

No.

T M JAW - First Draft

APPENDIX I

Share of Labor in claim on total output as decreasing function of life of capital: identity of single-cycle and going concern models.

A. Flowing (depreciating) capital.

The text shows (Fig.1, Table 4, pp. 29-30) that the share of labor in revenues from the service flow of a building or machine (flowing capital) is a decreasing function of life. Here we show the rule and the mathematics are exactly the same in a "going concern" model when an investment in machines is normalized or staggered.

There is one machine of every age. We build or buy one new machine each year, and scrap another one. The number of extant machines is n , where n is the number of years each machine lasts. As in Eqn. (1), p. 29, each one yields a flow of \$1 per year, so the yearly flow is \$ n .

The yearly labor input is no more than the warranted investment (I) in one machine when new. Dividing this by the yearly service flow (n) we arrive at the same result as when we analyze one machine over life, that is labor's share, is

$$\frac{I}{n} = \frac{1-e^{-in}}{ni} \quad [\text{same as (2), p. 29}] \quad (2)$$

The share going to capital as interest must be the complement of (2), of course. To check this, let us derive it directly.

The capital value remaining unrecovered in each machine at the start of every year, y , is the present value, (V_y) of future cash flows from that time until scrapping.

$$V_y = \frac{1-e^{-i(n-y)}}{i} \quad (3)$$

Total unrecovered capital (K_y) is:

$$K_y = \sum_{y=0}^n V_y = \frac{1-e^{-in}}{i} + \frac{1-e^{-i(n-1)}}{i} + \frac{1-e^{-i(n-2)}}{i} + \frac{1-e^{-i(n-n)}}{i} =$$

$$\frac{1}{i} \left[n-e^{-in} \sum_{y=0}^n e^{iy} \right] = \frac{1}{i} \left[n- \frac{1-e^{-in}}{i} \right] \quad (4)$$

Interest on K_Y as a share of total output flow is:

$$\frac{i K_Y}{n} = 1 - \frac{1-e^{-in}}{in} \quad (5)$$

Q. E. D.

In the going concern, capital recovery is the excess of cash flow (n) above interest cost (Ki). Rearranging (5)

$$n - Ki = \frac{1-e^{-in}}{i} \quad (5A)$$

But the right side of (5A) is I , the warranted investment to build a new machine. So capital recovery from all the machines together is just enough to finance one new machine each year.

Now we add land to the inputs. The capital is a building on a good site. The yearly service flow is now \$1 plus \$a, where a is the annual value of the site. Fig. 4 shows the share of land:

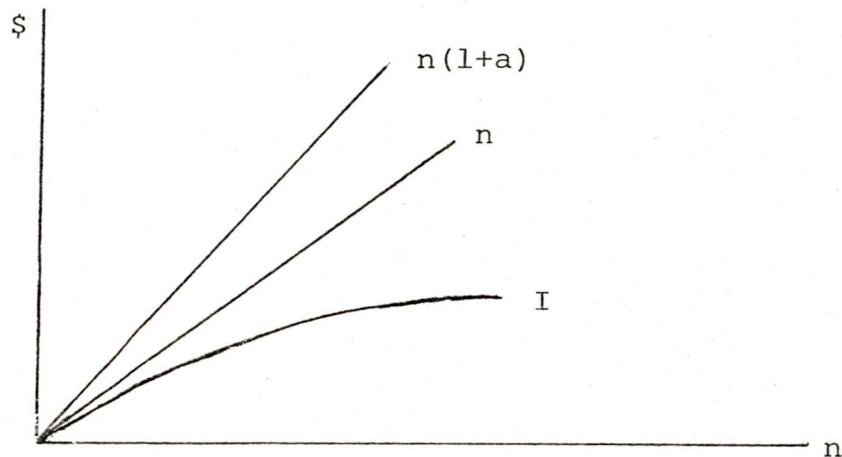


Figure 4: Division of value of service flow of n houses of evenly staggered ages among warranted investment (I), rent (a) and interest.

$\bar{K}$ remains the same as before. So does $\bar{I}$. Only service flow is greater. The shares of labor and capital are accordingly smaller.

The share of land is

$$\frac{na}{n(1+a)} = \frac{a}{1+a} \quad (6)$$

(6) is not a function of n , i.e. is unaffected by life, (under the present simplifying assumption of level service flow.) Accordingly, the share of land rises relative to labor, and falls relative to capital, with increasing life of capital.

The last point seems a little surprising because in the last years of an old building the cash flow covers little more than land rent. In part the surprising result comes from my artificial simplifying assumption of level cash flow over "life". In fact, there is a geriatric terminal life of old buildings, after the service life is over, when the carcass survives and yields just enough cash flow to give a return on the site. When we include this terminal period in "life", the share of land rises with life, and moreso when site value rises over time.

But that is not the whole story. In the main the surprising result represents a real phenomenon: to build for long sustained service flow of capital calls for heavy inputs of capital from the beginning of life, inputs which must be paid from the beginning. Note we are not here treating what happens from year to year to the individual building as it ages and the unrecovered capital approaches zero. Of course capital's share then also approaches zero. We are treating of shares over the whole life of buildings, and, what amounts to the same thing, shares generated from a normalized collection of buildings of every age.

