



Maximizing Site Value in Perpetuity

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Timing Replacement to
1 Maximizing Site Value in Perpetuity

M. Gaffney, 1992; rev. 1995

1. Overview

Now we are tooled up to deal with more complete models, and more complex problems. We can:

a. Show the interplay of several variables as they contribute to (or subtract from) meeting our performance standard.

i. The first standard we use is to maximize value in perpetuity. This is "T-max" for short, where T is Terra.¹

ii. A second, alternative standard is maximizing ROR (rate of return) where T is taken as a parameter, or "given." We can analyze this using the "redeemable bond" model. We have done this elsewhere - it is not the method pursued here. See the paper "Basic Mathematics of Finance, pp. 19 ff., on the redeemable bond; and the paper on Reserve, "Land Gains, Fast Write-off, and Incentives to Build."

b. Analyze and measure sensitivity of T to each parameter: play "what-if," to forecast the results of alternative actions like investing more, or trading off cash flow for longevity, etc.

c. Show how to add more parameters to the basic framework of the investment analysis. Given time, we should add income taxation, showing the effect of different tax lives, tax rates, and differential rates between "ordinary" income and "capital gains." See paper on Reserve, "Land Gains, Fast Write-off, and Incentives to Build."

¹The reasons for choosing Terra instead of an English word like Land, or Ground, are explained in the paper "Land Prices and Appreciation," n.2

2. Modifying Faustmann to handle future generations that differ from Generation-1.

In Faustmann,² we assumed all future generations were clones of Generation-1.³ That let us use a land value multiplier (LVM), based on perpetual replications. Here, rather, we take future land uses as exogenous, different from and independent of Generation-1. Call this the "orange-grove model," where later generations of use are to be urban: housing, shopping, warehousing, industry, transportation, institutions, recreational use, etc. More generally, we are looking at the universal case of Progressive Obsolescence. I say "orange-grove" because it is familiar, and "mnemonic" (easy to remember).

Let G_1 = PV of Generation #1.

$$G_1 = -K + a \cdot \text{DCFF} \quad (1)$$

where K = capital invested at time zero

a = cash flow (or service flow, or avoided cost)

DCFF = discounted cash flow factor.

Let F = Future Value of land at the end of n years of Generation-1. I remind you that $T = \text{Terra}$, the PV of land. T is the sum of the PVs of all future Generations of land use.

$$T = G_1 + FV^n \quad (2)$$

Figure 1, below, shows how these factors generally work together, as functions of n , to create $T\text{-max}$ (max w.r.t. n , that is). Other parameters than n may also be varied, for example raising K to raise a . Whatever parameters you vary, the objective, or "performance standard," is to max T . If it weren't for F , we would just let G_1 live on forever. F is what makes this an optimizing problem. See the paper "Intro to Optimizing."

²For explanation of Faustmann, see the paper, "When to Cut a Tree."

³I would call each Generation a Cycle, except that cycle starts with "c". "C" is most often used to mean "Cost," so "c" might be confusing. Remember, though, that each Generation is indeed a cycle: a discrete cycle of investing capital (K) and recovering it with enough surplus to cover interest and rent plus, the investor hopes, something extra as a token that somebody up there likes her.



Figure 1: T and its components as functions of n

The curve showing FI^n is not shown back to the origin, because it does not become a viable option until G_1 has been entered into irreversibly. Sometimes the case arises that one can consider omitting G_1 altogether - that case is not considered here, although it deserves consideration.

3. " a " falls over time.

