the mean period of investment times the interest rate (cf. (6)). As the mean period approaches $\frac{1}{i}$, capital comes to resemble land which is neither produced nor replaced, and the share of capital approaches 100%.

The share of labor is of course the complement, $(1+i)^{-n}$.

The relations that hold in the basic, simple case must also hold in complex cases because the simple case is the mathematical building block from which complex cases are derived by summation. Contradictions can arise mainly from errors in summation, or other blundering. A good systematic framework for analysis is the "cash-flow account," treated next.

V. Cash-flow accounts

A common way to keep track of sunk capital is the cash-flow account, so-called because depreciation is not handled by formula but (implicitly) by capital recovery; and the demand for cash is clearly shown for every year. It is a tool used by corporate headquarters which dole out cash for keeping tabs on divisions which operate with cash. It is disarmingly simple.

It proceeds through a sequence of years, the life of an "investment" (which may be a small unit like a fork-lift, or a large complex like a shopping center). We begin with the first capital outlay. Calculate current cash flow (receipts less expenses) and subtract it.* Charge a year's interest on the balance; add this to the balance.** Carry forward to next year and repeat, until end of life. (We omit determination of end of life under all conditions, but take the simplest normal case where the operation ceases when all capital is recovered.)

This procedure leaves a record of cash sunk, at each year-end. Each of these values is a column on Fig. 3. Summing them, we get the area under the curve, the number of dollar-years of capital used.

There is also a record of the total positive cash flow. Combining these two, we have period and frequency.

$$\text{Mean period} = \frac{\Sigma(\$\text{-yrs.})}{\Sigma \text{ Inputs}} \tag{9}$$

$$\text{Turnover or frequency} = \frac{\Sigma \text{ Inputs}}{\Sigma(\$\text{-yrs.})} \tag{10}$$

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*For negative cash flows, see below.

**To simplify, assume here all payments are concentrated on January 1. A finer account goes monthly, weekly, or even continuously.

The account also leaves a record of interest charged each year. Summing the interest flow, we have capital's yield and income.

The share of capital is:

$$\text{Share of capital} = \frac{\Sigma \text{ Interest Charges}}{\Sigma \text{ Positive Cash Flows}} \qquad (11)$$

The share of labor is:

$$\text{Share of labor} = \frac{\Sigma \text{ Capital Outlays}}{\Sigma \text{ Positive Cash Flows}} \qquad (12)$$

It may be puzzling to have "capital outlays" imputed to labor, but remember what is happening in each such outlay is labor is paid off, and the product carried then forward by the capitalist.

We use "cash flow" as a measure of output in order to subtract out associated costs. Cash flow is the residual that is imputable to the specific capital we choose to analyze in the account. It is understood that "expenses" are not used to build up inventory, but are strictly operating expenses. Inventory buildup calls for another capital account (which could be consolidated with the present one). We here assume no inventory buildup.

Negative cash flows are common in the early life of large investments. They are just like another capital outlay, and are simply folded in as such. This calls for an adjustment when summing cash flows to find total output: only add the positive flows. The negative ones are inputs.

Refer to Table 1.

TABLE 1: A Cash-Flow Account

1	2	3	4	5	6	7	8	9
			Current Cash Flow					
Year	K_{n-1} $(9)_{n-1}$	New K	Receipts	Expenses	Net (4)-(5)	K in Use Year n (2)-(3)-(6)	Interest Plowback	ΣK_n (7)+(8)
0	---	100	2	12	-10	110	11	121
1	121	50	20	10	10	161	16.10	177.10
2	177.10	25	23	9	14	188.10	18.81	206.91
3	206.91	--	96	26	70	136.91	13.69	150.60
4	150.60	--	130	30	100	50.60	5.06	55.66
5	55.66	--	75.66	20	55.66	---	---	---
Totals	711.27	175			239.66	646.61	64.66	711.27

$$\text{Mean Period} = \frac{\Sigma \text{ K in Use}}{\Sigma \text{ Investments}} = \frac{\Sigma(7)}{\Sigma(3) + \Sigma(8) + 10} = \frac{646.61}{249.66} = 2.59 \text{ years} \qquad (13)$$

$$\text{Turnover} = \frac{\Sigma \text{ Cash Flow}}{\Sigma \text{ K in Use}} = \frac{\Sigma(16) + 10}{\Sigma(17)} = \frac{249.66}{646.61} = .386 \qquad (14)$$

$$\text{Share of Capital} = \frac{\Sigma(8)}{\Sigma(6) + 10} = \frac{64.66}{249.66} = .259 \qquad (15)$$

$$\text{Share of Labor} = \frac{\Sigma(3) + 10}{\Sigma(6) + 10} = .741 \qquad (16)$$

VI. Evaluating Hunt's examples of reswitching

To be discussed Monday.

VI. An alleged "reswitching" paradox, and the solution.

A numerical case based on E. K. Hunt, pp. 406-07.

Hunt compares two "techniques," A and B. They are (by implication) mutually exclusive, or rival; they produce the same output; and they yield this output only at the end of the carcass life -- i.e., the capital is all tied up until the last year.

They differ as follows:

Carcass-life: A is shorter.

Labor input: A uses less.

Timing of labor input: A - labor goes in all at t_0.

B - labor goes in mostly near end of life, but partly at t_0.

At a high rate of interest, A is preferred, because of lower total costs (labor plus compound interest).