The share of labor falls with increasing life. But land tempers this effect by claiming a share of the product even when life is short, like the life of corn plants in rich Champaign County, Illinois; or when there is little capital, as on a parking lot. Appendix III develops this aspect further.

B. Growing (appreciating) capital

Assume we plant a tree on marginal land, at a cost of \$1, and harvest it n years later (when the growth of its stumpage value as a percentage of the base equals the rate of interest.) Stumpage revenue (R) is e^{ni} . Labor's share is no more than e^{-ni} , and less if planting cost is not all wages. e^{-ni} is a decreasing

function of $\underline{n}$. Labor gets a lesser share when life is longer.

This conclusion does not depend on using a single cycle as an example. Let the operation be normalized by staggering the cycles to keep a constant flow of input and output through a going concern. Each year we plant one new tree, and harvest the oldest one. The number of trees in our inventory must equal $\underline{n}$, an increasing function of $\underline{n}$. In addition, the average tree must be older and contain more capital.

The capital stock (K) is the sum of the value of all the trees:

$$K = e^i + e^{2i} + \dots + e^{ni} = \sum_{n=1}^n e^{ni} = \frac{e^{ni} - 1}{i} \quad (7)$$

The share of capital in receipts is:

$$\frac{K_i}{e^{ni}} = 1 - e^{-ni} \quad (8)$$

(8) is an increasing function of $\underline{n}$, and of course the complement of labor's share, e^{-ni} , a decreasing function of $\underline{n}$. This is exactly the same result as for a single cycle.

Now we add land. Let the tree grow on a good site, of value $\underline{S}$. Now growth must cover interest on $\underline{S}$ as well as on capital in the tree, by reaching at maturity a value on the function:

$$R_n = e^{ni} + S[e^{ni} - 1] = (1 + S)e^{ni} - S \quad (9)^1$$

The share of labor is no more than $1/R$, obviously a decreasing function of $\underline{n}$ since R grows with $\underline{n}$.

1. (9) may be derived from $(1 + S)e^{ni} = R + S$, which simply says we put in land at the beginning and get it back at the end, along with the mature tree.

The share of land is Sin/R . We will see ahead that this is an arcade function, an increasing and then decreasing function of n . The increase comes first because labor's share drops almost inversely with n for low values of n . The decrease comes later because K increases logarithmically with n so Ki takes nearly all.

The share of capital is the sum of interest on planting cost plus interest on ground rent, divided by R .

Interest on planting cost is $e^{in} - 1$.

Interest on ground rent is $S(e^{in} - 1 - in)^1$.

The share of capital is:

$$\frac{e^{in} + S(e^{in} - 1) - 1 - Sin}{e^{in} + S(e^{in} - 1)} = 1 - \frac{1 + Sin}{R} \quad (10)$$

(10) is also the complement of land's and labor's shares, as we would expect.

To normalize the single cycle, assume a fixed piece of land divided into n cells or tracts. Each year we plant trees in one cell, and harvest another. Payroll to plant one cell is $1/n$. Receipts each year are R/n . Yearly land cost is Si , and for each cell is Si/n .

The capital stock at all times is:

$$K_Y = \sum_{y=1}^n \frac{(1+S) e^{yi} - S}{n} = (1+S) \frac{e^{ni} - 1}{ni} - S \quad (11)$$

1. Note in passing that when S is large relative to planting cost (here equal to one), most of the capital is geogenic (land-derived). Some reforestation is entirely voluntary and geogenic, representing no labor at all.

Receipts each year are:

$$\frac{R}{n} = \frac{(1 + S) e^{ni} - S}{n} \quad (12)$$

Labor's share is:

$$\frac{l/n}{R/n} = \frac{1}{R} \quad (13)$$

(13) is the same as the single cycle case.

Land's share is:

$$\frac{Si}{R/n} = \frac{Sin}{R} \quad (14)$$

(14) too is the same as the single cycle case.

Capital's share is:

$$\frac{Ki}{R/n} = \frac{(1 + S) (e^{ni} - 1) - Sin}{R} \quad (15)$$

But (15) is the same as (10), the share of capital in the single cycle case.

So we have normalized tree-growing operations and shown that the shares of land, labor and capital are the same as when we look at one cycle over its life.

Now let us get a better idea of how the shares vary as functions of life (n). Table 5 is an example worked out for assumed values, with $i = .05$, and $S = 800$. Instead of assuming the original investment is \$1, as in the algebra above, it is \$200, assumed to be all payroll (P).