The simple, basic DCFF is based on the simplifying premise that " a " is constant until suddenly life ends. It is like a light-bulb that provides a constant level of service until suddenly it pops out. Most capital goods are not like that. Rather, they are like an old auto that progressively serves less well, and costs more to operate and maintain, until one day you painfully decide its time has come. Its service flow "decays" over time. I am going to make a fall over time, to resemble the auto. To keep things as simple⁴ as possible, let it fall by a constant annual rate, " d ." d stands for "decay factor." This

⁴I am sacrificing realism for simplicity here. People who build math models usually do so - it's their curse. The curse comes when they forget the simplifying premises are only expository devices to make a point in a few short words or symbols. Let's you and I remember. When you try to express real decay and depreciation paths in algebraic form you may find the math gets so complex it is unwieldy and incomprehensible. Then it stops helping you, but just gets between you and your subject. This is when you're better off to get some numbers from observation, and just tabulate them.

lets us use the same DCFF we are familiar with, modified in this way: add d to i. Thus:

$$G_1 = -K + a \frac{1 - (I+d)^{-n}}{i+d} \quad (3)$$

This makes the curve marked [a · DCFF] flatten out more quickly as it grows.

4. Finding n*

Every maximizing solution is also a marginal equality. (Remember how you max profit by equalizing MR and MC?) In (2), G_1 is rising with n while the other term is falling. Max T is where the rise of one equals the fall of the other.

The rise of G_1 is a times the rise of the DCFF. In an earlier week we learned the rise of DCFF is a times the discount factor.⁵

$$\Delta G_1 = a(I+d)^{-n} \quad (4)$$

We need to equate (4) with the fall of discounted F.

$$\Delta(FI^{-n}) = -FiI^{-n} \quad (5)$$

Equating (4) with the fall of (5), i.e. changing the sign of (5), we have our marginal condition for maximizing T:

$$a(I+d)^{-n} = FiI^{-n} \quad (6)$$

We simplify (6) by fudging just the tiniest bit, noting that $(I+d)$ is almost equal to $I(1+d)$. Now we can cancel out I^{-n} on both sides, leaving us with:

$$a(1+d)^{-n} = Fi \quad (7)$$

Pause, reflect, and remember. (7) says that the time to demolish and replace an old Defender (the left side) with a new Challenger (the right side) is when the current CF (cash flow) from the Defender falls to equal or less than interest on the

⁵See the paper "Basic Math of Finance," pp. 3 and 14, where "cume curves" are discussed.

value of the land as used by the Challenger (F_i).⁶ See the papers on Reserve, "Property Taxes and the Frequency of Urban Renewal," and "When to Build What."

To find n in any problem, just solve (7) for n . It turns out to be:

$$n = \frac{-\ln(F_i/a)}{\ln(1+d)} \quad (8)$$

Evaluate T , using the value of n gotten from (8), and you have $\max T$ (w.r.t. n).

5. Sensitivity, and best combinations

A manager's job is to find the best combinations of K , a , and d . He or she can raise a , or lower d , at the cost of raising K . Sensitivity tells you how much (and what sign) the effect of ΔK is; maximizing tells you exactly what amount is best.

Mind you, this comes down the line a bit, after the "imagineering" stage. It is no substitute for creativity, artistry, or salespersonship. As Bloody Mary sang, "You got to have a dream, if you no got a dream, how you going to have a dream come true?" Rather, what we are doing complements dreams by keeping them in touch with reality. Managerial economics may even be abused and stifle good dreams - uninspired bean-counters have suppressed a lot of creative talent. At the same time, a lot of creative people have blown a lot of other people's money chasing their own private dreams. You have to use judgment in each case, and balance the dreams and the beans. Analysis is just one step in developing judgment - keep it in perspective.

A lot of "analysis" in Managerial Economics is pretty primitive in regard to combining inputs optimally. See any standard Managerial text and look up "Capital Budgeting." Many just say, in effect, "invest in anything with a positive present value," meaning $DCF > K_0$. The trouble with that is, on any given piece of ground all alternative investments of K are mutually

⁶When F is growing over time a new factor is added, not treated here: it is known as the "ripening" problem. From various papers herein we know $F_i = a + F_g$, a fascinating bypath we will not explore here, for lack of time.

exclusive. To choose among them you must go a step more, and maximize G_1 .

Now we are going another step, taking account that time-slots on any one site are also mutually exclusive: you must choose among them at the margins. That means you must terminate G_1 to make way for future generations. In planning G_1 you should look ahead and plan for this, too, lest you invest too much in G_1 at the start.