Hunt now lets the interest rate fall sequentially, and finds that there is a lower level where B is preferred, but a still lower level where A is once again preferred. This is "Reswitching."

Hunt gives no examples, and leaves many assumptions implicit. We can fill in most of these, as above, and see what he is driving at. I have constructed a numerical example below that exemplifies his paradox.

One question he leaves hanging, however, both explicitly and implicitly. He is comparing a short-lived technique with a longer-lived one, but there is no clue as to what happens at the end of A's life. If (as in our example, below) B's life is 5 times A's life, then we should compare 5 cycles of A with one of B, rather than one on one. This problem arises whenever we need to define the annual value of land and compare two different use plans, as in forestry (whence Hunt's examples sprung, as he is here discussing Böhm-Bawerk et al.).

The solution is to convert lump-sum values to annual equivalents. B uses more time than A on the land, and if the land has an annual value, B needs to pay for this, with interest.* If b is the yearly rent from use B, then the cumulated value, B, of this rent after 50 years is:

$$B = b \frac{e^{50i} - 1}{i} \qquad (1)$$

Conversely, if we know B and want to find b,

$$b = B \frac{i}{e^{50i} - 1} \qquad (2)$$

(2) is called the "Sinking Fund formula," a standard tabulated parameter of finance.

In Hunt's case, we sum up all costs with compound interest at end of life, to find his "lower cost" technique. But then we need two more steps to find the economically preferred technique:

1. Subtract costs from revenues
2. Annualize this net figure, using (2)

Doing so, the reswitching paradox is solved rationally.**

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*Hunt's example does not let us assume land is marginal, as we did before, Because if a technique is viable at all at a high interest rate, it must yield rent at lower rates.

**Whether this supports or amends a mythical monolithic body of doctrine called "neoclassic theory" is a misleading question. We should first be concerned with whether something makes sense. If it does, and authorities have been overlooking it, the consequences of your promulgating it will be satisfyingly "radical." Many writers who call themselves neo-calssical are very weak on handling rent, or even admitting it exists. This may be why they have stumbled so badly on "reswitching."

I deplore the attitude expressed by some radical writers that mathematics is a weapon of "the other side." Math is everyone's tool, powerful and internally consistent. It can be abused, but pedants and high priests always make mysteries to confuse the issues -- if not math, they will find something else.

Following Hunt, we assume a case as follows.

	n	C_0	C_{49}	R_n
A	10	50	--	100
B	50	3	48	100

Now we tabulate the sum of costs with interest, and net revenues, and annualize net revenues at 6%, 3% and 0%.

i		e^{ni}	C_0e^{ni}	$C_{49}e^{i}$	$\sum_{0}^{n}\left[C_t e^{(n-t)i}\right]$	100-(5)	$\frac{i}{e^{in}-1}$	Annual Rent
(1)		(2)	(3)	(4)	(3) + (4) (5)	(6)	(7)	(6) x (7) (8)
.06	A	1.82	91.0	---	91.0	9.0	.0729	.657
	B	20.09	60.3	51	111.3	-11.3	.0031	-.034
.03	A	1.35	67.5	---	67.5	32.5	.0857	2.79
	B	4.48	13.4	49.5	62.9	37.1	.0086	.32
.00	A	1	50	---	50	50	.10	5
	B	1	3	48	51	49	.02	1

Column 5 is Hunt's case. Technique A is lower cost at 6%, higher cost at 3%, and again lower cost at 0%. But when we annualize, Column 8, there is no such effect. Lower interest rates make B look better and better, but never as good as A.

One could make examples where there is switching to B, as rates fall; but not reswitching. Try it. Go ahead prove me wrong if you can. Set up an example where the rent of B starts lower than A, but surpasses it as i falls. You can't make it reswitch. I guarantee it. But why should you believe me? Try it yourself.

There is another kind of "reswitching" that occurs. As cycles get longer, the share of rent first rises and then falls. But it rises by the fall of labor's share; and falls by the rise of capital's share. But capital's share always rises with length of cycle (n).

Hunt raises also the question of which technique is more labor-intensive. He stresses that B uses more labor. But again, when we make a fair comparison it is 5 cycles of A for one of B, and A uses much more labor. Next time, if there is a next time, we can walk through a cash flow analysis and calculate factor shares. When we do, we shall have a new kind of capital input to incorporate, that is land-derived (or "geogenic") capital. Because the unrealized rent of early years is plowed back, just like unrealized interest, and becomes part of capital stock.* We will handle this by adding the price of land to inputs at t_0; and withdrawing the same amount as salvage value of land at t_n.

But this is enough for today.

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*E.g. when you start a new stand of citrus, 5 years of land service is tied up in the trees before they bear. With jojoba, they're saying it may be 10.

This is how the factor which Hunt omits varies.

Values of Sinking Fund formula, $\frac{i}{e^{in} - 1}$

i =	.06	.04	.03	.02	0*
n					
0					
1	.97	.98	.985	.99	1.0
2	.47	.48	.485	.49	.5
3	.30	.314	.319	.323	.33
4	.22	.23	.235	.24	.25
5	.17	.18	.185	.19	.20
-					
10	.073	.081	.086	.09	.10
-					
20	.026	.033	.036	.041	.05
-					
30	.012	.017	.021	.024	.033
-					
40	.00599	.010	.013	.016	.025
-					
50	.00314	.0063	.0086	.012	.020

*The last column is found by using l'Hôpital's Rule for evaluating $\frac{0}{0}$, and equals $\frac{1}{n}$.