Applying the principle of leverage it is evident from Table 5 that a slight rise of interest rates will screen out or shorten long cycles, as we would expect from the simpler earlier demonstration. A rise of wage rates will screen out or lengthen short cycles -- we economize on labor by using it less often. For very long cycles, interest cost dominates everything. Such long cycles are however unusual outside of forestry, and often pathological there, permitted only by sheltering from real interest

Table 5: Factor shares as functions of life of trees, normalized operation

n	Interest factors		Receipts	Wages		Land Rent		Interest on Cap. Stock		Sum of Shares (check)	Intensiveness
				Am't.	Share	Am't.	Share	Am't.	Share		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	e^{ni}	$\frac{e^{ni}-1}{i}$	$\frac{1000(2)-800}{(1)}$	$\frac{200}{(1)}$	$\frac{(5)}{(4)}$	40	$\frac{40}{(4)}$		$\frac{(9)}{(4)}$	$(6)+(8)+(10)$	$(6)+(10)$
1	1.05	1.00	250	200	.80	40	.16	10	.04	1.00	.84
7	1.41	8.14	87	29	.33		.46	20	.23	1.02	.56
14	1.97	19.6	84	14	.17		.48	30	.36	1.01	.53
21	2.79	35.7	94	9.5	.10		.43	45	.48	1.01	.58
28	3.92	58.4	111	7.1	.06		.36	65	.59	1.01	.65
35	5.52	90.3	135	5.7	.04		.30	90	.67	1.01	.71
42	7.76	135	166	4.8	.03		.24	120	.72	.99	.75
49	10.9	198	206	4.1	.02		.19	160	.78	.99	.80
56	15.4	288	261	3.4	.01		.15	215	.82	.98	.83
63	21.5	410	328	3.2	.01		.12	285	.87	1.00	.88
70	30.4	588	423	2.9	.007		.09	380	.90	1.00	.91
77	42.7	834	544	2.6	.005		.07	500	.92	1.00	.93
84	60.2	1184	707	2.4	.003	40	.06	665	.94	1.00	.94

Notes by Column Numbers

(1) Life

(2) Amount of 1 at compound interest, 5% (9) $Ki = .05 \cdot 1000 \cdot \frac{(3)}{(1)} - .05 \cdot 800 = 50 \frac{(3)}{(1)} - 40$

(3) Amount of annuity at 5%

(11) Discrepancies due to rounding

(4) $\frac{(P + S) e^{ni} - S}{n}$

(12) Intensiveness

P = 200

S = 800

(7) $Si = 800 \times .05 = 40$

costs. For most growing capital, cycles are under 15 years, and land cost is a powerful force. High land rents screen out or shorten medium cycles. An example that the model partly fits is cattle breeding, a very land-using business. High interest rates are not enough to pry land away from cattle and sheep and feed grain, the great historical depopulators of farm land. High holding costs must be brought directly to bear on the tenure of land.¹

Historically, farm labor has had to clear away trees to find jobs working the land, and then to push back sheep and range cattle. These organisms of slow maturity create virtual economic deserts for labor. Even the capital in them is mostly geogenic (land-derived). Now for some decades, forests and livestock have been repossessing land formerly tilled, while farm labor left for the cities, and food prices rose. Something is scrambling the market signals; we have seen that tax bias is part of that something.

Aside from the intuitive leverage principle, how do higher rents and interest rates shorten life? It is geometrically obvious that any tree is mature financially in equilibrium when its growth curve touches the R function, where it has the same slope. Differentiating (9):

$$\frac{dR}{dn} = \frac{ie^{in}}{\downarrow} (1 + S) = i(R + S) \quad (16)$$

(16) says a tree is mature in the year when its growth just covers interest on its achieved value (iR) plus rent (iS). Raising rent and interest thus results in shorter cycles.

¹ The lives of cattle are foreshortened in the U.S.A. as compared with Argentina, where rangeland and cash grain are cheaper, for example. But in Western Europe and Japan, lives are even shorter, if cattle are grown at all.

APPENDIX II

Investor's rate of return after tax on realization of income from growing capital.¹

The rate of return after tax, ($\underline{r}$), is an increasing function of life, ($\underline{n}$).

$$e^{rn} = e^{in} (1 - t) + t \equiv \theta \quad (1) \quad 17$$

where $\underline{t}$ is the nominal tax rate.

To prove: $\frac{dr}{dn} > 0$

$$g \equiv \frac{d \ln \theta}{dn} \text{ is } \frac{\theta'}{\theta}, \text{ the growth rate of } \theta(n) \quad (2) \quad 18$$

$$\ln \theta = \int_0^n \frac{d \ln \theta}{dn} = \int_0^n g \, dn \quad (3) \quad 18 \quad 19$$

but from (1), $\ln \theta = rn$, so

$$r = \frac{1}{n} \int_0^n g \, dn \quad (4) \quad 19 \quad 20$$

By inspection of (4) and the Theorem of the Mean, $\underline{r}$ must be rising if $\underline{g}$ is monotonically rising, i.e. if

$$\frac{dg}{dn} > 0.$$

$$\frac{dg}{dn} = \frac{d}{dn} \left[i - \frac{ti}{\theta} \right] = \frac{ti \theta'}{\theta^2} > 0 \quad (5) \quad 20 \quad 21$$

$$\therefore \frac{dr}{dn} > 0$$

Q. E. D.

¹ Matthew Gaffney, Jr., John Hoven, and Steve Hanke helped refine this proof. Further discussion of this and related points are in the writer's "Tax-induced Slow Turnover of Capital", op. cit.

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~~(1A)~~

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APPENDIX III

"Intensity" of land use and "labor intensity" of land use are two different things. Capital on land may displace more labor than it requires to produce it. Much capital is partly geogenic (earth-derived), by the operation of compound interest.

We will compare the overall Intensity Quotient (Q) and labor-intensity (Q_w). We begin with a point-input-point-output (PIPO) model, and easily modify it to cover all cases.