Most analysis never gets this far. We are lucky, as noted, if it even gets as far as maximizing G_1 . If it does, the analyst has exhausted herself and looks no farther, ignoring F . Such analysis thus usually overstates the benefit of higher K , because, to recover higher K , you usually want to spread K over more time. (In our model, that means lowering $\underline{d}$, making capital last longer.) When you spread it over more time, that time costs something: the deferral of F .

Look at it this way. You must fit G_1 into a time slot. The slot is not absolutely fixed - not like the bed of Procrustes, (the big friendly giant of Greek mythology). This slot is stretchable - but at a cost. Our model lets us handle this side of things.

6. The productivity of capital: higher $\underline{a}$; slower decay ($\underline{d}$).

As you ΔK you normally Δa , which is why you do it. However, normally $(\Delta a/a) < (\Delta K/K)$, meaning a given % ΔK results in a lesser % Δa . E.g., you raise K by 20%, and the result is to raise $\underline{a}$ by 12%. That is an aspect of diminishing returns to Capital. Any given ΔK will raise G_1 , in spite of diminishing returns, so long as $\Delta a \cdot DCFF > \Delta K$.

ΔK may also have its payoff in longer life. In our present model, that means lowering $\underline{d}$, the rate of decay. Again, that raises G_1 so long as $[\Delta a \cdot DCFF] > \Delta K$.

In either case the result will be longer life, $\underline{n}$, of G_1 . The invisible cost of that (which our model makes visible) is deferral of F . To raise T , the rise of G_1 must be enough to offset the $-\Delta FI^{-n}$.

7. Numerical Example

We get numerical results, using algebra, and a calculator or computer, and playing "what if?". Again, it's time for "imagineering." Try out all kinds of combinations in the planning stage - look before you leap. Table 1 shows some cases where I have solved for n^* using Eqn. (8).⁷

Table 1: Values of n^* from given parameters

<u>Row</u>	<u>K</u>	<u>a</u>	<u>i</u>	<u>d</u>	<u>F</u>	<u>n^*</u>	<u>T</u>	<u>G_1</u>	<u>FI^{-n}</u>
1	9	10	.04	.15	100	6	104	28	76
2	27	20	"	"	"	11	127	67	60
3	"	15	"	.10	"	14	121	63	58
4	36	"	"	.075	"	19	125	80	45
5	"	"	"	.09	"	15	116	61	56
6	Over to You								

8. PROBLEMS

Get a sheet of tabulating paper (reticulated), with little cells for making tables. Extend Table 7 indefinitely, playing "what if?" until you feel you understand the sensitivities involved. Also, double check the T-max values by calculating T values for values of n just above and below the max. Remember, the n values are a little fuzzy, just as in the great outside world. Also remember, the max is flat-topped, so it is easy to miss the max due to the use of approximations, simplifications, interpolations, fudge factors, or idiosyncracies that creep in.

Most maxes in economics are flat-topped, actually. You may find it exasperating to search for the absolute max, and wonder if it's worth the trouble. You may miss the peak by a bit, and scarcely notice it. You may even say, as many managers do, that it doesn't matter enough to fuss over. This attitude, however, leads them into considerable losses. Maxes are flat-topped because they are averages. They average your big winners with your little winners. They will also, if you get careless,

⁷There is a little bit of "fudge" in my solution. Also, the max usually falls between the given points, i.e. in middle of the year intervals. Also, the max is flat-topped. In result, the given n^* values are really $[n^* \pm 1]$.

average in your losers and make them invisible. You could die by a thousand cuts, each one too little to notice.

This is where the marginal approach is so valuable. It focuses your attention on the marginal fringe of a choice, and thus keeps you from covering marginal losses by melding them with overall gains. Hiding mistakes in the averages is one of the cardinal sins of managers everywhere, and one of the major diseconomies of scale. See the paper on "Pros and Cons of Scale."

To apply the marginal approach here, use Eqn. (7) when n is at or near the max. Over to you.