We invest $\$I$ at the beginning of year one, on land worth $\$S$, and realize $\$R$ after n years.

In equilibrium

$$R = Ie^{in} + S(e^{in} - 1) \quad (17) \quad 21$$

$$\text{Labor input} \leq I \quad (18) \quad 22$$

$$\text{Land input} = Sin \quad (19) \quad 23$$

$$\text{Capital input} = I(e^{in} - 1) + S(e^{in} - 1) - Sin \quad (20) \quad 24$$

$$\text{Labor intensity, } Q_w \leq \frac{I}{R} \quad (21) \quad 25$$

$$\begin{aligned} \text{But total intensity, } Q &= \frac{R - Sin}{R} \\ &= 1 - \frac{R - Ie^{in}}{e^{in} - 1} \frac{in}{R} = 1 - \frac{R - Ie^{in}}{R} \left[\frac{in}{e^{in} - 1} \right] \quad (22) \quad 2627 \end{aligned}$$

The coefficient on the right [in brackets for easy reference], is the ratio of simple interest to compound interest. Call it θ .

For low values of n , $\theta \rightarrow 1$, and $\therefore Q \rightarrow \frac{Ie^{in}}{R}$.

But $e^{in} \rightarrow 1$ also, so $Q \rightarrow \frac{I}{R}$. That is, $Q \rightarrow Q_w$ and intensity and labor-intensity are the same, virtually, for short lives.

But for higher values of n , i.e. longer lives, $\theta \rightarrow 0$ and $\therefore Q \rightarrow 1$, even if labor-intensity, I/R , is low. This reflects the buildup of geogenic and autogenetic capital that occurs whenever recovery of principal is deferred.

To adapt this definition of intensity to flowing capital, we need only date all inputs, discount them to year 0, and add these present values to I . Similarly compound all revenues to year n and add them to R . Most investors above the lowest level of sophistication go through some such analysis whatever they may call it. Cash-flow analysis, annualizing, present value analysis, land appraisal, etc., all involve this kind of intertemporal comensuration.

Normally w , the share if I paid out for onsite workers, is much less than one, so true labor-intensity

$$wI/R < \frac{I}{R}.$$

27 28
(23)

There is on a priori grounds a strong negative relationship between w and n , because building durably calls for heavy materials, of high resource content.

(22) is based on deriving site value, S , from the current use, which is to assume the site is in its highest use. Often it is not, in which case (22) overstates intensity. If we have reason to put ~~our~~^{an} opportunity cost value on S higher than

$\frac{R - Ie^{in}}{e^{in} - 1}$, then we should not substitute this in (22), but stop at

$$Q = 1 - \frac{Sin}{R}$$

28 29A
(22A)

(22A) gives a lower value which can easily even be negative when R fails to cover simple interest on S , or rent.

The only good reason for managing land this way would be to save it for a higher future use. Yet the foregone revenues of these n years must then be regarded as plowed back into the site as "intangible geogenic capital". The present property tax is biased by virtue of failing to tax such capital, thus favoring it over buildings.

Landholders like some government agencies and corporations who are sheltered from social opportunity costs are doubly disinclined to turn over their capital fast enough. The land cost itself is a pressure to shorten cycles; but so is interest on the intangible geogenic capital which they do not feel. The last point has been entirely neglected and is very important.

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APPENDIX IV

Inflation, phantom profits, and tax bias against shorter investments.

We compare two $\frac{I}{P/PO}$ investments, one maturing in one year, the other in ten. $i = .07$, and there is annual inflation at 7%. $t = 50\%$.

	$\frac{I}{P}$	R	$R-I$	$\frac{R-Ie^{.07n}}{7}$	$\frac{(R-I)t}{7}$
$n = 1$	100	114	14	7	7
$n = 10$	100	400	300	200	150

When $n = 1$, half the profit of \$14 is phantom, and the 50% tax rate consumes the other half.

When $n = 10$, only one-third of the \$300 profit is phantom, so the tax leaves $\frac{1}{3}$ of ~~untaxed~~ ^{real} profit, *after tax*.

More generally,

$$\text{Taxable income} = R(n) - I_0 \quad (28) \quad 30$$

$$\text{Adjusted basis} = Ie^{jn} \quad (29)$$

where j is the rate of annual inflation.

In equilibrium,

$$R(n) = Ie^{(i+j)n} \quad (30)$$

Income adjusted for inflation is:

$$R(n) - Ie^{jn} = Ie^{(i+j)n} - Ie^{jn} = Ie^{jn}(e^{in} - 1) \quad (31)$$

Taxable income as a multiple of adjusted income is:

$$\beta = \frac{I[e^{(i+j)n} - 1]}{Ie^{jn}(e^{in} - 1)} = \frac{e^{(i+j)n} - 1}{e^{(i+j)n} - e^{jn}} \quad (32)$$

of $\beta = \frac{i+j}{i}$ β is a decreasing function of n , ranging from a high when $n = 1$, to a low of one when $n = \infty$.

The case is often made that it is the opposite, that inflation hits the longer investments harder. The argument is that the short investment lets principal be recovered at a lower price level, while it is worth more. It is an unbalanced argument, overlooking the larger fact that the long investment lets revenue be taxed later, in softer dollars. That is when $n = 1$ the \$7 phantom profit is taxed in year one, and paid in dollars worth twice as

much as those of year ten. When the phantom profit merely accrues in year one it is not taxed until year ten, when the dollar is worth half as much.

As we have seen, most long investments are written off faster than they actually depreciate. The intertemporal bias here is magnified because costs are deducted from hard dollars and taxes are later levied on soft ones.

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	<u>I</u>	<u>R</u>	<u>R - I</u>	<u>$R - Ie^{.07n}$</u>	<u>$(R - I)t$</u>
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$n = 1$	100	114	14	7	7
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$n = 10$	100	400	300	200	150
----------	-----	-----	-----	-----	-----

When $n = 1$, half the profit^{of \$14} is phantom, and the
50% tax rate consumes the other half.

When $n = 10$, only one third of the \$300
profit is phantom, so the tax leaves 50% of
untaxed profit.

More generally,

$$\text{Taxable income} = R(n) - I_0$$

$$\text{Adjusted basis} = I e^{in}$$

where j is the rate of ^{annual} inflation.

In equilibrium,

$$R(n) = I e^{(\frac{i+j}{1+j})n}$$

Income adjusted for inflation is:

$$R(n) - I e^{in} = I e^{(\frac{i+j}{1+j})n} - I e^{in} = I e^{in} (e^{in} - 1)$$

Taxable income as a multiple of adjusted income is:

$$\beta = \frac{I [e^{(\frac{i+j}{1+j})n} - 1]}{I e^{in} (e^{in} - 1)} = \frac{e^{(\frac{i+j}{1+j})n} - 1}{e^{(\frac{i+j}{1+j})n} - e^{in}}$$

β is a decreasing function of n , ranging from a high of $\beta = \frac{i+j}{i}$ when $n=1$, to a low of one when $n = \infty$.

The case is often made that it is the opposite, that inflation hits the longer investments harder. The argument is that the short investment lets principal be recovered at a lower price level, while it is worth more. It is an unbalanced argument, overlooking the larger fact that the long investment lets revenue be taxed later, ^{in softer dollars.} ~~at a higher price level when ^{the tax dollar} it is worth less.~~ That is, when $n=1$ the \$1 phantom profit is taxed in year one, and paid in dollars worth twice as much as those of year 10. When the phantom profit merely accrues in year one it is not taxed until year ten, when the dollar is worth half as much.

As we have seen, most long investments are

written off faster than they actually depreciate. The
intertemporal bias here is magnified because ~~the~~ costs are
deducted from hard dollars and taxes are ^{later} levied on soft ones.

Appendix III

"Intensity" of land use and "labor-intensity" of land use are two different things. Capital on land may displace more labor than it requires to produce it.

Much capital is partly geogenic (earth-derived) and all capital is partly autogenetic (self-derived), by the operation of compound interest.

We will compare ^{the overall} Intensity ^{Quotient} (Q) and labor-intensity (Q_w). We begin with a point-input point-output (P(PO)) model, and easily modify it to cover all cases.

We invest $\$I$ at the beginning of year one, ~~and~~ realize on land worth $\$S$, and realize $\$R$ after n years.

In equilibrium,

$$R = Ie^{in} + S(e^{in} - 1) \quad (1)$$

$$\text{Labor input} \leq I \quad (2)$$

$$\text{Land input} = Sin \quad (3)$$

$$\text{Capital input} = I(e^{in} - 1) + S(e^{in} - 1) - Sin \quad (4)$$

$$\text{Labor intensity, } Q_w \leq \frac{I}{R} \quad (5)$$

$$\text{But total intensity, } Q = \frac{R - Sin}{R}$$

$$= 1 - \frac{R - \frac{I}{e^{in} - 1} e^{in}}{R} = 1 - \frac{R - \frac{I}{e^{in} - 1} e^{in}}{R} \left[\frac{in}{e^{in} - 1} \right] \quad (6)$$

~~The share of~~

The coefficient on the right, [in brackets for easy reference], is the ratio of simple interest to compound interest, ~~and~~ ~~for~~ Call it θ . For

low values of n , $\theta \rightarrow 1$, and $\therefore Q \rightarrow \frac{Ie^{in}}{R}$.

But $e^{in} \rightarrow 1$ also, so $Q \rightarrow \frac{I}{R}$. That is,

$Q \rightarrow Q_w$ end intensity and labor-intensity are the same, virtually, for short lives. ~~P_1~~

But for higher values of $\underline{n}$, i.e. longer lives, $Q \rightarrow 0$ and $\therefore Q \rightarrow 1$, even if ~~the~~ labor-intensity, I/R , is low.

This reflects the buildup of geogenic and autogenetic capital, that occurs whenever recovery of principal is deferred.

To adapt this definition of intensity to flowing capital, we need only date all inputs, ~~and~~ discount them to $\frac{\text{year}}{n} 0$, and add these present values to I . Similarly, ~~date all~~ compound all revenues to ~~year~~ $\underline{n}$ and add them to R . Most investors above the lowest level

of sophistication go through some such analysis.

~~it~~ whatever they may call it: cash-flow analysis, annuallizing, present value analysis, ~~on site~~ ^{land} appraisal, etc., all involve this kind of intertemporal commensuration.

Normally, w , the share of I ~~paid~~ ^{paid} out for on-site ~~labor~~ workers, is much less than one, so ^{the} labor-intensity, $wI/R < \frac{I}{R}$. There is on a priori grounds a strong ~~negative~~ relationship between w and u , because building durably calls for heavy materials, of high resource content.

The formula above () is based on deriving site value, S , from the current use, which is to assume ~~assuming~~ the site is in its highest use. Often it is not, in which case () overstates intensity.

If we have reason to put our opportunity cost value

on S higher than $\frac{R - Ie^{in}}{e^{in} - 1}$, then we

should ~~redefine intensity~~: not substitute this in (), but stop at:

$$Q = 1 - \frac{Sin}{R} \quad (A)$$

(A) gives a ~~higher~~ lower value which can easily even be negative when R fails to cover ~~int~~ simple interest on S, or rent.

The only good reason for managing land this way would be to save it for a higher future use.

Let the foregone revenues of these n years must then be regarded as plowed back into the site as "intangible geogenic capital." ~~It is a~~ The present property tax is biased by virtue of failing to tax such capital.

~~xxx~~ Landholders like some govt. agencies and corporations ~~whom~~ do not feel a social oppor-

are sheltered from social opportunity costs
are doubly ^{dis}inclined to ~~use too little labor on~~
turn over their capital fast enough. The land
cost itself is a pressure to shorten cycles; but so is
interest on the ~~of~~ intangible geogenic capital
which they do not feel. The last point has
been entirely neglected and is very important.

APPENDIX I

Share of Labor in claim on total output as decreasing function of life of capital: identity of single-cycle and going concern models.

A. Flowing (depreciating) capital.

The text shows (Fig. 1, Table 4, pp. 29-30) that the share of labor in revenues from the service flow of a building or machine (flowing capital) is a decreasing function of life. Here we show the rule and the mathematics are exactly the same in a "going concern" model when an investment in machines is normalized or staggered.

There is one machine of every age. We build or buy one new machine each year, and scrap another one. The number of extant machines is n , where n is the number of years each machine lasts. As in Eqn. (1), p. 29, each one yields a flow of \$1 per year, so the yearly flow is \$ n .

The yearly labor input is no more than the warranted investment (I) in one machine when new. Dividing this by the yearly service flow (n) we arrive at the same result as when we analyze one machine over life, that is labor's share, is

$$I/n = \frac{1 - e^{-in}}{ni} \quad [\text{same as (2), p. 29}]$$

The share going to capital as interest must be the complement of (2), of course. To check this, let us derive it directly.

The capital value remaining unrecovered in each machine at the start of every year, v_y , is the present value, (V_y) of future cash flows from that time until scrapping.

$$v_y = \frac{1 - e^{-i(n-y)}}{i} \quad (3)$$

Total unrecovered capital (K_y) is:

$$K_y = \sum_{y=0}^n V_y = \frac{1-e^{-in}}{i} + \frac{1-e^{-i(n-1)}}{i} + \frac{1-e^{-i(n-2)}}{i} + \dots$$

$$+ \frac{1-e^{-i(n-n)}}{i}$$

$$= \frac{1}{i} \left[n-e^{-in} \sum_{y=0}^n e^{iy} \right] = \frac{1}{i} \left[n - \frac{1-e^{-in}}{i} \right] \quad (4)$$

Interest on K_y as a share of total output flow is

type in

$$\frac{i K_y}{n} = \frac{1-e^{-in}}{in} \quad (5)$$

Q. E. D.

In the going concern, capital recovery is the excess of cash flow (n) above interest cost (Ki). Rearranging (5)

$$n - Ki = \frac{1-e^{-in}}{i} \quad (5A)$$

But the right side of (5A) is I , the warranted investment to build a new machine. So capital recovery from all the machines together is just enough to finance one new machine each year.

Now we add land to the inputs. The capital is a building on a good site. The yearly service flow is now \$1 plus \$a, where a is the annual value of the site. Fig. 4 shows the share of land:

\$ (1+a)

— n

Fig. 4: Division of value of service flow of n houses of evenly staggered ages among warranted investment (I), rent (a) and interest.

$\bar{K}$ remains the same as before. So does $\bar{I}$. Only service flow is greater. The shares of labor and capital are accordingly smaller.

The share of land is

$$\frac{na}{n(1+a)} = \frac{a}{1+a} \quad (6)$$

(6) is not a function of n , i.e. is unaffected by life. (under the present simplifying assumption of level service flow.) Accordingly, the share of land rises relative to labor, and falls relative to capital, with increasing life of capital.

The last point seems a little surprising because in the last years of an old building the cash flow covers little more than land rent. In part the surprising result comes from my artificial simplifying assumption of level cash flow over "life". In fact, there is a geriatric terminal life of old buildings, after the service life is over, when the carcass survives and yields just enough cash flow to give a return on the site. When we include this terminal period in "life", the share of land rises with life, and moreso when site value rises over time.

But that is not the whole story. In the main the surprising result represents a real phenomenon: to build for long sustained service flow of capital calls for heavy inputs of capital from the beginning of life, inputs which must be paid from the beginning. Note we are not here treating what happens from year to year to the individual building as it ages and the unrecovered capital approaches zero. Of course capital's share then also approaches zero. We are treating of shares over the whole life of buildings, and, what amounts to the same thing, shares generated from a normalized collection of buildings of every age.

The share of labor falls with increasing life. But land tempers this effect by claiming a share of the product even when life is short, like the life of corn plants in rich Champaign County, Illinois; or when there is little capital, as on a parking lot. Appendix III develops this aspect further.

B. Growing (appreciating) capital

Assume we plant a tree on marginal land, at a cost of \$1, and harvest it n years later (when the growth of its stumpage value as a percentage of the base equals the rate of interest.) Stumpage revenue (R) is e^{ni} . Labor's share is no more than e^{-ni} , and less if planting cost is not all wages. e^{-ni} is a decreasing

function of $\underline{n}$. Labor gets a lesser share when life is longer.

This conclusion does not depend on using a single cycle as an example. Let the operation be normalized by staggering the cycles to keep a constant flow of input and output through a going concern. Each year we plant one new tree, and harvest the oldest one. The number of trees in our inventory must equal $\underline{n}$, an increasing function of $\underline{n}$. In addition, the average tree must be older and contain more capital.

The capital stock (K) is the sum of the value of all the trees:

$$K = e^i + e^{2i} + \dots + e^{ni} = \sum_{n=1}^n e^{ni} = \frac{e^{ni} - 1}{i} \quad (7)$$

The share of capital in receipts is:

$$\frac{K_i}{e^{ni}} = 1 - e^{-ni} \quad (8)$$

(8) is an increasing function of $\underline{n}$, and of course the complement of labor's share, e^{-ni} , a decreasing function of $\underline{n}$. This is exactly the same result as for a single cycle.

Now we add land. Let the tree grow on a good site, of value $\underline{S}$. Now growth must cover interest on $\underline{S}$ as well as on capital in the tree, by reaching at maturity a value on the function:

$$R_n = e^{ni} + S[e^{ni} - 1] = (1 + S)e^{ni} - S \quad (9)^1$$

The share of labor is no more than $1/R$, obviously a decreasing function of $\underline{n}$ since R grows with $\underline{n}$.

1. (9) may be derived from $(1 + S)e^{ni} = R + S$, which simply says we put in land at the beginning and get it back at the end, along with the mature tree.

The share of land is Sin/R . We will see ahead that this is an arcade function, an increasing and then decreasing function of $\underline{n}$. The increase comes first because labor's share drops almost inversely with $\underline{n}$ for low values of $\underline{n}$. The decrease comes later because K increases logarithmically with $\underline{n}$ so Ki takes nearly all.

The share of capital is the sum of interest on planting cost plus interest on ground rent, divided by $\underline{R}$.

Interest on planting cost is $e^{in} - 1$.

Interest on ground rent is $S(e^{in} - 1 - in)$.¹

The share of capital is:

$$\frac{e^{in} + S(e^{in} - 1) - 1 - Sin}{e^{in} + S(e^{in} - 1)} = 1 - \frac{1 + Sin}{R} \quad (10)$$

(10) is also the complement of land's and labor's shares, as we would expect.

To normalize the single cycle, assume a fixed piece of land divided into $\underline{n}$ cells or tracts. Each year we plant trees in one cell, and harvest another. Payroll to plant one cell is $1/\underline{n}$. Receipts each year are $R/\underline{n}$. Yearly land cost is S , and for each cell is $S/\underline{n}$.

The capital stock at all times is:

$$K_Y = \sum_{y=1}^n \frac{(1+S) e^{yi} - S}{n} = (1+S) \frac{e^{ni} - 1}{ni} - S \quad (11)$$

1. Note in passing that when S is large relative to planting cost (here equal to one), most of the capital is geogenic (land-derived). Some reforestation is entirely voluntary and geogenic, representing no labor at all.

Receipts each year are:

$$\frac{R}{n} = \frac{(1 + S) e^{ni} - S}{n} \quad (12)$$

Labor's share is:

$$\frac{l/n}{R/n} = \frac{l}{R} \quad (13)$$

(13) is the same as the single cycle case.

Land's share is:

$$\frac{S_i}{R/n} = \frac{S_{in}}{R} \quad (14)$$

(14) too is the same as the single cycle case.

Capital's share is:

$$\frac{K_i}{R/n} = \frac{(1 + S) (e^{ni} - 1) - S_{in}}{R} \quad (15)$$

But (15) is the same as (10), the share of capital in the single cycle case.

So we have normalized tree-growing operations and shown that the shares of land, labor and capital are the same as when we look at one cycle over its life.

Now let us get a better idea of how the shares vary as functions of life (n). Table 5 is an example worked out for assumed values, with $i = .05$, and $S = 800$. Instead of assuming the original investment is \$1, as in the algebra above, it is \$200, assumed to be all payroll (P).

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Table 5: Factor shares as functions of life of trees, normalized operation

n	Interest factors		Receipts	Wages		Land Rent		Interest on Cap. Stock		Sum of Shares (check)	Intensiveness
				Am't.	Share	Am't.	Share	Am't.	Share		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	e^{ni}	$\frac{e^{ni}-1}{i}$	$\frac{1000(2)-800}{(1)}$	$\frac{200}{(1)}$	$\frac{(5)}{(4)}$	40	$\frac{40}{(4)}$		$\frac{(9)}{(4)}$	(6)+(8)+(10)	(6)+(10)
1	1.05	1.00	250	200	.80	40	.16	10	.04	1.00	.84
7	1.41	8.14	87	29	.33		.46	20	.23	1.02	.56
14	1.97	19.6	84	14	.17		.48	30	.36	1.01	.53
21	2.79	35.7	94	9.5	.10		.43	45	.48	1.01	.58
28	3.92	58.4	111	7.1	.06		.36	65	.59	1.01	.65
35	5.52	90.3	135	5.7	.04		.30	90	.67	1.01	.71
42	7.76	135	166	4.8	.03		.24	120	.72	.99	.75
49	10.9	198	206	4.1	.02		.19	160	.78	.99	.80
56	15.4	288	261	3.4	.01		.15	215	.82	.98	.83
63	21.5	410	328	3.2	.01		.12	285	.87	1.00	.88
70	30.4	588	423	2.9	.007		.09	380	.90	1.00	.91
77	42.7	834	544	2.6	.005		.07	500	.92	1.00	.93
84	60.2	1184	707	2.4	.003	40	.06	665	.94	1.00	.94

Notes by Column Numbers

(1) Life

(2) Amount of 1 at compound interest, 5%

(3) Amount of annuity at 5%

(4) $\frac{(P + S) e^{ni} - S}{n}$

P = 200

S = 800

(7) $Si = 800 \times .05 = 40$

(9) $Ki = .05 \cdot 1000 \cdot \frac{(3)}{(1)} - .05 \cdot 800 = 50 \frac{3}{1}$

(11) Discrepancies due to rounding

(12) Intensiveness

still not right

Applying the principle of leverage it is evident from Table 5 that a slight rise of interest rates will screen out or shorten long cycles, as we would expect from the simpler earlier demonstration. A rise of wage rates will screen out or lengthen short cycles - we economize on labor by using it less often. For very long cycles, interest cost dominates everything. Such long cycles are however unusual outside of forestry, and often pathological there, permitted only by sheltering from real interest costs. For most growing capital, cycles are under 15 years, and land cost is a powerful force. High land rents screen out or shorten medium cycles. An example that the model partly fits is cattle breeding, a very land-using business. High interest rates are not enough to pry land away from cattle and sheep and feed grain, the great historical depopulators of farm land. High holding costs must be brought directly to bear on the tenure of land.¹

Historically, farm labor has had to clear away trees to find jobs working the land, and then to push back sheep and range cattle. These organisms of slow maturity create virtual economic deserts for labor. Even the capital in them is mostly geogenic (land derived). Now for some decades, forests and livestock have been repossessing land formerly tilled, while farm labor left for the cities, and food prices rose. Something is scrambling the market signals; we have seen that tax bias is part of that something.

Aside from the intuitive leverage principle, how do higher rents and interest rates shorten life? It is geometrically obvious that any tree is mature financially in equilibrium when its growth curve touches the R function, where it has the same slope. Differentiating (9):

$$\frac{dR}{dn} = ie^{in} (1 + S) = i(R + S) \quad (16)$$

(16) says a tree is mature in the year when its growth just covers interest on its achieved value (iR) plus rent (iS). Raising rent and interest thus results in shorter cycles.

1. The lives of cattle are foreshortened in the U.S.A. as compared with Argentina, where rangeland and cash grain are cheaper, for example. But in Western Europe and Japan, lives are even shorter, if cattle are grown at all.

Total unrecovered capital ~~capital~~ (K_y) is

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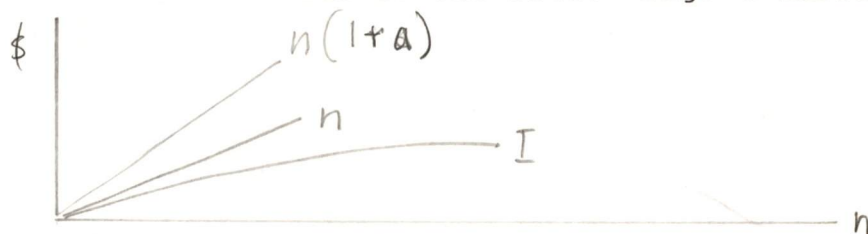


Fig. 4: Division of value of service flow of n houses of evenly staggered ages among warranted investment (I), rent (a) and interest.

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Use this space, avoid p. 7

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Inflation ~~is biased~~ hurts the most those whose need for liquidity is high relative to their real assets. There are those whose ~~capital turns~~ ~~or~~ volume is high relative to their capital, because ^{That is, (over)} liquidity needs vary with volume. ~~Accordingly~~

Lumpsum's study of the concentration of wealth found that money ^{and near-money} as a share of wealth declines with total wealth, ~~as does~~ so inflation as a tax has ^a regressive quality ~~while~~ compared to ~~at~~ taxes on real wealth.

diffusion is ~~low~~ low the more the more

well for spreading is high relative to the

and over there are those whose capital turns

there are those whose capital turns over fast.

the volume is high relative to their capital, the more

(the more)

the more the more the more the more

the more the more the more the more

found that the more the more the more the more

the more the more the more the more

the more the more the more the more

the more the more the more the more